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PROCEEDINGS

OF THE

PHYSICAL SOCIETY OF LONDON

From December 1922 to August 1923

VOL. XXXV.

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY OF LONDON.

SESSION 1922-1923.

Meetings held at the Imperial College of Science,

The Chair being taken at all the meetings by the President,
ALEXANDER RUSSELL, M.A., D.Sc.

October 27, 1922.

1. The Presidential Address was delivered by **ALEXANDER RUSSELL**, M.A., D.Sc.

2. A Demonstration of A Radio-Telegraphic Device was given by T. D. **PARKIN**, of Marconi's Wireless Telegraph Co., Ltd.

3. A Paper on "The Problem of Two Electrified Spheres" was read by **ALEXANDER RUSSELL**, M.A., D.Sc.

November 10, 1922.

The following Papers were read :—

1. "The Homographic Treatment of the Symmetrical Optical Instrument," by G. **TEMPLE** (Birkbeck College).

2. "On the Structure of the Sulphur Dioxide Molecule," by Prof. A. O. **RANKINE**, D.Sc., and C. J. **SMITH**, M.Sc., A.R.C.S., D.I.C.

3. "The Thermal Effect of Vapours on Rubber," by A. S. **HOUGHTON**, B.Sc., A.I.C.

A DEMONSTRATION of An Apparatus for Testing the Tensile Strength of Gas Mantles, was given by J. T. **ROBIN**.

November 24, 1922.

The following Papers were read :—

1. "The Theory of the Singing Flame," by E. G. **RICHARDSON**, B.Sc.

2. "Unit Surfaces," by Miss **ALICE EVERETT**, M.A.

3. "Vibration Galvanometers with Asymmetric Moving Systems," by Prof. R. Ll. Jones.

A Demonstration of Some Applications of the Gyroscope, was given by Paul Schilowsky, Chairman of the Gyroscopic Society, Petrograd.

A Demonstration of A New Balance for Compensating the Temperature Error of Watches and Chronometers, was given by Paul Ditisheim, La Chaux de Fonds, Switzerland.

December 8, 1922.

The following Papers were read :—

1. "The Relation between Molecular and Crystal Symmetry as shown by X-Ray Crystal Analysis," by G. Shearer, M.A., B.Sc.

2. "Modification of the Powder Method of Determining the Structure of Metal Crystals," by E. A. Owen, M.A., D.Sc., and G. D. Preston, B.A.

3. "The Cathode Ray Oscillograph," by A. B. Wood, D.Sc.

A Demonstration of A Low Voltage Cathode Ray Oscillograph, was given by R. Webb, International Western Electric Company.

January 3 and 4, 1923.

The Annual Exhibition of Apparatus of the Physical and Optical Societies was held from 3-6 p.m. and 7-10 p.m.

The following Lectures were delivered each day :—

"Reproduction of Colour by Photographic Processes," by Mr. W. Gamble.

"Recent Photo-Elastic Researches on Engineering Problems," by Prof. E. G. Coker, F.R.S.

January 26, 1923.

The following Papers were read :—

1. "A Supposed Relationship between Sunspot Frequency and the Potential Gradient of Atmospheric Electricity," by Dr. C. Chree, F.R.S.

2. "A Further Improvement in the Sprengel Pump," by J. J. Manley, M.A.

3. "Null Methods of Measurement of Power-factor and Effective Resistance in Alternating Current Circuits by the Quadrant Electrometer," by Dr. D. **Owen**.

A **Demonstration** of An Electro-capillary Relay for "Wired Wireless," was given by Major C. E. **Prince**, O.B.E.

February 9, 1923.

Annual General Meeting.

GENERAL BUSINESS.

The Annual Report was read by the Secretary, Mr. F. E. **Smith**, O.B.E., F.R.S., and the Report of the Treasurer was read by the Treasurer, Mr. W. R. **Cooper**, M.A., B.Sc. Both Reports were unanimously adopted.

ANNUAL REPORT.

During the year thirteen ordinary Science Meetings and one Special Meeting of the Society were held. The former were held at the Imperial College of Science, and the latter, by the kind invitation of Sir Joseph Petavel, F.R.S., at the National Physical Laboratory.

In all 30 separate Papers were presented to the Society, and there were 17 demonstrations of special devices.

The Seventh Guthrie Lecture was delivered by Professor N. Bohr on March 24, the subject being "The Effect of Electric and Magnetic Fields on Spectral Lines." The lecture was attended by 165 Fellows and Visitors.

On May 26 Dr. F. W. Aston, F.R.S., delivered a special lecture on "Atomic Weights and Isotopes," and on October 27 Dr. Alexander Russell gave his Presidential Address, in which he described recent developments in physical science, and the part the Physical Society had played therein.

Excluding the Guthrie Lecture and the visit to the National Physical Laboratory, which was attended by about 150 Fellows and their friends, the average attendance at the meetings was 72. The demonstrations during the year were undoubtedly attractive, and the Council takes this opportunity of reminding Fellows that demonstrations are welcome at all meetings of the Society. Expenses of transport of apparatus can, in most cases, be paid by the Society.

In co-operation with the Optical Society the Twelfth Annual Exhibition of Scientific Apparatus was held on January 4 and 5. Fifty-five firms exhibited apparatus, and in many cases interesting demonstrations were arranged. Two special demonstrations were given by Dr. F. L. Hopwood. The Exhibition was the most successful ever held by the Society, the attendance reaching nearly 3,000. Mr. A. A. Campbell Swinton, F.R.S., gave two lectures on the "Johnsen-Rahbek Electrostatic Telephone and Its Predecessors"; Sir Frank Dyson, F.R.S., Astronomer Royal, gave a lecture on "The Employment of Coarse Wire Gratings in Astronomy," and Mr. F. Harrison Glew delivered a lecture on "Radium; Its Applications in Peace and War." The lectures were illustrated by experiments and were attended by large audiences. The Society is greatly indebted to the Governing Body of the Imperial College for allowing the Exhibition to be held in its rooms; it is also indebted to many members of the staff of the College for valuable assistance.

Mr. C. E. S. Phillips and Mr. F. E. Smith were appointed by the Council to represent the Society on the Council of the Institute of Physics; Dr. D. Owen and Mr. Cooper represented the Society on "Science Abstracts" Committee until October 27 last. On the appointment of Mr. Cooper as Editor of Science Abstracts, Mr. T. Smith replaced Mr. Cooper as the Society's representative.

The Institution of Electrical Engineers having reported that the financial position of "Science Abstracts" necessitated a reconsideration of the Society's contribution, a small Committee was appointed to make recommendations. As a first action Council agreed that from January 1, 1923, Fellows subscribing for Section B shall be charged ten shillings per annum; this appears to be slightly less than the cost price.

The first edition of Mr. Jeans' Report on the Quantum Theory has been sold out; there are many inquiries for copies, and Mr. Jeans has kindly agreed to revise and extend the Report. The new edition will be ready shortly. Professor Fowler's Report on "Series in Line Spectra" has already been issued to Fellows.

The question of arranging for occasional meetings of the Society in provincial towns has received careful consideration, and is viewed with favour. It is hoped to arrange for one such meeting during 1923.

In 1921 the Council informed the Conjoint Board that it would welcome any reciprocal arrangement by means of which the publications of any Scientific Society might be sold at a slightly reduced rate to Fellows of other Societies. The Conjoint Board obtained the opinions of other scientific bodies; of 40 definite replies, 16 Societies would agree to a reduction, and 24 were not in favour. No further action for the present is recommended.

Owing to some difficulty in the preparation of the Duddell Memorial Medal, the first award was not possible in 1922. The Committee dealing with the memorial have informed Council that the difficulty has been overcome, and an award will be made in 1923.

The number of Honorary Fellows on the Roll on December 31, 1922, was ten, and the number of ordinary Fellows and Students was 572. During the year 45 new Fellows and 5 Students were elected, and there were 5 resignations.

The Society has to record with regret, the deaths of Mr. C. J. Lambert, Mr. F. W. Sanderson, Mr. H. M. Elder, Professor Trouton, and Mr. L. W. Shave. Mr. C. J. Lambert was elected a Fellow during the first session of the Society, 48 years ago; he was a Life Fellow and took great interest in the proceedings of the Society. Mr. F. W. Sanderson, the late Headmaster of Oundle School, was also a Life Fellow, having been elected in 1885. Mr. H. M. Elder was elected a Fellow in 1886, and had served as a Secretary and as a Vice-President. Professor Trouton was made a Fellow in 1895 and had served on the Council. Mr. L. W. Shave was one of the youngest Fellows, having been elected but 12 months ago; he took a very keen interest in the work of the Society.

The Council have also to record, with regret, the death of one Honorary Fellow, Dr. R. Benoît, of Paris. Dr. Benoît was made an Honorary Fellow in 1909.

REPORT OF THE TREASURER.

The Accounts this year show the very unusual position of a debit balance, the expenditure having exceeded the income by £60 13s. 9d. Fortunately, this does not mean that the Society is in an unsatisfactory financial position. The deficit is due entirely to the publication of Prof. Fowler's Report on Spectra, of which the cost of printing was £358 5s. 5d. Against this cost must be set the receipts from sales of the Report, and as these will continue there is not much doubt that the present deficit will be made good in the following year.

The income has increased by nearly £120, being £1,778 19s. 8d., as compared with £1,660 5s. 4d. for the previous year. This is chiefly due to sales of publications, which realised £539 4s. 11d. The increase in subscriptions was small.

The expenditure on the "Proceedings" and the ordinary publications has decreased by £59 3s. 11d., but owing to the special publications the total cost of printing and distribution has risen to £1,256 1s. 1d., as compared with £1,095 15s. 7d. in the previous year.

An unsatisfactory feature in the Balance Sheet is a substantial increase in the "subscriptions in arrears." The investments have been valued at market prices, through the courtesy of the Manager of the London County Westminster & Parr's Bank at Charing Cross, and at length they show a marked appreciation. During the year the sum of £199 12s. 6d. was invested by the purchase of £200 5 per cent. War Loan, this investment being in respect of composition fees.

The Trust Fund of £500 London, Brighton & South Coast Railway Ordinary Stock, which was transferred to the Society by the late W. F. Stanley in 1901 for the establishment of the "Bulletin," has been set out separately so as to indicate clearly that this stock is distinguished from the other investments by being in trust.

ELECTION OF OFFICERS AND COUNCIL.

Dr. Lewis Simons and Mr. W. Shaw having been nominated scrutators, a ballot was held for the Officers and Council for the year 1923-1924. The following were elected :—

President : Alexander Russell, M.A., D.Sc.

Vice-Presidents : E. H. Barton, D.Sc., F.R.S. ; Prof. T. Mather, F.R.S. ; T. Smith, B.A. ; C. R. Darling, F.I.C.

Secretaries : F. E. Smith, O.B.E., F.R.S. ; D. Owen, B.A., D.Sc.

Foreign Secretary : Sir Arthur Schuster, Ph.D., Sc.D., F.R.S.

Treasurer : W. R. Cooper, M.A., B.Sc.

Librarian : Prof. A. O. Rankine, D.Sc.

Other Members of Council : R. W. Paul ; Prof. C. L. Fortescue, O.B.E. ; W. S. Tucker, D.Sc. ; E. H. Rayner, M.A., D.Sc. ; J. H. Brinkworth, B.Sc. ; J. Guild, A.R.C.Sc., D.I.C. ; F. L. Hopwood, D.Sc. ; E. A. Owen, B.A., D.Sc. ; J. H. Vincent, D.Sc., M.A. ; G. B. Bryan, D.Sc.

INCOME AND EXPENDITURE ACCOUNT.

From January 1st to December 31st, 1922.

INCOME.	£	s.	d.	£	s.	d.
Entrance Fees	44	2	0			
Subscriptions by Fellows *	728	8	11			
" Voluntary	12	12	0			
" by Students	4	4	0			
" Arrears paid	25	19	9			
" Paid in Advance	38	2	9			
" for "Science Abstracts" and Advance Proofs	30	11	9			
				884	1	2
Composition Fees				52	10	0
Sales of Publications (Fleetway Press, Ltd.)				539	4	11
Dividends:—						
Furness Debenture Stock	11	7	10			
Midland Railway	28	15	0			
Metropolitan Board of Works	7	0	0			
Lancaster Corporation Stock	8	14	0			
New South Wales Stock	8	17	10			
London, Brighton & South Coast Railway	19	9	8			
Great Eastern Railway	14	5	0			
India 3½% Stock	17	10	0			
Funding Loan	18	17	0			
War Stock 5%	5	0	0			
				139	16	4
Income Tax refunded				42	1	6
Interest on deposit account				6	14	6
Optical Society (for Exhibition)				18	6	3
Exhibitors' payments on account of catalogue				96	5	0
				1,778	19	8
Balance, being excess of Expenditure over Income				60	13	9
				£1,839	13	5

EXPENDITURE.	£	s.	d.	£	s.	d.
"Science Abstracts" (Inst. El. Eng.)				309	4	6
Ordinary Publications:—						
"Proceedings"	568	1	11			
"Bulletin"	56	8	11			
General, including printing for Exhibition	168	4	6			
				792	15	4
Special Publications:—						
Discussion on Hygrometry	24	18	7			
Report on Spectra	358	5	5			
				383	4	0
Postage of Publications				80	1	9
Reporting	79	10	0			
Refreshments and Attendance	96	14	7			
Petty Cash—						
Secretaries' Expenses	20	18	7			
Treasurer's Expenses	20	10	0			
Guthrie Lecture (Honorarium)	10	0	0			
Report on Spectra (Honorarium)	20	0	0			
Periodicals	3	17	6			
Insurance	1	15	0			
F. S. Spiers (registered address)	2	2	0			
Brokerage and Bank Charges	1	2	2			
Advertising	7	18	0			
				264	7	10
Conjoint Board of Scientific Societies				10	0	0
				£1,839	13	5

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,

ROBT. W. PAUL
F. J. W. WHIPPLE } *Honorary Auditors.*

February 1st, 1923.

* Forty-one Fellows paid reduced subscriptions by the arrangement with the Institute of Physics, the total discount amounting to £15 17s. 11d. "Voluntary Subscriptions" are subscriptions paid by Fellows who compounded for the low sum of £10.

BALANCE SHEET AT DECEMBER 31ST, 1922.

ASSETS.	£	s.	d.	£	s.	d.
Subscriptions in arrears	84	15	0			
Less reserve for subscriptions probably unrealisable	44	15	0			
				40	0	0
Investments (valued at Dec. 31):—						
£533 Furness Railway 3 per cent. Debenture Stock	336	0	0			
£1,600 Midland Railway 2½ per cent. Perpetual Preference Stock	800	0	0			
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock	186	0	0			
£400 Lancaster Corporation 3 per cent. Redeemable Stock	268	0	0			
£254 2s. 9d. New South Wales 3½ per cent. Ordinary Stock	249	0	0			
£500 London, Brighton & South Coast Railway Ordinary Stock	405	0	0			
£500 Great Eastern Railway 4 per cent. Debenture Stock	410	0	0			
£500 India 3½ per cent. Stock	315	0	0			
£650 4% Funding Loan, 1960-90	559	0	0			
£200 5% War Loan 1929/47 inscribed (purchased for £199 12s. 6d. in 1922)	200	0	0			
				3,728	0	0
Stock of Publications (Treasurer's valuation)				400	0	0
Cash at Bank, on Deposit	100	0	0			
Cash at Bank, Current Account	113	10	9			
	213	10	9			
Adjustment for outstanding cheques, &c.	46	14	0			
				166	16	9
Cash in hand (Treasurer's Petty Cash)					5	3
				£4,335	2	0

LIABILITIES.	£	s.	d.
Life Compositions	1,995	10	0
Balance, General Fund	2,339	12	0
	£4,335	2	0

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,

ROBT. W. PAUL
F. J. W. WHIPPLE } *Honorary Auditors.*

February 1*st*, 1923.

Note.—The value of the Society's library has not been brought into the Balance Sheet.

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1922.

	£	s.	d.
130 Fellows paid £10	1,300	0	0
3 Fellows paid £15	45	0	0
1 Fellow paid £20 10s	20	10	0
12 Fellows paid £21	252	0	0
12 Fellows paid £31 10s.	378	0	0
158	£1,995	10	0

W. F. STANLEY TRUST (FOR THE "BULLETIN").

	£	s.	d.		£	s.	d.
£500 London, Brighton & South Coast Railway...	405	0	0	Carried to Balance Sheet	405	0	0

Audited and found correct,

W. R. COOPER, *Honorary Treasurer.*

February 1st, 1923.

ROBT. W. PAUL
F. J. W. WHIPPLE } *Honorary Auditors.*

ELECTION OF HONORARY MEMBER.

A ballot was held for a new honorary member, and the following was elected: Dr. S. W. Stratton, President of the Massachusetts Institute of Technology, and formerly Director of the Bureau of Standards, Washington, D.C.

VOTES OF THANKS.

The following votes of thanks were carried unanimously: To the Hon. Auditors (Messrs. R. W. Paul and F. J. W. Whipple), proposed by Dr. Allan Ferguson and seconded by Dr. W. S. Tucker. To the retiring Officers and Council, proposed by Dr. R. T. Beatty and seconded by Mr. B. W. Clack. To the Governing Body of the Imperial College of Science, proposed by Dr. C. Chree and seconded by Dr. H. Borns.

SCIENTIFIC BUSINESS.

The following Paper was read :—

"The Eötvös Torsion Balance," by Capt. H. SHAW, M.Sc., and E. LANCASTER-JONES, B.A.

A DEMONSTRATION of The Flame Phone (Scientific and Projections, Ltd.), was given by H. W. HEATH, B.Sc.

February 23, 1923.

DEMONSTRATIONS AND DISCUSSION ON "X-RAY MEASUREMENTS."

At the AFTERNOON MEETING the following Demonstrations were shown :—

1. "Method of Measuring X-Ray Intensity," by Major C. E. S. PHILLIPS.
2. "Intermittent Discharge from Sectorless Static Machine," by E. J. EVANS, B Sc.
3. "An X-Ray Balance," by L. H. CLARK, M.Sc.
4. "Dr. Solomon's Ionometer," by H. B. GOUGH, Sunic Research Laboratories.
5. "A Spectrometer for Measuring End-radiation," by W. E. SCHALL, B.Sc.
6. "The Ondoscope," by F. L. HOPWOOD, D.Sc.

At the EVENING MEETING an introductory Address was given by Sir WILLIAM H. BRAGG, K.B.E., F.R.S., and the following Papers were then read :—

1. "The Measurement of X-Ray Intensity, and the Necessity for an International Method," by Professor SIDNEY RUSS.

2. "The Quality of X-Rays Produced by Various Types of High-tension Generators and an Incandescent X-Ray Bulb," by F. J. HARLOW, B.Sc., F.Inst.P., and E. J. EVANS, B.Sc.

3. "Practical X-Ray Measurements for Medical Purposes," by Dr. MARTIN BERRY.

4. "X-Ray Protective Materials," by Dr. G. W. C. KAYE and Dr. E. A. OWEN.

March 9, 1923.

The following Papers were read :—

1. "The Crystalline Structure of Anthracene," by Sir W. H. BRAGG, K.B.E., F.R.S.

2. "On the Frequency of Vibration of Circular Diaphragms," by J. H. POWELL, M.Sc., and J. H. T. ROBERTS, D.Sc.

3. "Radio-acoustic Method of Locating Positions at Sea," by A. B. WOOD, D.Sc., F.Inst.P., and Capt. H. E. BROWNE, O.B.E., R.N.

March 23, 1923.

The following Papers were read :—

1. "On a New Moving-coil Galvanometer of Rapid Indication," by Dr. W. J. H. MOLL, University of Utrecht.

2. "A Thermopile for Measuring Radiation," by Dr. W. J. H. MOLL.

3. "Note on Aberration and the Doppler Effect as Treated in the Theory of Relativity," by Capt. C. W. HUME, M.C., B.Sc.

The following Experimental Demonstrations were shown :—

"The Production of Electromotive Forces by Heating Junctions of Single Metals," by Mr. C. R. DARLING, F.I.C., F.Inst.P., and the Hon. C. W. STOPFORD.

"The Double Refraction due to Motion in a Vanadium Pentoxide Sol, and Some Applications," by Mr. R. H. HUMPHRY, M.Sc.

April 27, 1923.

The following Papers were read :—

1. "The Analysis of Bubbles in Glass," by J. W. RYDE and R. HUDDART, of the Research Staff of the General Electric Company, London.

2. "A Simple Regenerative Vacuum Device and Some of its Applications," by H. P. WARAN, M.A., Ph.D.(Cantab.), F.Inst.P.

3. "Application of the Eötvös Torsion Balance to the Investigation of Local Gravitational Fields," by Capt. H. SHAW and E. LANCASTER-JONES, B.Sc.

The following Demonstrations were shown :—

"An Electromagnetic Inductor," by L. F. RICHARDSON, F.Inst.P.

"An Experiment Demonstrating Time-lag in Vision," by F. LL. HOPWOOD, D.Sc., A.R.C.Sc., F.Inst.P.

May 11, 1923.

The Duddell Medal was handed over to the President of the Society by Sir William H. Bragg, K.B.E., F.R.S., Chairman of the Duddell Memorial Committee.

The Eighth Guthrie Lecture was delivered by J. H. JEANS, D.Sc., LL.D., F.R.S., who took as his subject "The Present Position of the Radiation Problem."

May 25, 1923.

The following Papers were read :—

1. "The Effect of Torsion on the Thermal and Electrical Conductivities of Metals," by Prof. C. H. LEES, D.Sc., F.R.S., and J. E. CALTHROP, B.A., B.Sc.

2. "The Use of the Wien Bridge for the Measurement of the Losses in Dielectrics at High Voltages, with Special Reference to Electric Cables," by A. ROSEN, B.Sc., A.M.I.E.E.

The following Demonstrations were shown :—

"An Experiment on the Production of an Intermittent Pressure by Boiling Water," by C. R. DARLING, F.I.C., F.Inst.P.

"A Novel Instrument for Recording Wireless Signals," by N. W. MCLACHLAN, D.Sc., M.I.E.E.

June 8, 1923.

Prof. James G. Gray, D.Sc., F.R.S.E., Cargill Professor of Applied Physics in the University of Glasgow, delivered a Lecture entitled " A General Solution of the Problem of Finding the True Vertical for all Types of Marine and Aerial Craft." The Lecture was accompanied by Demonstrations with Gyroscopes.

June 22, 1923.

Prof. F. Horton, M.A., D.Sc., F.R.S., Royal Holloway College, University of London, delivered a Lecture entitled "The Excitation and Ionisation Potentials of Gases and Vapours."

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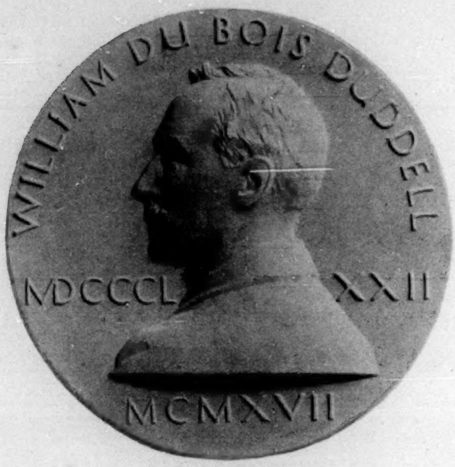
THE DUDDELL MEMORIAL MEDAL.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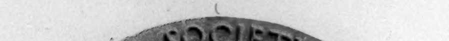
THE PHYSICAL SOCIETY OF LONDON
RERUM NATVRAM EXPANDERE

REVERSE.

I. *Presidential Address.* *By* Dr. Alexander Russell.

Delivered October 27, 1922.

In this address I shall first give my views of some recent developments in physical science. Not because I am specially qualified to talk on these subjects—some of you have studied them far more deeply than I have—but because it may be of value as showing how the average scientific man is struggling to get a sure foundation for his knowledge.

New Physical Laws.—Since our Society was founded 48 years ago, several new physical laws have been discovered. Newton pointed out that it redounds to the credit of theory to have as few fundamental principles as possible. Physicists, therefore, have expended almost as much thought in trying to deduce these laws from primary causes as in discovering new practical applications for them. There is nothing that gives an investigator greater pleasure than discovering that some apparently complicated phenomenon can be rigorously deduced from fundamental principles. As a rule, however, this discovery requires ability of the highest order. For example, when Snow Harris found that two spheres electrified with like electricities sometimes attracted and sometimes repelled one another, it needed the genius of a William Thomson to prove that Harris's phenomena followed rigorously from the laws of electricity enunciated by Coulomb. The physical explanation of some of the phenomena discovered in recent years still baffles us, but there can be little doubt, judging by the ability and perseverance of the present generation of physicists, that their secrets will soon be wrested from Nature.

The main functions of the Physical Society are to disseminate the knowledge of new discoveries and examine critically new theories from every point of view. A look through our Proceedings and through the Reports written by eminent physicists which have been issued from time to time will show how well it has fulfilled its functions.

Quantum Theory.—It was no easy matter for Planck's Quantum Theory to make headway 20 years ago. The Society, therefore, invited Mr. J. H. Jeans, Sec. R.S., to write a Report on Radiation and the Quantum Theory. This Report has been of the greatest value and has been helpful to workers all over the world. Planck's constant is now considered to be one of the fundamental constants in Nature. The apparent difficulty that energy is radiated in quanta will probably disappear when the atomic mechanism of matter is more fully understood. Physicists are looking forward with great interest to the appearance of the new edition of this report.

Relativity.—Although the theory of Relativity has been widely discussed for several years, little progress has been made in presenting the theory in a form readily intelligible by the public. Even the most elementary explanations of the subject finish up by referring to a four dimensional continuum or a four-dimensional manifold. These words convey a meaning to nobody but a mathematician. A simple explanation without mathematical conceptions seems to be impossible. The Report on the "Relativity Theory of Gravitation" which Professor Eddington wrote for the Society in 1918 is still one of the best introductions to the practical side of this difficult theory. The clear way in which he states the crucial phenomena by means of which Einstein's gravitational theory will be adjudged can be appreciated by everyone. It is well known that a charged body in motion behaves as if it had an additional mass, the reason being that the moving charge generates a magnetic field in the surrounding region. This increases the energy of the moving system. Hence it is equivalent to the effect produced by an increase in the mass of the body. This can be readily understood. But on the theory of relativity the mass, whether it have an electric charge or not, varies with the speed. A certain amount of experimental evidence can be deduced in support of this conclusion, but its foundations certainly want strengthening.

Realities.—In his address to Section A of the British Association this year, Professor Hardy said that the old-fashioned geometry of Euclid and the space-times of Einstein and Minkowski were "realities." On the other hand he pointed out that no philosopher or physicist had ever put forward any convincing account of what "reality" was. I hope it will be a long time before any physicist wastes time in trying to define physical "reality." Physicists only claim to know what they have learned from experience.

The Position of the Physicist.—In the opening sentence of Fourier's "Theory of Heat," our position is clearly stated:—

"Primary causes are unknown to us but are subject to simple and constant laws which may be discovered by experiment and the study of which is the object of Natural Philosophy." Thomson and Tait prefixed this sentence in the original French to their Treatise on Natural Philosophy. They point out that most useful and interesting results follow by mathematical processes from known physical laws. This seems to contradict Professor Hardy's statement that it is impossible to produce by mathematical reasoning any proposition at all concerning the physical world. I think that every one who sees the progress that physical science, helped by mathematics, is making at the present time must admit that it is hourly improving man's mastery over Nature.

Series in Line Spectra.—This year a very valuable and exhaustive report on "Series in Line Spectra," by Professor Fowler, was published by the Society. To physicists the subject is of absorbing interest. The discovery by Balmer in 1885 of the law according to which the hydrogen lines arrange themselves in a spectrum marks an epoch in the history of spectrum analysis. The simplicity and accuracy of the law was a great inducement to workers to search for possible physical causes, and most satisfactory progress has been made.

The great importance of the study of spectra lies in the fact that it must give us valuable information about the constitution of matter. This was early recognised. As far back as 1867, Sir William Thomson (Kelvin) suggested that the two principal lines in the sodium spectrum might be due to the fact that the sodium atom had two fundamental modes of vibration whose periodic times were the same as that of the lines they produced. At that time he was elaborating a vortex theory of the ether and so he suggested that the sodium atom consisted of two approximately equal vortex rings passing through one another like links in a chain. It is difficult to see how this assumption could be elaborated so as to explain the 65 lines given for Sodium in Fowler's Report. Rutherford's nuclear theory, however, will probably furnish in the future far simpler and more satisfactory explanations.

The Electron and the Atom.—The discovery that external magnetic and electrostatic fields could alter the position of the lines in the spectrum (the Zeeman and

Stark effects) is strong evidence that electricity plays a leading part in the atomic constitution of matter. During the last century the theory of electricity was as unsatisfactory as the experimental verification of the results obtained from a few fundamental assumptions was satisfactory. Consider, for instance, the Paper on the electrification of two spherical conductors read by Poisson in 1812 to the French Academy. He made the supposition that electricity consisted of two fluids. Every molecule of one fluid attracted every molecule of the other fluid with a force inversely proportional to the square of the distance between them. It also repelled every molecule of a like fluid according to the same law. By rigorous and elementary mathematical reasoning he showed how to find the capacities of the two spheres in the presence of each other, and for particular cases he worked out their accurate numerical values. Although he supposed the layer of electricity on each sphere to be practically infinitely thin, yet he found formulæ to give the density, and consequently the electric stress, at points on the two surfaces. The accuracy of his results probably still far exceeds the limits of accuracy of the most sensitive present-day apparatus.

The great advance that has been made in recent years is the discovery of the electron by Sir J. J. Thomson. The importance of this discovery can hardly be over-estimated. The electron undoubtedly plays a leading part in the constitution of the atom. It gave us our first insight into what is happening in the ultra-microscopic region of matter. Judging by the important results obtained by Rutherford's nuclear theory of the constitution of matter, it must be given the leading place amongst atomic theories. Accepting this hypothesis and Planck's theory of quanta, Bohr has shown that the position of the lines in the hydrogen spectrum can be computed with an accuracy as great as that of the most refined experimental measurements. It is difficult to believe that his theory is not substantially accurate. His theory also applies to ionised helium, where we have a single electron rotating round a nucleus of four times the mass of the hydrogen nucleus. In the Guthrie lecture which Professor Bohr gave us this year it was indicated how the theory could be expanded so as to explain the spectra of more complex elements. According to Bohr and Sommerfeld, the nucleus is surrounded by various groups of orbits. For all orbits of one group the energy of the electrons is substantially the same, but as some of the orbits are elliptical the energy given up when an electron jumps from one orbit to another may not be an exact quantum. The frequency of the radiation, therefore, may have two or more values, differing slightly from one another. This would explain how a single spectrum line is sometimes doubled or trebled, giving us what Sommerfeld calls the fine structure of the lines. Dr. Silberstein has recently shown that if we make the assumption that the mutual forces between the electrons of a neutral helium atom are negligibly small, then by the theory of quanta we can compute the positions of at least 40 lines in the helium spectrum with the highest accuracy. It is difficult to see any reason why such accurate results should follow from an erroneous assumption. W. M. Hicks has also given analytical relations connecting the lines in the spectrum of hydrogen. There are now so many relations to explain that any theory which explains them all will be very convincing. It may be possible to do without planetary electrons by the use of an atomic model such as that devised by Professor Whittaker, but many of us believe that Bohr's theory has a substantial foundation.

It has to be remembered that no satisfactory explanation has yet been given of the stability of molecules in terms of the mutual actions taking place between their atoms. Lodge suggests that we should abandon the idea of electrical action between atoms producing chemical force, and substitute for it the forces produced by the interlacing magnetic fields produced by the rapidly revolving electrons. It is possible that chemical affinity might be explained in this way. It is advisable, therefore, for physicists to keep an open mind and see how far the various hypotheses that have been made can explain physical phenomena. They at least suggest new lines of research.

The Moseley classification of the elements according to the electric charges on the nuclei of their atoms has produced a wonderful simplification in theory. In his series of ordinal numbers for the chemical elements ranging from hydrogen to uranium (1 to 92) there are now only three numbers missing—namely, 43, 61 and 75. It is probable that the elements corresponding to these numbers will soon be discovered. The discovery of isotopes—that is, atoms which have the same nuclear charge, but which have different masses—will be a great help in furthering our knowledge. They prove that the physical and chemical properties of the atom depend only on its nuclear charge, and are independent of its mass.

Text Books.—The time has now come when the text books of physics in everyday use will have to be largely re-written. Recent advances in physics are but feebly reflected in some of them. A few books, like Newton's *Principia Mathematica*, are written for all time ; but in the light of recent researches there is a large field of work open to writers in reorganising our knowledge. There is always a large amount of inertia to be overcome before any change can be made in the orthodox ways of teaching science, but it is of national importance that the community should recognise the potential value of recent discoveries. Rutherford has induced artificial radio-activity in boron, nitrogen, fluorine, sodium, aluminium and phosphorus by means of the high-speed helium nucleus. He has proved that the hydrogen atoms leave the nucleus with more energy than that possessed by the helium projectile which caused the emission. Theoretically, therefore, it is possible to obtain some of the atomic energy stored in matter. Manufacturers should recognise that knowledge of the constitution of matter obtained by research in pure physics will probably be of the greatest practical value to them.

Applied Research.—One of the great differences between pure and applied research is that the former is generally concerned with the simple elements, whilst the latter has to study the materials used in everyday work. Consider rubber, and the composite materials used for insulating electric cables. We desire to know their electric strength, their inductivity, their dielectric losses at high and low frequencies, their capacity for moisture, their ignitability, their insulating, their mechanical and their thermal properties, and lastly their life under normal conditions of working. With simple elements we can generally make the assumption that their physical properties remain constant. It is not so with some of the materials used in commerce.

In the old days, a good buyer with the aid of a pocket-knife and a careful visual examination was able to purchase material which was generally found to be satisfactory by his firm. Now the manufacturers have combined together, and with the help of the Department of Scientific and Industrial Research, of the National Physical Laboratory, and the leading research laboratories, are carrying out elaborate researches to find out which of the physical qualities of a material are of the greatest importance in practice. They are also devising methods to enable these qualities to be easily and accurately measured. The material may offer great resistance to the passage of a disruptive spark through it, but engineers have to be satisfied that this electric strength is a permanent quality of the material. Some cables when buried in the ground deteriorate rapidly. It is necessary, therefore, to make costly and lengthy experiments to investigate these and similar phenomena.

Methods of Duality.—It is found sometimes that the experimental determination of a problem in one branch of science enables us to solve the analogous problem in another. Kelvin always laid the greatest stress on these analogies. He pointed out, for example, the analogy between the motion of a vortex filament in a perfect fluid and the magnetic field round a circular current. The expression for the total kinetic energy of the vortex ring and the moving fluid can be written down at once from the expression for the electro-magnetic energy stored up in the field round the circular current. Similarly, the electric resistance between two spheres in a homogeneous medium can be written down when we know the electrostatic capacity between them.

These analogies are of practical importance. For instance, when we find out experimentally the electrostatic capacity between the cores and the sheath of a polycore cable we can deduce the thermal resistance to a steady flow of heat from the cores to the sheath in terms of the thermal conductivity of the insulating material. Hence by measuring the thermal resistance we get the true value of the thermal conductivity of the material. I do not know whether this method has yet been adopted. In the ordinary experimental method a mathematical formula is used, but as this formula neglects, amongst other things, the twist of the cores round the central axis of the cable it is affected by errors the magnitude of which are unknown; hence the combined electrostatic and thermal method is preferable. Similarly, if the insulating material of the cable obeys Ohm's law its insulation resistance per mile can be found either from the electrostatic or from the thermal measurements.

As a help in discovering new theorems and applications the methods of duality are of great value in physics. Some of the analogies are obvious. Consider, for example, the method of images and the phenomena of refraction in light, sound, magnetism, electrostatics, and electrokinetics. In some cases the analogies have to be laboriously constructed. In electrical science, duality theorems can be deduced by interchanging resistance and conductance, current and pressure, inductance and capacity, star and mesh, electrostatic and electromagnetic instruments, and dynamos and motors. In many cases these theorems are as rigorous as the theorems in duality deduced in geometry.

As an example of duality on a large scale we can take Mr. George Constantinesco's method of transmitting power by transmitting vibrations through water contained in pipes. The rapid development of this method followed at once when the analogy between the vibrations of liquids in pipes and alternating currents in wires was seen. Practically the whole theory of the transmission of power by alternating currents together with that of all the accessory generators, motors, transformers and condensers can be directly applied both to give a working theory of the transmission of power by hydraulic waves and to develop a system of hydraulic machines exactly analogous to those used in electrical work. We have, for instance, hydraulic generators and motors of the single and polyphase type. We also have both synchronous and asynchronous machines.

The Electrical Units.—Pure scientific research has probably been of greater help to the electrical than to any other industry. The invaluable work done by the British Association Committee, both in defining our electrical units and in devising methods of determining them in terms of the centimetre, the gramme and the second is described in Gray's "Absolute Measurements." The electrical industry rightly reveres the memory of Maxwell, Kelvin and Rayleigh. Their thanks are also due to our secretary, Mr. F. E. Smith, whose skill and physical insight enabled him to measure the electrical units with an accuracy which is at present in excess of all commercial requirements. I agree with him in thinking that all electrical students should be taught how to measure resistance in electromagnetic measure by means of a rotating coil. They can also measure current in absolute measure by a tangent galvanometer or by weighing the attraction between coils carrying currents. Hence by Ohm's law they can determine the unit of E.M.F. in absolute electromagnetic measure. I should suggest also that they weigh the electrostatic attraction between two spherical conductors maintained at a high potential difference. They can then easily compute the E.M.F. in electrostatic measure, and measuring it in electromagnetic measure by a voltmeter, they find that the electrostatic unit of pressure is 300 volts, and the ratio of the electrostatic to the absolute electromagnetic unit of pressure is 3×10^{10}, which gives the velocity of light in centimetres per second.

Precipitation of Vapours.—A very useful field of research is in connection with the electrical precipitation of vapours. In many industries smoke vapours and dust laden gases are discharged into the atmosphere. In some cases they carry off valuable by-products. The commercial efficiency of the works is thereby appreciably lowered. In all cases the smoke and vapours constitute a nuisance, and in some cases they constitute a danger. The nuisance may be mitigated and the danger removed by electrical precipitation. In this process charged electrodes which exercise forces of attraction and repulsion on neighbouring particles are used. The electrodes are so shaped that one promotes discharge and the other makes it a minimum. In works the deposit is usually removed from the electrodes by tapping them every three or four hours with hammers actuated by electric motors. The insulating framework is also periodically vibrated. In the Cottrell process when the flow of gases is about 170 cubic ft. per second, an input of 2 kw. at a pressure ranging from 25 to 100 thousand volts is required. Precipitation is also of great value in smelting works and in cement plants. In the former case the recovery of the waste metals—copper, silver, or tin, for instance—is the main object in view. In America the Cottrell plant for copper smelters treats 130,000 cubic ft. of gases per second and recovers about 5 million pounds of copper per annum. The method can also be used to precipitate coal smoke, where unfortunately the by-product is of little or no value. As gas or electricity can now always be used to heat buildings in towns, the time has come when coal consumers should be penalised. When coal is burnt some form of soot precipitation plant should be made compulsory. It is better policy to prevent the causes of fogs than to invent remedies for the fogs themselves.

Acoustics.—There has recently been considerable discussion as to the best method of securing good acoustical properties in a large room. It is agreed that good conditions for hearing can only be secured by eliminating reverberation due to reflexion from the boundaries of the room. If we knew more about the acoustical properties of matter it should not be difficult to prevent this reverberation without the necessity of altering the design of the room or of appreciably increasing the cost. In this connection Sir Arthur Schuster has pointed out that experimental investigation of the acoustical properties of materials with regard to absorption, scattering and rate of transmission is still required. A somewhat analogous and even more pressing problem which has to be solved is the construction of noiseless and vibrationless sub-stations for power and railway supply in cities and large towns. The converse problem of making a sound-proof room for use in telephonic research also needs investigation. The Wallace Sabine laboratory of acoustics in America has done excellent work, but much further work remains to be done.

Radio Communication.—Radio-communication is now a great and popular branch of applied science. There can be no doubt that broadcasting when it becomes general will have a great influence on national development. The fascination of the art is attracting many able young men to take up applied electricity as a career. The labours of physicists have placed this branch of science on a sound foundation. When once Professor Fleming realised the mode of action of the electrons in the Edison effect, the thermionic valve came into existence and subsequent developments followed with almost bewildering rapidity.

The main difficulty yet to be overcome is how to eliminate the effects of atmospherics from radiophones. There are two ways of minimising these effects—namely, by increasing the sharpness of the tuning, and by directional reception—that is, by taking the signals from the aerial at the best point. In order that the signals may be heard in any kind of weather it is necessary to erect the sending stations on the most suitable site and make them powerful. As unanimity of control has been secured we can hope that the mistakes which have been made in America, where broadcasting is in rather a chaotic condition, will be avoided. It is estimated that in America there are about two million radio-receiving sets in use. In the neighbourhood of New York there are no less than 20 broadcasting stations, each using 1½ kw. and emitting a wave-length of 360 metres.

It will be of interest to see whether the other method of broadcasting, namely, through the mains of the electric lighting supply companies, will make progress. It is certainly scientifically feasible.

Electrons in Engineering.—Richardson's researches proving that electrons are emitted by hot metal by a process similar in all respects to evaporation were invaluable in the development of thermionic devices. At first sight these investigations appear to have little to do with heavy electrical engineering. Leblanc, however, has recently shown how the large amount of power required by a high speed electric train can be communicated to it without rubbing contacts by magnetic induction. He proposes to use alternating current having a frequency of 20,000. This is carried over the track by a series of tubular condensers adjusted to resonance. The current in the locomotive circuit is converted to low frequency by means of valve transformers which operate ordinary induction motors as in other alternating-current railway systems. Besides the advantage of doing away with rubbing contacts, it has the advantage that any interference produced in neighbouring telegraph and telephone circuits can easily be remedied by means of condensers. Leblanc's suggestions show how recondite mathematical theorems and apparently unpractical physical investigations can be combined to devise a new and promising system for heavy electric traction.

Individual versus Collective Effort.—Progress in new branches of industry needs in the first place, individual effort. Individual workers with simple apparatus and a whole-hearted enthusiasm are more likely to obtain valuable results than the best organised collective effort with costly and elaborate plant. Collective effort is only advisable when it is almost certain that valuable results will be obtained. Brilliant discoveries are usually made in out-of-the-way fields where few ever penetrate and where the prospects of a successful issue are too remote to justify a collective effort.

Development Departments.—It has to be remembered, however, that when a new discovery is made which may have commercial applications, then, as a rule, it can be developed far better by the Development Department of a large industrial undertaking than by the individual inventor. I think, therefore, that a closer co-operation between physicists and industrial engineers is highly desirable and will be to their mutual advantage. In pre-war days the discoveries of British scientists were often first commercially exploited in Germany. For example, Sir James Dewar's vacuum flask for preventing bodies from gaining or losing heat was first made on a commercial scale in Germany. This was due to the more thorough organisation of the Development Departments in German works. I am glad to see that there are signs that the great importance of the work done by these departments is being recognised in this country.

The Physical Society.—It is interesting to remember that the Physical Society of London was founded in the same year that Graham Bell began his researches in telephony. The invention of the telephone was a boon to humanity. It shows how pure research can benefit the world and should encourage physicists to develop the practical side of their discoveries. The first Paper to the Society was read on March 21, 1874, by Professor J. A. Fleming, whom every physicist and engineer delights to honour. It was entitled "On the New Contact Theory of the Galvanic Cell."

We have recently had to lament the death of two of our Fellows, Professor F. T. Trouton and F. W. Sanderson. Professor Trouton, who has served on the Council of this Society, was one of Fitzgerald's most distinguished pupils. He did excellent work on Hertzian waves and, in conjunction with Professor Rankine, carried out an important experiment to try and detect the Fitzgerald shrinkage in the theory of relativity. Amongst other work, he discovered the connexion between latent heat and molecular weight.

F. W. Sanderson did invaluable work in promoting pure and applied science teaching in this country. For 30 years he was headmaster of Oundle School and carried out many novel and successful experiments in education. His greatest success lay in interesting practically every boy in his school in some branch of study. By the deaths of Sanderson, of Oundle, and McClure, of Mill Hill, the country has lost two great educational experimentalists. Their success proves the wisdom of allowing teachers to develop their own individual methods.

Conclusion.—I should like to emphasise that there has seldom, if ever, been such an interesting period in the history of physical science as the present time. It was a great shock to scientists to find that the velocity of light was independent of the motion of the observer. Some explained it by saying that the earth dragged the ether along with it, but this is contradicted by the phenomenon of the aberration of light. Fitzgerald and Lorentz suggested independently the contraction hypothesis which gives a rational explanation, but which is very difficult to verify by experiment. The relativity theory does not explain the nature of the phenomenon, but it indicates that the hypothesis of the ether is not a necessity. So many exciting problems have arisen in recent years that Rutherford's experiments showing the transmutation of gases have not received the attention they deserve. These experiments mark a new epoch in the history of science, and in the future they may have marvellous commercial developments. The further our knowledge of physical science extends the more closely does it affect us in our normal everyday mental and physical life. A country that neglects the study of physical science cannot take a leading place amongst civilised nations. It is our duty to the human race as well as to our own country to do our utmost to extend the knowledge of the universe in which we live.

II. *The Problem of Two Electrified Spheres.*

By ALEXANDER RUSSELL, *M.A., D.Sc., Principal of Faraday House.*

RECEIVED SEPTEMBER 21, 1922.

ABSTRACT.

It is shown that Poisson's method can be readily applied to get the complete solution of the problem of two electrified spherical conductors. The case of a spherical condenser is first considered and simplified methods of computing its capacity are given. Simple expressions for the potentials at the two inverse points due to the charges on each of the spheres are found. Poisson's method is then used to show that the capacity coefficients for two spheres external to one another can be very simply expressed in terms of the capacities of certain spherical condensers and can thus be readily computed in all cases. Formulæ for the attraction and repulsion of two electrified spheres are given. When the charges are of like sign the electrostatic energy of the system has a maximum value when the spheres are at a given distance apart. Similarly, when the potentials are of like sign and maintained constant, the electrostatic energy of the system has a minimum value for a definite distance between them. The particular case when the spheres are close together is considered in detail. The exactly analogous problems in thermal and electric conduction are also considered, and it is shown how their solutions can be written down at once. Finally, the formula for the maximum value of the electric stress in the medium surrounding the spheres is given and its practical importance is pointed out. For equal spherical electrodes at a fixed distance apart there is a definite size of radius which makes the disruptive potential difference a maximum. The distance between them where the attraction has a maximum value is also found.

THE problem of two electrified spherical conductors has attracted the study of physicists for over a hundred years. From the point of view of the mathematician it may be considered that the complete solution has been obtained. The solutions, however, are usually given in the form of series which are in general very laborious to compute numerically, and hence they are rarely used by the physicist. The author has therefore thought it advisable to give what he considers to be the simplest solution of the problem, and to show by numerical examples that in practically every case the numerical solution can be readily obtained. The methods of computing the capacity of a spherical condenser are first considered. It is then shown how the self and mutual capacity coefficients of external spheres can always be written down by simple formulæ in terms of the capacities of certain spherical condensers, and can thus be readily computed. Finally their attractions and repulsions, the maximum value of the potential gradient at their surfaces and the value of the potentials at all points are given.

In calculating the capacity of a spherical condenser* the author formerly used Kelvin's method of images. Difficulties arose as in some cases the Kelvin points were at infinity. In what follows he gives a modification of the method which Poisson† gave 112 years ago. He now thinks that this method is the simplest that has so far been devised.

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* Proc. Roy. Soc., Vol. 94, p. 206.

† Memoires de l'Institut (1812).

LEMMA.

Consider an electrified spherical conductor of radius a with its centre at O (Fig. 1). The charge q on it is distributed over its surface. Let the potential due to this charge at a point P distant x from O be denoted by $\varphi(x/a)$. Then if $OP\,.\,OQ=a^2$, we have

$$\varphi\left(\frac{x}{a}\right)=\Sigma\frac{\delta q}{AP}=\frac{a}{OP}\Sigma\frac{\delta q}{AQ}=\frac{a}{x}\varphi\left(\frac{a}{x}\right). \qquad (1)$$

This is true, whether x is greater or less than a. Notice also that $\varphi(0)=\frac{q}{a}$.

THE CAPACITY OF A SPHERICAL CONDENSER.

Let A (Fig. 2) be the centre of a spherical cavity in a conducting solid, and let B be the centre of a spherical conductor in this cavity. Let a and b be the radii of the two spheres, and let c be the distance AB between their centres. Let the inner sphere have a charge q. There will be a charge $-q$ over the surface of the cavity.

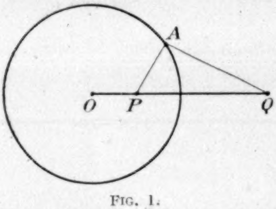

FIG. 1.

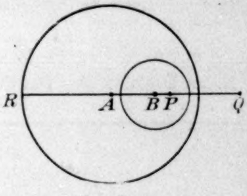

FIG. 2.

The potential at a point in the outer conductor at a great distance away will be zero, for its distances from the charges $+q$ and $-q$ will be practically the same. Hence every point on the outer conductor must be at zero potential. We shall suppose that the inner conductor is at potential V. Let P (Fig. 2) be any point on the line $RABQ$, inside the sphere B, and let $AP=x$. Let $f_1(x/a)$ be the potential at P produced by the charge $-q$, and let $f_2\{(x-c)/b\}$ be the potential at the same point produced by the charge $+q$. Since the outer conductor is at potential zero, we have for the potential at a point Q in it, on AB produced

$$f_1\left(\frac{AQ}{a}\right)+f_2\left(\frac{BQ}{b}\right)=0$$

Hence if $AQ=\xi$ we have by (1)

$$\frac{a}{\xi}f_1\left(\frac{a}{\xi}\right)+\frac{b}{\xi-c}f_2\left(\frac{b}{\xi-c}\right)=0 \qquad (2)$$

where ξ lies outside the limits $+a$ and $-a$.

We have also at every point P on RQ inside the sphere B

$$f_1\left(\frac{x}{a}\right)+f_2\left(\frac{x-c}{b}\right)=V \qquad (3)$$

where x lies in value between $c-b$ and $c+b$.

Now let $\frac{x}{a}=\frac{a}{\xi}$. This is always possible since a must be greater than $b+c$.

Hence eliminating $f_1\left(\frac{x}{a}\right)$ between (2) and (3) we get

$$f_2\left(\frac{x-c}{b}\right)-\frac{ab}{a^2-cx}f_2\left(\frac{bx}{a^2-cx}\right)=V. \quad \ldots \ldots \quad (4)$$

If we write

$$\frac{x_n-c}{x_{n-1}}=\frac{b^2}{a^2-cx_{n-1}} \quad \ldots \ldots \ldots \ldots \quad (5)$$

we deduce by successive applications of (4) that

$$f_2\left(\frac{x_1-c}{b}\right)=V\left\{1+\frac{ab}{a^2-cx_1}+\frac{a^2b^2}{(a^2-cx_1)(a^2-cx_2)}+\ldots\right\}. \quad \ldots \quad (6)$$

The formulæ (6) and (5) enable us to compute the potential due to the charge $+q$ at all points on the diameter of the inner sphere which passes through A and B.

It is convenient to express a, b and c in terms of certain functions α, β and r, which may be defined* by the equations

$$a=r/\sinh\alpha,\ b=r/\sinh\beta,\ c=r\sinh(\beta-\alpha)/(\sinh\alpha\sinh\beta) \quad \ldots \ldots \quad (7)$$

It can be shown that r is the radius of the sphere which cuts the two spheres A and B orthogonally, and which has its centre on AB produced. If we denote $\beta-\alpha$ by ω, we have

$$\cosh\alpha=\frac{a^2+c^2-b^2}{2ac},\ \cosh\beta=\frac{a^2-b^2-c^2}{2bc},$$

$$\cosh\omega=\frac{a^2+b^2-c^2}{2ab},\ \text{and}\ \sinh\omega=\frac{cr}{ab}.$$

Hence also $c=a\cosh\alpha-b\cosh\beta=a\varepsilon^{\alpha}-b\varepsilon^{\beta}=a\varepsilon^{-\alpha}-b\varepsilon^{-\beta}$ . . . . (8)

The potential at B (Fig. 2) due to the charge q is obviously q/b, and this equals KV/b, where K is the capacity of the spherical condenser. Hence, putting $x_1=c$, and noting that $f_2(o)$ is the potential at B, we get

$$\frac{K}{b}=1+\frac{ab}{a^2-c^2}+\frac{a^2b^2}{(a^2-c^2)(a^2-cx_2)}+\ldots \ldots \ldots \quad (9)$$

where $x_2, x_3, \ldots$ are found successively by (5).

Now
$$\frac{ab}{a^2-cx_1}=\frac{\sinh\beta}{\sinh(\beta+\omega)}$$

and
$$\frac{ab}{a^2-cx_2}=\frac{\sinh(\beta+\omega)}{\sinh(\beta+2\omega)},\ \text{etc.}$$

Hence
$$K=\sum_0^\infty\frac{r}{\sinh(\beta+n\omega)} \quad \ldots \ldots \ldots \quad (10)$$

This agrees with formula (5), given in the Proc. Roy. Soc. A., Vol. XCIV., p. 209, which was obtained by Kelvin's method of images. We see that (10) can be written in the form

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* Russell, Alternating Currents, Vol. 1, p. 166.

$$K=\frac{r}{\sinh\beta}+K_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Where K_1 is the capacity of a spherical condenser for which the inner radius b_1 equals $r/\sinh(\beta+\omega)$, the outer radius a_1 equals $r/\sinh(\beta+\omega-\omega)$ and the distance between their centres c_1 equals $\frac{a_1 b_1}{r}\sinh\omega$.

We shall denote the capacity of a spherical condenser by $K(a, b, c)$. Hence

$$K(a,b,c)=b+K\left(b,\ \frac{ab^2}{a^2-c^2},\ \frac{cb^2}{a^2-c^2}\right)=b+\frac{b^2}{a^2-c^2}K\left(\frac{a^2-c^2}{b},\ a,\ c\right) \quad . \quad . \quad . \quad . \quad (12)$$

for $K(ma, mb, mc)=mK(a, b, c)$.

As the condenser on the right-hand side of (12) is always more nearly concentric than the one on the left-hand side, (12) is very useful for computing purposes.

The author has previously shown (*l.c. ante*) that

$$K(a,\ b,\ c)=\frac{ab}{a-b}\left\{1+A_2\frac{c^2}{(a-b)^2}+A_2A_4\frac{c^4}{(a-b)^4}+\ .\ .\right\}$$

where $A_2=\frac{ab}{a^2+b^2+ab}$, and A_2A_4 has its maximum value $\frac{1}{5}$ when $a=b$.

Hence, when $c^4/\{5(a-b)^4\}$ can be neglected compared with unity, we have

$$K(a,\ b,\ c)=\frac{ab}{a-b}\left\{1+\frac{ab}{a^2+b^2+ab}\cdot\frac{c^2}{(a-b)^2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

with high accuracy.

When $c/(a-b)$ is large, it is better to use the formula (*l.c. ante*)

$$K(a,\ b,\ c)=\frac{b}{2}+\frac{r}{2\omega}\log_e\frac{\cosh\beta+1}{\cosh\beta-1}+\frac{r\omega}{12}\ \frac{\cosh\beta}{\sinh^2\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Writing $2s=a+b+c$, we get

$$cr=2[s(a-s)(s-b)(s-c)]^{\frac{1}{2}}$$

$\cosh(\omega/2)=\{s(s-c)/ab\}^{\frac{1}{2}}$; $\sinh(\omega/2)=\{(a-s)(s-b)/ab\}^{\frac{1}{2}}$; $\omega/2=\log_e\{\cosh(\omega/2)+\sinh(\omega/2)\}$; and $\cosh(\beta/2)=\{(s-b)(s-c)/bc\}^{\frac{1}{2}}$.

The formulæ give the capacities in electrostatic units (centimetres). To find the capacities in microfarads we divide by 900,000.

NUMERICAL EXAMPLES.

Using (12), we get

$$K(10,\ 1,\ 6)=1+\frac{1}{64}K(64,\ 10,\ 6)$$

$$=1+\frac{1}{32}K(32,\ 5,\ 3)$$

Similarly

$$K(32,\ 5,\ 3)=5+\frac{5}{203}K(203,\ 32,\ 3)$$

We see by (13) that the error made in computing $K(203, 32, 3)$ as if it were the concentric spherical condenser $K(203, 32, 0)$ is very small. Writing, therefore,

$$K(203, 32, 3)=\frac{203 \,.\, 32}{203-32}$$

and, substituting, we get

$$K(10, 1, 6)=1{\cdot}185490 \ldots$$

The true value is $1{\cdot}185491 \ldots$

As a further example of the use of (12)

$$K(3, 1, 1)=1+\frac{1}{8} K(8, 3, 1)$$

$$K(8, 3, 1)=3+\frac{1}{7} K(21, 8, 1)$$

$$K(21, 8, 1)=8+\frac{8}{55} K(55, 21, 1)$$

Thus $$K(3, 1, 1)=1+\frac{3}{8}+\frac{1}{7}+\frac{1}{385} K(55, 21, 1)$$

$$=1{\cdot}6061 \ldots \text{approx.}$$

Using (14), we get

$$K(10, 7, 1)=3{\cdot}5+20{\cdot}520+0{\cdot}204$$

$$=24{\cdot}224 \ldots \text{approx.}$$

More simply, by (12) and (13)

$$K(10, 7, 1)=7+\frac{7}{99} K(99, 70, 7)$$

$$=24{\cdot}21 \ldots \text{approx.}$$

When $c/(a-b)$ is very nearly unity we can use the formula

$$K(a, b, c)=\frac{ab}{a-b}\left\{\frac{1}{2}\log_e \frac{2ab}{(a-b)(a-b-c)}-\psi\left(\frac{a}{a-b}\right)\right\} \quad . \quad . \quad . \quad . \quad (15)$$

where $\psi\left(\frac{a}{a-b}\right)$ is the logarithmic derivate of the gamma function, tables of the values of which are given in Russell's "Alternating Currents," Vol. I., p. 241.

For example, by (15)

$$K(8, 4, 3{\cdot}99)=27{\cdot}82 \ldots$$

It will be seen that the capacities of spherical condensers can be computed in all cases with little labour.

THE POTENTIALS AT THE INVERSE POINTS OF THE TWO SPHERES.

If P and Q be the inverse points of the two spheres in Fig. 2, then $AP \,.\, AQ=a^2$ and $BP.BQ=b^2$. It easily follows that $AP=a\varepsilon^{-\alpha}=c+BP=c+b\varepsilon^{-\beta}$. From equation (4) we can readily find simple expressions for the potentials at P and Q due to the charge q. Equation (4) may be written

$$f_2\left(\frac{x-c}{b}\right)-\frac{ab}{a^2-cx}f_2\left\{\frac{x-c}{b}+\frac{c(x-a\varepsilon^{-\alpha})(x-a\varepsilon^{\alpha})}{b^2(a^2-cx)}\right\}=V.$$

Hence, if V_P be the potential at P due to the charge q, we have

$$V_P\left\{1-\frac{ab}{a^2-ca\varepsilon^{-\alpha}}\right\}=V,$$

and thus by (8)

$$V_P=V\frac{\varepsilon^{\omega}}{\varepsilon^{\omega}-1}. \qquad (16)$$

By (1) we get

$$V_Q=V\frac{\varepsilon^{-\alpha}}{\varepsilon^{\omega}-1}. \qquad (17)$$

If we denote the potentials at P and Q due to $-q$ by V'_P and V'_Q, we obviously have

$$V_P+V'_P=V, \quad \text{and} \quad V_Q+V'_Q=0.$$

Thus
$$V'_P=-V\frac{1}{\varepsilon^{\omega}-1} \quad \text{and} \quad V'_Q=-V\frac{\varepsilon^{-\alpha}}{\varepsilon^{\omega}-1}. \qquad (18)$$

Note also that

$$\varepsilon^{\frac{\beta}{2}}=\left\{\frac{s-b)(s-c)}{bc}\right\}^{\frac{1}{2}}+\left\{\frac{s(a-s)}{bc}\right\}^{\frac{1}{2}} \qquad (19)$$

and
$$2\frac{\varepsilon^{\omega}}{\varepsilon^{\omega}-1}=1+\left\{\frac{s(s-c)}{(a-s)(s-b)}\right\}^{\frac{1}{2}}. \qquad (20)$$

For instance, in the condenser $K(5, 2, 2)$, $BP=b\varepsilon^{-\beta}=\frac{1}{2}$, $BQ=8$, and if the potential of the inner sphere be V, $V_P=2V$, $V_Q=V/2$, $V'_P=-V$, and $V'_Q=-V/2$. By (12) also the charges on the inner and outer conductors are $3{\cdot}892\ V$ and $-3{\cdot}892\ V$ approximately.

RELATIONS BETWEEN THE CAPACITIES OF SPHERICAL CONDENSERS.

There are an infinite number of algebraical relations connecting the capacities of various condensers with one another. For example, provided that c is greater than 2, we can show by (9) and (7) that

$$K(c^2-1, c, 1)-cK(c^2-1, 1, c)=\frac{c^2-1}{c^2-2}K(c^2-2, 1, 1). \qquad (21)$$

Putting $c=4$, we get

$$K(15, 4, 1)-4K(15, 1, 4)=\frac{15}{14}K(14, 1, 1) \text{ exactly}=1{\cdot}154 \text{ (approx.)}.$$

THE FORCE ON THE INNER SPHERE OF A SPHERICAL CONDENSER.

If W denote the electrostatic energy stored in the condenser we have

$$W = q^2/2K = \tfrac{1}{2}KV^2.$$

As K always increases as the distance c between the centres of the spheres increases, we see that if the charge q be maintained constant, W diminishes as c increases, but that if V be maintained constant, W increases as c increases. We have now to find out whether the mutual force tends to increase or diminish c. When the charge is maintained constant it follows from the conservation of energy that the work done by the mutual force F must be supplied from the electrostatic energy. Hence F must act so that W diminishes and consequently c increases. When c is zero the equilibrium is unstable. It follows also that when the P.D. V is maintained constant c increases, and the electrostatic energy increases. Since

$$W_2 - W_1 = \int_1^2 F\delta c,$$

we see that when the P.D. is kept constant the increase in the electrostatic energy $W_2 - W_1$ equals the concurrent work done $\int_1^2 F\delta c$. The battery which maintains the P.D. constant has therefore to supply an amount $2\,(W_2 - W_1)$ of energy.

If F denote the force tending to increase c we have $F = \frac{V^2}{2}\frac{\delta K}{\delta c}$, and thus by (10),

$$\begin{aligned}\frac{2F}{V^2} &= \Sigma\frac{n\cosh(\beta+n\omega)}{\sinh^2(\beta+n\omega)} \\ &-\frac{\cosh\alpha\cosh\beta}{\sinh\omega}\Sigma\frac{1}{\sinh(\beta+n\omega)} \\ &+\frac{\sinh\beta\cosh\alpha}{\sinh\omega}\Sigma\frac{\cosh(\beta+n\omega)}{\sinh^2(\beta+n\omega)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (22)\end{aligned}$$

the summation being taken from $n=1$ to $n=\infty$.

It will be noticed that so long as V is constant the attractive force is the same whether the geometrical constants are a, b and c, or ma, mb and mc.

By substituting q/K for V in (22) we see that the force when q remains constant varies inversely as K^2, when the relative values of a, b and c, and therefore α and β, remain constant. Hence, for a given value of q the force when the constants are a, b and c would be m^2 times the force when the constants are ma, mb and mc.

When $c^2/(a-b)^2$ can be neglected compared with unity, we get, by (13)

$$F = \frac{\delta W}{\delta c} = \frac{a^2b^2c}{(a^3-b^3)(a-b)^2}V^2 = \frac{c}{a^3-b^3}q^2. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23)$$

Hence in this case the attractive force varies as the distance between the centres.

When the distance x $(=a-b-c)$ between the spheres is very small, we get (*l.c. ante*)

$$F = \frac{ab}{4(a-b)x}V^2. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (24)$$

In this case, therefore, the force varies inversely as the distance between the spheres. If V be measured in electrostatic units (one electrostatic unit$=$300 volts), F will be given in dynes. By means of (15) we can readily express F in terms of q^2, but the formula is not so simple.

External Spheres.

Let us consider two conducting spheres (Fig. 3) with their centres at A and B. Let their radii be a and b, and let c be the distance between their centres. We shall denote their charges by Q_1 and Q_2, and their potentials by v_1 and v_2 respectively. We have, therefore,

$$Q_1=k_{11}v_1+k_{12}v_2 \text{ and } Q_2=k_{22}v_2+k_{12}v_1,$$

where k_{11} and k_{22} are the self capacity coefficients and k_{12} is the mutual capacity coefficient.

We shall first show how these coefficients can be computed in terms of a, b and c. To shorten the formulæ, we shall commence with the assumption that $v_1=1$ and $v_2=0$.

Let $f_1(x/a)$, where $AP=x$, denote the potential due to Q_1 at any point P on AB which is inside the sphere A. Similarly, let $f_2\{(c-x)/b\}$ be the potential due to Q_2 at the point P.

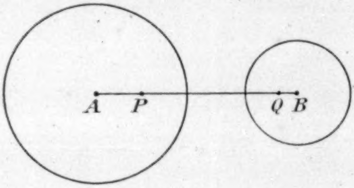

Fig. 3.

Then, if x lies between $+a$ and $-a$, we have

$$f_1(x/a)+\{b/(c-x)\}f_2\{b/(c-x)\}=1 \quad \text{. . . . . (25)}$$

Similarly, if x_1 lies between $c+b$ and $c-b$, we have

$$(a/x_1)f_1(a/x_1)+f_2\{(c-x_1)/b\}=0 \quad \text{. . . . . (26)}$$

Now, writing

$$\frac{c-x_1}{b}=\frac{b}{c-x}$$

so that

$$\frac{a}{x_1}=\frac{a(c-x)}{c^2-b^2-cx}$$

and eliminating the second function between (25) and (26), we get

$$f_1\left(\frac{x}{a}\right)=1+\frac{ab}{c^2-b^2-cx}f_1\left\{\frac{a(c-x)}{c^2-b^2-cx}\right\}. \quad \text{. . . . . (27)}$$

At A, where $x=0$, we obviously have

$$f_1(0)=\frac{Q_1}{a}=\frac{k_{11}}{a}.$$

Thus

$$\frac{k_{11}}{a}=1+\frac{ab}{c^2-b^2}f_1\left(\frac{ac}{c^2-b^2}\right). \quad \text{. . . . . . (28)}$$

Let us introduce, as before, functions α, β and r, where r is the radius of the sphere which cuts A and B orthogonally, and

$$\sinh \alpha = \frac{r}{a},\quad \sinh \beta = \frac{r}{b}.\quad \text{We write also}$$

$$\sinh \omega = \sinh (\alpha+\beta) = \frac{cr}{ab},\ \text{and}$$

$$c = a \cosh \alpha + b \cosh \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (29)$$

We find that (28) becomes

$$k_{11}\frac{\sinh \alpha}{r} = 1 + \frac{\sinh \alpha}{\sinh (\alpha+\omega)} f_1\left\{\frac{\sinh \omega}{\sinh (\alpha+\omega)}\right\}.$$

And by (27)
$$f_1\left\{\frac{\sinh \omega}{\sinh (\alpha+\omega)}\right\} = 1 + \frac{\sinh (\alpha+\omega)}{\sinh (\alpha+2\omega)} f_1\left\{\frac{\sinh 2\omega}{\sinh (\alpha+2\omega)}\right\}$$

Proceeding in this way, we get by substitution that

$$k_{11} = \sum_0^\infty \frac{r}{\sinh (\alpha+n\omega)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (30)$$

Hence, from symmetry

$$k_{22} = \sum_0^\infty \frac{r}{\sinh (\beta+n\omega)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

At the centre of the sphere B, the potential due to the charge Q_2 is Q_2/b, that is, k_{12}/b. Hence from (26), putting $x=c$, we get

$$f_1(0) = \frac{k_{12}}{b} = -\frac{a}{c} f_1\left(\frac{a}{c}\right)$$

Therefore
$$-k_{12} = \frac{r}{\sinh \omega} f_1\left(\frac{\sinh \beta}{\sinh \omega}\right)$$

And from (27)

$$f_1\left(\frac{\sinh \beta}{\sinh \omega}\right) = 1 + \frac{\sinh \omega}{\sinh 2\omega} f_1\left\{\frac{\sinh (\beta+\omega)}{\sinh 2\omega}\right\}$$

$$f_1\left\{\frac{\sinh (\beta+\omega)}{\sinh 2\omega}\right\} = 1 + \frac{\sinh 2\omega}{\sinh 3\omega} f_1\left\{\frac{\sinh (\beta+2\omega)}{\sinh 3\omega}\right\}$$

and so on. Hence

$$-k_{12} = \sum_1^\infty \frac{r}{\sinh n\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (32)$$

Comparing (30) with (10) we get at once

$$k_{11} = \frac{r}{\sinh \alpha} + K\left\{\frac{r}{\sinh \alpha},\ \frac{r}{\sinh (\alpha+\omega)},\ \frac{r \sinh \omega}{\sinh \alpha \sinh (\alpha+\omega)}\right\} \ . \qquad (33)$$

This may be written

$$k_{11}(a, b, c) = a + \frac{a^2}{c^2-b^2} K\left\{\frac{c^2-b^2}{a}, b, c\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (34)$$

Similarly we find that

$$-k_{12}(a, b, c)=\frac{ab}{c}+\frac{a^2b^2}{c(c^2-a^2-b^2)}K\left\{\frac{c^2-a^2-b^2}{ab}, 1, 1\right\} \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$k_{22}(b, a, c)=b+\frac{b^2}{c^2-a^2}K\left\{\frac{c^2-a^2}{b}, a, c\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

The computation of the capacity coefficients, therefore, reduces to calculating the capacities of spherical condensers and so we can calculate them by the formulæ given above.

NUMERICAL EXAMPLES.

Poisson (*l.c. ante*) considers the case for which $a=1$, $b=3$ and $c=5$. By (34) we get

$$k_{11}(1, 3, 5)=1+\frac{1}{5^2-3^2}K(16, 3, 5)$$

and by (12)

$$K(16, 3, 5)=3+\frac{3}{77}K(77, 16, 5)$$

If we only want a four-figure accuracy for k_{11} we can assume that

$$K(77, 16, 5)=K(77, 16, 0)=\frac{77.16}{77-16}$$

Hence

$$k_{11}=1+\frac{3}{16}+\frac{3}{61}=1\cdot2367$$

Similarly, by (35),

$$-k_{12}=\frac{3}{5}+\frac{3}{25}K(5, 1, 1)$$

$$=\frac{3}{5}+\frac{3}{25}+\frac{1}{200}K(24, 5, 1)$$

$$=\frac{3}{5}+\frac{3}{25}+\frac{3}{85}=0\cdot7516.$$

These numbers agree exactly with Poisson's calculations.

Similarly $k_{22}=3+\frac{3}{8}K(8, 1, 5)$

Proceeding as before, we get $k_{22}=3\cdot4719$.

As another example, take the system (1, 7, 10).

In this case

$$k_{11}=1+\frac{1}{51}K(51, 7, 10)= \ . \ . \ .$$

Thus

$$k_{11}=1\cdot160\ 112\ 4 \ . \ . \ .$$

$$k_{22}=7\cdot576\ 596\ 7 \ . \ . \ .$$

and

$$-k_{12}=0\cdot814\ 326\ 5 \ . \ . \ .$$

THE CAPACITY BETWEEN TWO SPHERES.

When the charges on the two spheres are equal but opposite in sign, the ratio of the positive charge to the potential difference between the spheres is called the capacity between the spheres. We shall denote it by $C\ (a, b, c)$ or by C. It is easy to show that

$$C(a, b, c) = \frac{k_{11}k_{22} - k^2_{12}}{k_{11} + k_{22} + 2k_{12}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (37)$$

As a rule C is computed by first calculating the values of k_{11}, k_{22} and k_{12} and then substituting in (37). When, however, the spheres are equal to one another $k_{11} = k_{22}$ and (37) becomes

$$C(a, a, c) = \tfrac{1}{2}(k_{11} - k_{12}) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (38)$$

It readily follows from (34) and (35) that

$$C(1, 1, c) = \tfrac{1}{2} + \frac{1}{2c}\ K(c, 1, 1) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (39)$$

For example, take the case of two spheres, the radius of each being 1 centimetre and the distance between thir centres being 4 cm. We have

$$\begin{aligned} C(1, 1, 4) &= \tfrac{1}{2} + \tfrac{1}{8}K(4, 1, 1) \\ &= \tfrac{1}{2} + \tfrac{1}{8} + \frac{1}{120}\ K(15, 4, 1) \\ &= \tfrac{1}{2} + \tfrac{1}{8} + \frac{1}{30} + \frac{1}{1680}\ K(56, 15, 1) \\ &= 0{\cdot}670\ 529\ 88\ .\ .\ . \end{aligned}$$

SPHERES CLOSE TOGETHER.

When the spheres are so close together that it is laborious to apply (34) and (35) we can use the formulæ.*

$$k_{11} = \frac{r}{\omega}\left\{ -\psi\left(\frac{a}{\omega}\right) - \log_e \frac{\omega}{2} + \frac{\omega^2}{72} + \frac{7\omega^4}{43200} + \ .\ . \ -\frac{\alpha\beta}{12} - \frac{7\alpha^2\beta^2}{1440} - \ .\ .\ . \right\} \quad . \quad (40)$$

and $$-k_{12} = \frac{r}{\omega}\left\{ \gamma - \log_e \frac{\omega}{2} + \frac{\omega^2}{72} + \frac{7\omega^4}{43200} + \ .\ .\ . \right\} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (41)$$

where $\psi(x)$ denotes the logarithmic derivate of the gamma function and γ is Euler's constant (0·577216). We have

$$\psi(x) + \gamma = -\frac{1}{x} + S_2x - S_3x^2 + S_4x^3 - \ .\ .\ .$$

where $S_m = \Sigma\frac{1}{n^m}$, the summation being taken from $n=1$ to $n=\infty$.

Let C_a and C_b denote the capacities of the two spheres when touching one another. In this case we have

$$C_a = k_{11} + k_{12} = -\frac{ab}{a+b}\left\{ \psi\left(\frac{b}{a+b}\right) + \gamma \right\} = a\left\{ 1 - \left(\frac{b}{a+b}\right)^2 S_2 + \left(\frac{b}{a+b}\right)^3 S_3 - \ .\ .\ .\ , \right\}$$

and $$C_b = b\frac{ab}{(a+b)^2}\left\{ S_2 + \frac{b}{a+b}S_3 + \frac{b^2}{(a+b)^2}S_4 + \ .\ .\ . \right\}\dagger$$

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* Russell, Alternating Currents, Vol. 1, p. 240.

† J. J. Thomson, Electricity and Magnetism, Chap. 5.

It is to be noticed that when $b^3/(a+b)^3$ can be neglected compared with unity, we can write $C_a+C_b=a$. When the spheres are touching the following formulæ are exact :—

$$a=b, \quad C_a=a\log 2=0{\cdot}6931a=C_b$$

$$a=2b, \quad C_a=\frac{a}{6}\left(3\log\ 3+\frac{\pi}{\sqrt{3}}\right);\ C_b=\frac{a}{6}\left(3\ \log 3-\frac{\pi}{\sqrt{3}}\right)$$

$$=0{\cdot}8516a \qquad\qquad =0{\cdot}2470a$$

$$a=3b, \quad C_a=\frac{a}{4}\left(3\log\ 2+\frac{\pi}{2}\right);\quad C_b=\frac{a}{4}\left(3\ \log 2-\frac{\pi}{2}\right)$$

$$=0{\cdot}9126a \qquad\qquad =0{\cdot}1272a$$

$$a=5b, \quad C_a=\frac{a}{12}(\log 2+3\ \log 6+\pi\sqrt{3}),\ C_b=\frac{a}{12}\ (\log 2+3\log 6-\pi\sqrt{3})$$

$$=0{\cdot}9592a \qquad\qquad =0{\cdot}0523a$$

When x/a and x/b can be neglected compared with unity, where $x=c-a-b$, we can show that

$$C_a+C_b=-\frac{ab}{a+b}\left\{\psi\left(\frac{a}{a+b}\right)+\psi\left(\frac{b}{a+b}\right)+2\gamma\right\} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (42)$$

and

$$C=\frac{r}{\omega}\log_e\frac{2}{\omega}+\frac{r^2}{\omega^2}\,\frac{\psi\left(\frac{\alpha}{\omega}\right)\psi\left(\frac{\beta}{\omega}\right)-\gamma^2}{C_a+C_b} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (43)$$

$$=\frac{ab}{2(a+b)}\log_e\frac{2ab}{(a+b)x}+\frac{a^2b^2}{(a+b)^2}\cdot\frac{\psi\left(\frac{a}{a+b}\right)\psi\left(\frac{b}{a+b}\right)-\gamma^2}{C_a+C_b} \quad . \ . \quad (44)$$

where C is the capacity between the spheres.

For example, when $a=b$, we have, since $\psi(1/2)=-\gamma-2\ \log\ 2$ and $C_a+C_b=2a\log 2$

$$C=\frac{a}{4}\log\frac{a}{x}+\frac{a}{4}(2\gamma+2\log 2)$$

$$=\frac{a}{4}\left\{\log\frac{a}{x}+2{\cdot}540\,726\ .\ .\right\} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (45)$$

When b/a is very small (44) becomes—

$$C=\frac{b}{2}\log\frac{2b}{x}+\gamma b. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (46)$$

This formula gives the capacity between a sphere of radius b and an infinite plane.

Hence, since $-k_{12}=C$ in this case, we get by (35)

$$K\left\{b,\frac{b^2}{2(b+x)},\frac{b^2}{2(b+x)}\right\}=\frac{b}{2}\log\frac{2b}{x}+(\gamma-1)b, \quad . \ . \ . \ . \ . \ . \quad (47)$$

when x is very small.

THE POTENTIALS AT THE INVERSE POINTS DUE TO Q_1 AND Q_2 RESPECTIVELY.

Looking back at (27) we see that the potential at the centre of the sphere A, where $x=o$, equals $1+$the potential at a point $x=a_1$, multiplied by $\sinh \alpha/\sinh(\alpha+\omega)$. Similarly the potential at the point $x=a_{n-1}$ equals $1+$the potential at the point $x=a_n$ multiplied by $\sinh(\alpha+\overline{n-1}\,\omega)/\sinh(\alpha+n\omega)$. We also see that $a_n=a\sinh n\omega/\sinh(\alpha+n\omega)$.

Hence, when n is very great

$$a_{n-1}=a_n=a\varepsilon^{-\alpha}=AP,$$

where P is the inverse point of the two spheres which is inside A. Hence, if V_P be the potential at this point due to Q_1, we have

$$V_P=1+\varepsilon^{-\omega}V_P, \text{ and so } V_P=\varepsilon^{\omega}/(\varepsilon^{\omega}-1).$$

Hence, also $V_Q=\varepsilon^{\beta}/(\varepsilon^{\omega}-1)$. If V'_P, V'_Q be the potentials due to Q_2, we see that

$$V'_P=-1/(\varepsilon^{\omega}-1) \text{ and } V'_Q=-\varepsilon^{\beta}/(\varepsilon^{\omega}-1).$$

We see, therefore, by superposition, that when the potentials of the spheres are V_1 and V_2, we have

$$(\varepsilon^{\omega}-1)V_P=\varepsilon^{\alpha}(V_1\varepsilon^{\beta}-V_2)\,;\ (\varepsilon^{\omega}-1)V_Q=V_1\varepsilon^{\beta}-V_2$$

$$(\varepsilon^{\omega}-1)V'_P=V_2\varepsilon^{\alpha}-V_1\,;\ (\varepsilon^{\omega}-1)V'_Q=\varepsilon^{\beta}(V_2\varepsilon^{\alpha}-V_1)$$

Hence, if $\quad V_1/V_2=\varepsilon^{-\beta},\ V_P=V_Q=o.$; and $V'_P=V_1,\ V'_Q=V_2$.

ELECTRICAL IMAGES OF SPHERES.

If the surfaces of two spheres A and B are reflecting there will be a series of images formed inside A, and a similar series formed inside B. From this optical analogy Kelvin called the inverse spheres $A_1, A_2, \ldots$ and $B_1, B_2, \ldots$ where A and B are the centres of inversion (Fig. 4), the electrical images of the two spheres. It has to be noted that these electrical images do not coincide with the optical images. A_1 is the electrical image of B formed by A, and B_1 is the image of A formed by B. Similarly A_2 is the image of B_1 formed by A, and B_2 is the image of A_1 formed by B. In general A_n is the image of B_{n-1}, and B_n is the image of A_{n-1}.

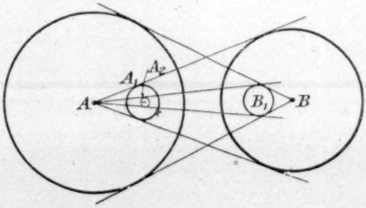

FIG. 4.

It is not difficult to show that A_2 is the image (inverse sphere) of A formed by A_1, and in general that A_n is the image of A_{n-2} formed by A_{n-1}. If R_n be the radius of A_n and a_n the distance of its centre from A, it can be shown that

$$R_n=r/\sinh(\alpha+n\omega) \text{ and } a_n=a\sinh n\omega/\sinh(\alpha+n\omega).$$

Hence, when n is very large $a_n = a\varepsilon^{-a}$ and the nth image practically coincides with the inverse point P.

By formula (33) we get

$$k_{11}(a, b, c) = a + K(a, R_1, a_1),$$

and by (12)
$$K(a, R_1, a_1) = R_1 + K(R_1, R_2, a_2 - a_1),$$

and
$$K(R_1, R_2, a_2 - a_1) = R_2 + K(R_2, R_3, a_3 - a_2), \text{ etc.}$$

Thus
$$k_{11}(a, b, c) = a + R_1 + R_2 + \dots \qquad (48)$$

which agrees with (30).

Interpreting the equations we see that k_{11} equals the radius of A, together with the capacity of the spherical condenser formed by A and A_1. The capacity of this condenser equals the radius of A_1 plus the capacity of the condenser formed by A_1 and A_2, and so on. Hence the images divide the sphere A into an infinite number of spherical condensers, the capacity between any consecutive pair A_{n-1} and A_n being equal to the radius of A_n plus the capacity between the next pair A_n and A_{n+1}.

In the corresponding case of two cylinders it will be found that the capacities between the consecutive cylindrical images (inverse cylinders) are all equal to one another, and they also equal the capacity between the two cylinders.

It is interesting to note that the centres of the sphere A and of the images A_1, A_2, . . . coincide with the points where we have to place Kelvin's charges q_0, q_1, q_2, . . . so that these charges may produce a field external to the sphere which is identical with the field produced by the charge on A when its potential is unity, and that of B is zero. Kelvin's charges* also are given by $q_0 = a$, $q_1 = R_1$, $q_2 = R_2$, . . .

Both Poisson's and Kelvin's methods give the magnitudes and positions of the electrical image spheres.

ATTRACTIONS AND REPULSIONS OF TWO ELECTRIFIED SPHERES.

Let W denote the electrostatic energy of the system. Then

$$W = \tfrac{1}{2}k_{11}v_1^2 + \tfrac{1}{2}k_{22}v_2^2 + k_{12}v_1v_2 \qquad (49)$$

This can also be written in the form

$$W = \tfrac{1}{2}p_{11}q_1^2 + \tfrac{1}{2}p_{22}q_2^2 + p_{12}q_1q_2 \qquad (50)$$

where
$$p_{11} = k_{22}/\Delta^2,\ p_{22} = k_{11}/\Delta^2,\ p_{12} = -k_{12}/\Delta^2 \text{ and } \Delta^2 = k_{11}k_{22} - k_{12}^2 \qquad (51)$$

If we suppose the spheres to move, the rate at which the potential energy increases is given by

$$\frac{\delta W}{\delta c} = \frac{1}{2}\frac{\delta k_{11}}{\delta c}v_1^2 + \frac{1}{2}\frac{\delta k_{22}}{\delta c}\cdot v_2^2 + \frac{\delta k_{12}}{\delta c}v_1v_2 = nv_1v_2 - \tfrac{1}{2}lv_1^2 - \tfrac{1}{2}mv_2^2 \qquad (52)$$

Where $l = -\delta k_{11}/\delta c$, $m = -\delta k_{22}/\delta c$ and $n = \delta k_{12}/\delta c$, and l, m and n are positive quantities. The value of $\delta W/\delta c$ gives the force acting between the two spheres.

Similarly we find from (50) that when q_1 and q_2 are kept constant,

$$\frac{\delta W}{\delta c} = \frac{1}{2}\frac{\delta p_{11}}{\delta c}q_1^2 + \frac{1}{2}\frac{\delta p_{22}}{\delta c}q_2^2 + \frac{\delta p_{12}}{\delta c}q_1q_2.$$

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* Russell, Alternating Currents, Vol. 1, p. 238.

Substituting for p_{11}, p_{22} and p_{12} their values from (51), and writing for q_1 and q_2, $k_{11}v_1+k_{12}v_2$ and $k_{22}v_2+k_{12}v_1$, we readily find that

$$\frac{\delta W}{\delta c}=-(nv_1v_2-\tfrac{1}{2}lv_1^2-\tfrac{1}{2}mv_2^2)\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (53)$$

Hence $$\frac{\delta W}{\delta c}\ (v_1,\ v_2 \text{ constant})=-\frac{\delta W}{\delta c}\ (q_1,\ q_2 \text{ constant})\quad . \quad . \quad . \quad . \qquad (54)$$

By the conservation of energy we see that when q_1 and q_2 are constant, the work done by the mutual force between them in moving the spheres must be taken from the electrostatic energy. Hence the spheres move so as to diminish the electrostatic energy. It follows also from (54) that when their potentials are maintained constant the spheres move so that the electrostatic energy is increased.

We see from (52) and (53) that $\delta W/\delta c$ vanishes when

$$\frac{v_1}{v_2}=\frac{n}{l}\pm\frac{\sqrt{n^2-lm}}{l}$$

and $$\frac{q_1}{q_2}=\frac{k_{11}(v_1/v_2)+k_{12}}{k_{22}+k_{12}(v_1/v_2)}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (55)$$

When for a given value of c either of these conditions is satisfied, the electrostatic energy at this distance is a maximum when the charges are kept constant, and a minimum when the potentials are kept constant. In order that (55) be satisfied it is necessary that v_1 and v_2 have the same sign. If they have opposite signs the electrostatic energy continually diminishes as the spheres approach one another when the charges are kept constant and continually increases when the potentials are maintained constant.

When the charges and therefore the potentials are of unlike sign, the mutual force is always attractive. When, however, they are of like sign the force is in general attractive when they are close together and repulsive when they are far apart. In an intermediate position determined by (55) the mutual force is zero.

SPHERES CLOSE TOGETHER.

When the spheres are at potentials v_1 and v_2, and the distance x between them is small compared with the radius of either, the force F between them is given by

$$\begin{aligned}F=&-\frac{(v_1-v_2)^2ab}{4x(a+b)}\left\{1+\frac{(a^2+b^2-ab)x}{3ab(a+b)}\log\frac{(a+b)x}{2ab}+\frac{(2a^2+2b^2+ab)x}{18ab(a+b)}\right\}\\ &-\frac{a^2+b^2-ab}{6(a+b)^2}\left\{\psi\left(\frac{b}{a+b}\right)v_1^2+\psi\left(\frac{a}{a+b}\right)v_2^2+2\gamma v_1v_2\right\}\\ &-\frac{ab(a-b)}{6(a+b)^3}\left\{\psi'\left(\frac{b}{a+b}\right)v_1^2-\psi'\left(\frac{a}{a+b}\right)v_2^2\right\}\\ &-\frac{ab}{6(a+b)^2}v_1v_2\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (56)\end{aligned}$$

When F is negative, the force is attractive, and when F is positive it is repulsive. It will be seen that when x is very small we only have repulsion when v_1 is very nearly equal to v_2. In all other cases the force is attractive.

If $v_1=v_2=v$, the force is repulsive. When $a=b$, $F=0{\cdot}07386v^2$; when $a=3b$, $F=0{\cdot}0430v^2$; and when a is very large compared with b, the force vanishes.

When $a=b$, (56) reduces to

$$F=-\frac{a(v_1-v_2)^2}{8x}\left\{1-\frac{12\gamma+1}{36}\cdot\frac{x}{a}-\frac{1}{6}\frac{x}{a}\log\frac{a}{x}\right\}+\frac{\log 2-\frac{1}{4}}{12}(v_1^2+v_2^2) \quad . \quad . \quad (57)^*$$

When $v_1=v_2=v$, we get

$$F=0{\cdot}073858v^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)\dagger$$

a formula that was given by Kelvin.

We see, however, that when x is very small the slightest disparity between the values of v_1 and v_2 will make the force attractive.

EQUAL SPHERES.

When the distance between two equal spheres is greater than the radius of either, the force F between them is given very approximately by

$$F=\left\{1+\frac{14}{y^5}+\frac{54}{y^8}\right\}\frac{q_1q_2}{c^2}-\left\{\frac{2}{y^3}+\frac{3}{y^5}+\frac{4}{y^7}+\frac{25}{y^3(y^2-1)^3}+\frac{75}{y^5(y^2-1)^3}\right\}\frac{q_1^2+q_2^2}{c^2} \quad . \quad . \quad (59)\ddagger$$

where $y=c/a$.

It is also given by

$$F=\left\{\frac{1}{y^2-4}-\frac{1}{y^4}-\frac{1}{y^6}-\frac{1}{y^8-4y^4}\right\}v_1v_2-\left\{\frac{1}{y(y^2-4)}-\frac{1}{y(y^3-2y)^2}\right\}(v_1^2+v_2^2) \quad . \quad (60)\ddagger$$

When F is negative the force is attractive, and when F is positive it is repulsive. For example, when $c=4a$, the formulæ become

$$F=1{\cdot}004\frac{q_1q_2}{c^2}-0{\cdot}03456\frac{(q_1^2+q_2^2)}{c^2}$$

and

$$F=0{\cdot}0792v_1v_2-0{\cdot}0208(v_1^2+v_2^2).$$

In this case F vanishes when $v_1/v_2=0{\cdot}283$, which is the same condition as $q_1/q_2=0{\cdot}0344$. When c/a is large, $F=\frac{q_1q_2}{c^2}-\frac{2a^3}{c^5}(q_1^2+q_2^2)$, and $F=\frac{a^2}{c^2}v_1v_2-\frac{a^3}{c^3}(v_1^2+v_2^2)$.

In using the formulæ given above it has to be remembered that when the pressure v and the charge q are given in electrostatic units the force F is given in dynes.

The electrostatic unit of pressure equals 300 volts and the electrostatic unit of charge (quantity) equals $1/(3\times10)$ coulombs.

ANALOGOUS PROBLEMS.

The formulæ given above can be usefully applied in the theory of heat, in electric and magnetic conduction, and in the theory of hydrodynamics. Consider, for instance, two spheres deeply embedded in a thermally homogeneous medium and maintained at constant temperatures θ_1 and θ_2. Let Q_1 and Q_2 denote the flow of heat per second from each sphere. When the flow has become steady, we get

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* Proc. Roy. Soc., Vol. 82, p. 531.

† Reprint, p. 83.

‡ Journal Inst. of El. Eng., Vol. 48, p. 257

$$Q_1=\frac{4\pi}{\sigma}(k_{11}\theta_1+k_{12}\theta_2) \text{ and } Q_2=\frac{4\pi}{\sigma}(k_{22}\theta_2+k_{12}\theta_1),$$

where σ is the thermal resistivity, and the values of k_{11}, k_{22} and k_{12} can be computed from the formulæ given above. In the particular case when $Q_1=-Q_2=Q$, we have

$$\frac{Q}{\theta_1-\theta_2}=\frac{4\pi}{\sigma}K \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (61)$$

where K is the electrostatic capacity between the two spheres.

Similarly, in electric conduction, we have, with the usual notation

$$I_1=\frac{4\pi}{\rho}(k_{11}V_1+k_{12}V_2)\,;\; I_2=\frac{4\pi}{\rho}(k_{22}V_1+k_{12}V_1)$$

and when $I_1=-I_2=I$ we get

$$\frac{I}{V_1-V_2}=\frac{4\pi}{\rho}K \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (62)^*$$

where ρ is the electric resistivity.

We see that when V_1 and V_2 are maintained constant and are of the same sign, the power supplied to the medium in which the spheres are immersed has, in general, a minimum value for a definite value of c.

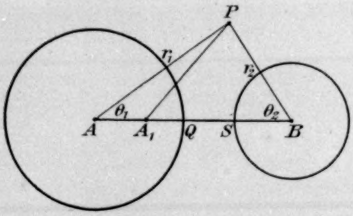

FIG. 5.

THE POTENTIAL AT ANY EXTERNAL POINT.

Let the point P (Fig. 5) be at distances r_1 and r_2 from A and B and let the angles PAB, PBA be θ_1 and θ_2 respectively. Let the potentials of the spheres be V_1 and 0 respectively. So far as the electric field outside the two spheres is concerned, we can replace the charges on the spheres by point charges q_0, q_1, q_2 . . . at A, A_1, A_2 . . . and q'_1, q'_2 . . . at B_1, B_2 . . . where

$$q_n=aV_1\frac{\sinh\alpha}{\sinh(n\omega+\alpha)},\; q'_n=-aV_1\frac{\sinh\alpha}{\sinh n\omega}$$

$$AA_n=a_n=a\frac{\sinh n\omega}{\sinh(n\omega+\alpha)} \text{ and } BB_n=b_n=b\frac{\sinh\{(n-1)\omega+\alpha\}}{\sinh n\omega}$$

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* Jeans, Electricity and Magnetism, p. 351.

Thus the potential V at the point P due to the given charges is got by

$$
\begin{aligned}
V = {} & \frac{q_0}{r_1} + \frac{q_1}{(r_1^2 - 2r_1a_1\cos\theta_1 + a_1^2)^{\frac{1}{2}}} \\
& + \frac{q_2}{(r_1^2 - 2r_1a_2\cos\theta_1 + a_2^2)^{\frac{1}{2}}} + \dots \\
& + \frac{q'_1}{(r_2^2 - 2r_2b_1\cos\theta_2 + b_1^2)} + \dots \\
= {} & \frac{k_{11}V_1}{r_1} + \frac{k_{12}V_1}{r_2} \\
& + \frac{a^2V_1P_1(\cos\theta_1)}{r_1^2} \sum_1^\infty \frac{\sinh\alpha\sinh n\omega}{\sinh^2(n\omega+\alpha)} \\
& + \frac{a^3V_1P_2(\cos\theta_1)}{r_1^3} \sum_1^\infty \frac{\sinh\alpha\sinh^2 n\omega}{\sinh^3(n\omega+\alpha)} + \dots \\
& - \frac{b^2V_1P_1(\cos\theta_2)}{r_2^2} \sum_1^\infty \frac{\sinh\alpha\sinh\overline{(n-1\omega+\alpha)}}{\sinh^2 n\omega} - \dots \qquad (61)
\end{aligned}
$$

where $P_1(\cos\theta_1)$, $P_2(\cos\theta_2)$ . . . are zonal harmonics.

If the second sphere be at potential V_2, we have to add to the series (61) a similar series which can be written down by symmetry at once.

The Maximum Value of the Electric Stress.

It is important in practice to know the maximum values of the electric stress at points on the surface of the sphere. These values obviously occur at the points Q and S (Fig. 5). Denoting the maximum value of the stress at Q by $R_{\text{max.}}$, we get*

$$
\begin{aligned}
R_{\text{max.}} = \frac{\cosh^2(\alpha/2)}{a\sinh(\alpha/2)} \Bigg[& V_1\left\{\frac{\sinh(\alpha/2)}{\cosh^2(\alpha/2)} + \frac{\sinh(\alpha/2+\omega)}{\cosh^2(\alpha/2+\omega)} + \dots\right\} \\
& - V_2\left\{\frac{\sinh(\omega-\alpha/2)}{\cosh^2(\omega-\alpha/2)} + \frac{\sinh(2\omega-\alpha/2)}{\cosh^2(2\omega-\alpha/2)} + \dots\right\}\Bigg] \quad . \quad . \quad (62)
\end{aligned}
$$

The value of the stress at S is got by writing V_2 for V_1, b for a and β for α in (62).

When the spheres are equal, the maximum value of the electric stress between them is given by

$$R_{\text{max.}} = \frac{V_1 - V_2}{x} f_1 + \frac{2V_2}{x}(f_1 - f) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

where

$$f = \sinh\alpha\cosh(\alpha/2)\left\{\frac{\sinh(\alpha/2)}{\cosh^2(\alpha/2)} + \frac{\sinh(3\alpha/2)}{\cosh^2(3\alpha/2)} + \dots\right\}$$

$$f_1 = 2\sinh\alpha\cosh(\alpha/2)\left\{\frac{\sinh(\alpha/2)}{\cosh^2(\alpha/2)} + \frac{\sinh(5\alpha/2)}{\cosh^2(5\alpha/2)} + \dots\right\}$$

and $\cosh\alpha = 1 + x/(2a)$.

Simplified formulæ for computing f and f_1 are given in the Journ. of the Inst. of El. Engin., Vol. LVII., p. 223.

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* **Russell, Alternating Currents, Vol. 1, p. 253.**

It is also shown that for a given pair of spherical electrodes the value of $R_{\text{max.}}$ at which a disruptive discharge takes place is constant for distances between the spheres varying from about 1 mm. up to a distance equal to three times the diameter of either electrode. The author has shown that the value of $R_{\text{max.}}$ at 25°C. and 76 cm. pressure for electrodes the diameters of which vary in size from half a centimetre to 50 cm. is given by

$$R_{\text{max.}} = 27 \cdot 4 + \frac{14 \cdot 1}{\sqrt{a}} \quad \ldots\ldots\ldots\ldots \quad (64)$$

when the potentials of the electrodes are equal and opposite at the instant of discharge; a is the radius of an electrode in centimetres and $R_{\text{max.}}$ is given in kilovolts per centimetre. The maximum inaccuracy of (64) over the range mentioned is approximately 1 per cent.

When $V_1 = -V_2 = E/2$, formula (63) becomes

$$R_{\text{max.}} = \frac{E}{x} f$$

and thus

$$E = \left(27 \cdot 4 + \frac{14 \cdot 1}{\sqrt{a}}\right) \frac{x}{f} \quad \ldots\ldots\ldots\ldots \quad (65)$$

If we fix the value of x it is easy to see from this formula that in certain cases E has a maximum value for a given value of a. For instance, if x is 1 cm., then E has the greatest value when $a = 2 \cdot 53$ cm., that is, approximately 1 in. Taking the experimental values of E from the Paper referred to above, we get :—

Radius of each electrode in cm.	0·25	0·5	1	3·125	6·25	12·5	25
Disruptive pressure in kilovolts	20·8	26·7	30·5	31·9	31·5	30·6	29·8

The results of the first three cases were given by A. Heydweiller* and the last four are taken from the standardisation Rules of the American Institution of Electrical Engineers.

THE MAXIMUM POSSIBLE ATTRACTION BETWEEN SPHERES IN AIR.

Taking Heydweiller's results for spheres 1 cm. in diameter, we readily find that the attractions between the electrodes at the instant of discharge are given by the following table :—

x in cm.	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
E in kilovolts...	8·37	11·37	14·55	17·31	19·92	22·05	24·09	25·59	27·00
F in dynes	193·1	219·0	250·0	264·6	274·5	271·7	268·7	256·3	244·4

The maximum attraction occurs when x equals $1 \cdot 2a$ approximately.

Let a pair of spherical electrodes each have a radius a and be at a distance x apart. Let F_1 be the attraction between them when the disruptive voltage is E_1. The potentials are supposed equal and opposite at the instant of discharge. Consider also a pair of electrodes the radii of which are each equal to na, and let the distance between them be nx. Let F_2 be the force between them at the instant of the disruptive discharge. Then $F_1=\alpha E_1^2$ and $E_1=(27\cdot4+14\cdot1/\sqrt{a})(x/f)$, where α and f depend only on the ratio x/a. It readily follows that

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* **Entladungspotentiale Ann. der Phys. und Chemie, Vol. 48, p. 235.**

$$\frac{F_2}{F_1}=\left\{n\frac{27\cdot4+14\cdot1/\sqrt{na}}{27\cdot4+14\cdot1/\sqrt{a}}\right\}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Hence this ratio is independent of x and so if x' be the distance between the first pair of electrodes which gives to F_1 a maximum value, F_2 will have a maximum value when the distance between the spheres in the second case is nx'.

We saw above that when $a=\frac{1}{2}$ cm. approx. F has its maximum value when $x'=0\cdot6$ cm. Let us take $n=10$, then if F_2 be the attraction for electrodes of 5 cm. radius at the instant of discharge $F_2=50\cdot8F_1$, and so the maximum value of F_2 is nearly 13940 dynes.

An electrostatic balance therefore having spherical electrodes each ten centimetres in diameter and 5 centimetres apart would be very suitable for measuring potential differences up to 100,000 volts.

DISCUSSION.

Sir OLIVER LODGE (communicated): I feel sure that Dr. Russell's Paper is a valuable contribution to knowledge; and I admire the way in which he has tackled the problem of the two electrified spheres. An electrostatic balance with spherical electrodes, instead of flat plates, would have many advantages, provided always that the surroundings did not exercise too much disturbance, and provided dust particles or irregularities on the spheres did not promote an undesired kind of brush discharge accompanied by air currents.

Mr. T. SMITH congratulated the President on a valuable piece of work. Having cleared up a subject at which physicists had been working for so long, the author would have been entitled to employ as his unit of length a characteristic wave-length of the most widely distributed element; which, as Clerk Maxwell had remarked, was the only unit fitted for those investigators whose work might outlast the earth! In regard to the use of "image" in an optical sense, in the course of the Paper, this hardly seemed the correct word to employ.

Dr. D. OWEN said that in investigating the attraction between conductors in the case where the disruption point was being reached, it had to be remembered that ions were present, which would interfere with the distribution of the electric field. This lent some uncertainty to practical methods based on this method for measuring voltages; the time factor entered into this question. The same remark applied to the estimation of voltage in terms of the distance between spheres at which sparking just occurred.

The AUTHOR, in reply, said that it might be more correct to speak of the "inverse" of one sphere with respect to the other. The equation (64) to which Dr. Owen referred was an empirical one. Possibly the presence of ions might account for this formula, as theoretically the value of R_{max} should be constant.

III. *The Homographic Treatment of the Symmetrical Optical Instrument.* By G. Temple, *Birkbeck College.*

Received May 23, 1922.
(Communicated by Dr. D. Owen.)

ABSTRACT.

This Paper gives a homographic treatment of the perfect, symmetrical optical instrument; discusses its focal planes, principal planes and nodal points; and shows that these latter are sufficient to define uniquely the optical properties of the instrument.

The object of this Paper is to replace the rather cumbrous algebra of Abbe's* theory of the symmetrical optical instrument by the more elegant methods of modern geometry.

Definition of the Instrument.

The perfect, symmetrical, optical instrument is defined by two conditions—the condition for stigmatism, and the condition for axial symmetry.

1. *Stigmatism.*—The effect of the instrument must be to form a point image of a point object. Now, to any ray through the point object, there corresponds a ray through the point image. Hence, rays coplanar in the object space correspond to rays coplanar in the image space. Hence the instrument sets up a homographic, or (1, 1) correspondence between the points, lines and planes of the object and image spaces.†

2. *Axial Symmetry.*—Associated with the instrument there must be a certain line—the optic axis—such that to rays in a plane π containing the axis, there correspond rays in the same plane; and to lines perpendicular to the axis, there correspond lines also perpendicular to the axis. Hence to points on the axis there correspond points on the axis, so that the axis is a self-corresponding or "double" line. Also since all lines perpendicular to the axis in the plane π meet in the same point W at infinity, W is a double point. Now the instrument sets up a (1, 1) correspondence between the points or the lines of the plane π. Therefore the plane π contains three double points, the vertices of a triangle whose sides are three double lines. Hence the other two double points lie on the axis and are either both real or both imaginary.

Focal Planes.

The line at infinity, w, of any plane perpendicular to the axis contains all the double points, W, and is therefore itself a double line. Now the focal planes φ, φ, of the image and object space are defined as the planes corresponding to the plane at infinity ω regarded as belonging to the object or image space respectively. But the double line w lies on ω. Therefore it also lies on φ, φ'. Hence the focal planes φ, φ' are perpendicular to the axis.

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* Drude: "The Theory of Optics," pp. 14-28 (1902). v. Rohr: "The Formation of Images in Optical Instruments," pp. 83-124 (1920). Gapski: "Theorie des Optischen Instrumente pack Abbe" (1894).

† Jessopp: "The Line Complex."

PRINCIPAL PLANES AND NODAL POINTS.

These are introduced so as to exhibit their duality.

Principal Planes.	*Nodal Points.*
Consider a pair of corresponding planes η, η' whose intersection is the line w.	Consider a pair of corresponding points N, N' whose join is the axis.
There is a (1, 1) correspondence between the lines on η, η' and the common line w is self-corresponding.	There is a (1, 1) correspondence between the lines through N, N', and the common line the axis is self-corresponding.
Hence corresponding lines on η, η' are in perspective.	Hence corresponding lines through N, N' are in perspective.
When the centre of perspective is at infinity η, η' are principal planes.	When the axis of perspective is at infinity N, N' are nodal points.
Then the join of corresponding points of η, η' meets the axis at infinity.	Then the intersection of corresponding lines through N, N' is at infinity.
If μ, μ' be another pair of principal planes, any line l meeting the axis at infinity, is met by η, η', μ, μ' in two pairs of corresponding points. Hence l is a double line.	If P, P' is another pair of nodal points any point L at infinity is joined to N, N', P, P' by two pairs of corresponding lines. Hence L is a double point.

But this is only true for telescopic systems of unit magnifying power. Hence in general there is only one pair of principal planes or nodal points.

CONSTRUCTION OF IMAGE.

Let A, A' be corresponding points ; and let AN, $A'N'$ meet φ, φ' in F, F', and η, η' in H, H'.

If Ω' is the point at infinity on $A'N'$, since A, A' ; N, N' ; F, Ω' ; H, H' are pairs of corresponding points, $A'N'$ is parallel to AN and $[ANFH]=[A'N'\Omega'H']$. (using the notation of Mobius for cross-ratios). Hence, φ, φ' ; η, η', N, N' suffice to determine A' corresponding to A.

MIRROR SYSTEMS.

The distinction between mirror systems and lens systems is that whereas in a lens system, a collineation is effected between the points, lines and planes of two different spaces : in a mirror system, the collineation is effected between the points, lines and planes of the same space.

Hence in a mirror system, the corresponding pairs of points on the axis form an involution, of which the pole of the mirror P is evidently one double point. Hence the other double point Q is also real, and the centre of the involution (*i.e.*, the point corresponding to the point at infinity, or the focus) is midway between P and Q ; and any two corresponding points are harmonically separated by P and Q. The principal points obviously coincide at P, and the nodal points at Q.

DISCUSSION.

Mr. T. **Smith** congratulated the author on the brevity with which his method covered ground that is usually treated at great length, especially in German textbooks. The importance of the perfect instrument lies not in its realisation in practice, which is unattainable except for the plane mirror, but in that it is the ideal to which actual instruments must approximate. In practice perfection is attainable with respect to a single plane object, but perfection with regard to more than one plane implies complete perfection. Clerk Maxwell employed a method independent of constructional details and applied it to the practical case where the principal and nodal points coincide; this case might also be of interest to the author. Clerk Maxwell further introduced one physical property, *viz.*, the ratio of the velocities of light in the object and image spaces, and deduced hence the properties of the focal lengths. In practical cases there is a (1, 1) correspondence between rays, but not between points, and this aspect of the matter has been elegantly treated by the method of characteristic functions, notably by Hamilton.

Mr. F. J. W. **Whipple** said that the Paper was extremely concise, and might be amplified with advantage. In presenting it the author had stated that correspondence between points implies correspondence between planes. Would he give the proof of this theorem?

The **Author**, in reply to Mr. Smith, said it had occurred to him that the consequences of a (1, 1) correspondence between rays might be deduced by means of the geometry of the line complex. The proof asked for by Mr. Whipple was as follows. Let A, B, C, D be four coplanar points in the object space and A', B', C', D' four corresponding points in the image space. Then the latter four points are also coplanar. For let AD meet BC in X. Since X lies on both AD and BC, the corresponding point X' must lie on both $A'D'$ and $B'C'$. Hence these must be coplanar.

ADDENDUM BY THE AUTHOR.

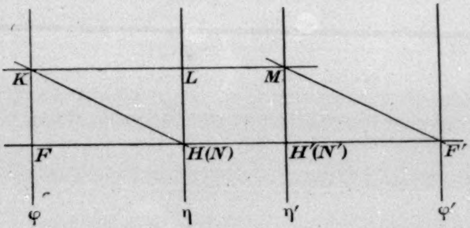
The following example bears upon Mr. T. Smith's remark as to focal lengths :—

Let the axis of the instrument meet the focal planes φ, φ' in F, F'; and the principal planes η, η' in H, H'.

Then the geometrical expression of the fact that the refractive indices of the material of those parts of the object and image spaces outside the instrument, are the same, is that H, H' respectively coincide with the nodal points N, N'.

Let K be any point on φ. Draw KLM parallel to the axis to meet η, η' in L, M.

Now F' corresponds to the point at infinity on the axis, which is the same as the point at infinity on KLM.



Hence the ray KL passes through F', and since L,M are corresponding points, it also passes through M.

Hence the ray KL corresponds to the ray MF'.

Hence the point K corresponds to the point at infinity on MF'.

Join KH, *i.e.*, KN. Then K also lies at infinity on a line through H' (N') parallel to KH (N). Therefore KH, MF' are parallel.

But $KF = MH'$.

Therefore $FH = H'F'$, *i.e.*, the focal lengths are numerically equal.

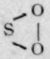
IV. *On the Structure of the Sulphur Dioxide Molecule.*

By A. O. RANKINE, *D.Sc., Professor of Physics, and* C. J. SMITH, *M.Sc., A.R.C.S., D.I.C., Research Student, Imperial College of Science and Technology.*

RECEIVED JULY 15, 1922.

ABSTRACT.

This Paper deals with the question of the arrangement of the atoms in the molecule of gaseous sulphur dioxide and the manner in which they are linked together. Two suggested arrangements are considered in detail, and evidence is brought forward, based on the determination of molecular dimensions as derived from viscosity measurements, which suggests that the more probable arrangement is that in which chemical bonds exist between *each* pair of the three atoms, corresponding to the formula



ACCORDING to the views put forward by Langmuir* sulphur dioxide is one of the numerous examples of chemical compounds in which the mode of linking together of the atoms is by electron sharing. The completed molecule is conceived to be an arrangement in which each oxygen atom, by holding electrons in common with the other constituents, has attained the configuration appropriate to a neon atom; and the sulphur atom, by a like process, has simulated an atom of argon. The extent of electron sharing is determined by the total deficiency of electrons as compared with the corresponding inert atoms. Thus there is a total deficiency of six, two on account of each oxygen atom, the atomic number of which is 8, as compared with the 10 of neon, and two on account of the sulphur atom, the atomic number being in this case 16, as compared with the 18 of argon. This requires that in the sulphur dioxide molecule six electrons altogether must be held in common by the constituent atoms taken in pairs; and we have to inquire into the probable distribution of the electron sharing, since it can take place in more ways than one.

Langmuir, although he does not give a diagram of the electron arrangement, represents the molecule by the symbols

$$O=S-O$$

The significance of the single bond is that two electrons are shared; the double bond signifies the sharing of four electrons; thus the necessary total of six electrons are held in common. This arrangement demands, however, that the two oxygen atoms should be on opposite sides of the sulphur atom. It also raises the puzzling question as to why the singly bound oxygen atom attaches itself at all, in view of the apparently saturated condition of the hypothetical molecule O=S. For present purposes we will not wait to attempt to answer this question, but will examine an alternative possible arrangement of the sulphur dioxide molecule which is worthy of consideration. It has, we believe, already been suggested—we are not sure whence the suggestion originally comes. It is a triangular arrangement which is represented thus

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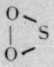
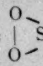
* I. Langmuir, Journ. Amer. Chem. Soc., Vol. 41, p. 868 (1919).



From the point of view of electron sharing this would signify that two electrons were shared at each link—or six altogether—so that the corresponding inert atom electron arrangements are approximately attained. There must, of course, be some distortion, on account of the sulphur atom being larger in size than those of oxygen ; but account cannot be taken of it, owing to insufficient knowledge of its character.

The recent measurements of one of us (C.J.S.)[*] of the mean collision area of the sulphur dioxide molecule in the gaseous state permit the production of evidence which helps us to estimate the alternative probabilities of one or the other of the two above molecular structures being the true one. This evidence, in so far as it is valid, decides very definitely in favour of the second alternative, and the purpose of the present Paper is to show how this comes to be the case.

As has already been pointed out, there are good grounds for believing that oxygen atoms in combination resemble neon atoms in size and shape, and that sulphur atoms in like manner correspond to argon atoms. Dimensionally, therefore, it may be assumed that the sulphur dioxide molecule will have the properties of a particular arrangement of the two neon atoms and one argon atom. If a possible arrangement is chosen we can calculate the mean area which the model so constituted would present in collision for all orientations, and compare the result with the actual mean collision area deduced from determinations of the viscosity of the gas. The general application of this device has already been described,[†] and need be repeated here only so far as the special case demands. The sizes of the collision spheres of argon and neon are known from viscosity measurements—their radii are respectively 1·44 Å and 1·15 Å. The distances between the centres of atoms in combination are also known from the crystal measurements of W. L. Bragg.[‡] For the kind of chemical combination operative in SO_2 this distance is the sum of the radii of the outer electron shells of the atoms in question. Between sulphur and oxygen this distance is 1·68 Å ; between two oxygens it is 1·30 Å.

It will be more convenient to deal first with the model corresponding to the structure  for the reason that it is quite determinate—a consideration which, as will be seen later, does not apply to the model representing $O \overset{-}{=} S - O$. The centres of the collision spheres—or the nuclei of the atoms—lie at the corners of an isosceles triangle, the equal sides of which are 1·68 Å long, and the base 1·30 Å. The centre of the argon sphere (corresponding to the sulphur atom) is the apex, and the neon (oxygen) centres are at the other corners. Fig. 1 represents the arrangement drawn to scale, all three centres, or nuclei, being in the plane of the diagram.

This is the model of which we have to find the mean collision area for all possible orientations. It is one which does not lend itself to analytical treatment of a reasonably simple character, on account of the absence of an axis of complete symmetry; and we have therefore dealt with the problem partially by graphical methods. The axis SO' is chosen, and the orthogonal projection in a direction making an angle θ with SO' is drawn. This consists, of course, of three overlapping circles with centres at the corners of the triangle into which the triangle $SO'O''$ projects. In finding the mean projection area for a fixed value of θ it is necessary to contemplate all the equally probable positions of O'' with respect to the axis SO', *i.e.*, we must imagine O'' describing a circle round SO', and find the projections for suitably chosen positions of O'' on this circle. The projections of S and O' in any particular case remain fixed, and the projection of O'' describes the ellipse into which the abovementioned circle projects. The necessary co-ordinates are readily found, and the corresponding circular projections can be drawn and measured. Owing to partial symmetry it is only necessary to examine the results of positions of O'' on half its circular path around SO', and the corresponding angle from 0 to π has been divided into the intervals $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$ in order to attain a sufficient accuracy in estimating the mean area of projection. The general form of projection for a fixed value of θ is two unvarying overlapping circles corresponding to S and O', and the circle of varying position corresponding to O''. In each position it was only necessary to measure with the planimeter that part of the latter circular projection which was not eclipsed by the other circles; for formulæ already exist* for dealing analytically with the mean projection of S and O' alone. By drawing the appropriate graph and integrating mechanically the additional mean area $\bar{A}$ for projection in the θ direction—additional, that is to say, to the calculable value due to S and O' alone—was determined. Values of $\bar{A}$ were thus obtained for several directions of projection with respect to SO'; actually the values of θ taken were

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[*] C. J. Smith, Phil. Mag., Vol. 44, p. 508 (1922).

[†] A. O. Rankine, Proc. Phys. Soc., Vol. 33, p. 362.

[‡] W. L. Bragg, Phil. Mag., Vol. 40, p. 169 (1920).

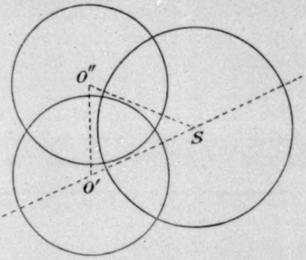

FIG. 1.—POSSIBLE ARRANGEMENT OF ATOMS IN MOLECULE.

$O'O'' \equiv 1{\cdot}30$Å; $SO' = SO'' \equiv 1{\cdot}68$Å.

Radii.—Large sphere $\equiv 1{\cdot}44$Å; small spheres $\equiv 1{\cdot}15$Å.

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* A. O. Rankine, Proc. Phys. Soc., Vol. 33. p. 362.

$0, \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$, and $\frac{\pi}{2}$. It has already been shown by one of us (A.O.R.)* that if $\overline{\overline{A}}$ is the final mean area of projection for all orientations in space which are equally probable,

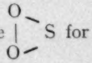
$$\overline{\overline{A}}=\int_0^{\pi/2}\overline{A}\sin\theta\,.\,d\theta.$$

The values of $\overline{A}$ above mentioned were therefore multiplied by the corresponding values of $\sin\theta$, and plotted against θ; and the integral was evaluated mechanically. The result obtained is

$$\overline{\overline{A}}=0{\cdot}127\times10^{-15}\text{ cm.}^2.$$

To this there has to be added, in order to obtain the total mean projection area of our model, the mean area of projection corresponding to S and O', obtained by calculation from the formula already mentioned. This proves to be

$$0{\cdot}865\times10^{-15}\text{cm.}^2,$$

so that it may be stated that the model corresponding to the structure  for sulphur dioxide has a mean collision area

$$0{\cdot}99\times10^{-15}\text{cm.}^2.$$

It is estimated that the accuracy of this calculation, having regard to the limitations of the planimeter measurements of area, is within one per cent. The actual mean collision area of sulphur dioxide, as derived from viscosity measurements is

$$0{\cdot}94\times10^{-15}\text{cm.}^2,$$

and this is sufficiently near to that of the model to enable the difference to be attributed to experimental errors and inexactitude in our assumptions.

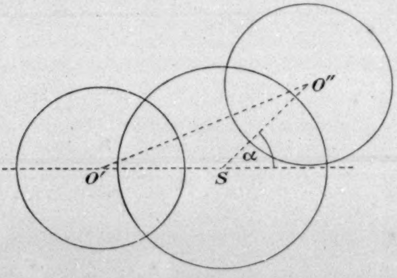


FIG. 2.—POSSIBLE ARRANGEMENT OF ATOMS IN MOLECULE.
$SO'=SO''\equiv1{\cdot}68$Å.
Radii.—Large sphere≡1·44Å; small spheres≡1·15Å.

We now proceed to consider the other model, namely, that corresponding to the structure suggested by Langmuir, O=S—O. Here, to a certain extent, the model is indeterminate; but fortunately it is possible to dismiss it without formal calculation, for reasons which will appear. According to what has already been said, this model may be represented by the full lines in Fig. 2, where the centres are, as before, in the plane of the diagram, and the spheres have the same sizes. SO' and SO'' are the same as before but, since it is no longer supposed that the two oxygens share electrons between themselves, the distance $O'O''$ is not known, nor is the angle α. Following Langmuir's usual method of representation α would be 45°, and the corresponding electron arrangement necessitates its being less than 90°; but there is no need to insist upon this particular value for the purpose of the following argument. It is certain that the mean area of projection of the model (which could be determined, were α known, in precisely the same way as that already indicated) would diminish as the angle α increased. For if α is so small that the plane touching the spheres O' and S does not cut the sphere O'', there is no possibility, in any orientation, of the spheres O' and O'' eclipsing one another except in regions covered by the sphere S. This means that the mean area of projection is the area of the circle with centre S plus two equal and entirely independent contributions arising from the excrescences O' and O''. This proves to be

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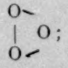
* A. O. Rankine, Proc. Roy. Soc., A. Vol. 98, p. 360.

$$1{\cdot}08 \times 10^{-15}\text{cm.}^2.$$

Preliminary drawings of projections, taking $\alpha = 45°$ made it manifest that the effect —a reduction of course—of the mutual eclipsing of O' and O'' independently of S was very small, since it occurs only at orientations of infrequent occurrence. Even if the angle is greater than 45°, the mean projection area does not become reduced to $0{\cdot}99 \times 10^{-15}$cm.^{2}—that of the former model considered—until $O'O''$ becomes the 1·30 Å appropriate to the sharing of electrons between the two oxygens; and this as we have seen, still gives a value somewhat in excess of the experimentally determined mean collision area of the sulphur dioxide molecule.

The calculations which we have made are thus quite definitely in favour of the view that in the sulphur dioxide molecule there is a single bond, *i.e.*, two electrons shared, between each pair of the three constituent atoms, as compared with that which regards electron linkage as absent between the oxygen atoms. It would be interesting to know how far sulphur dioxide is equivalent in this respect to ozone, which is frequently represented as



but there seems little hope of obtaining the necessary experimental data, owing to the instability of this gas.

This opportunity may be taken of mentioning that the graphical method described in this Paper has been applied to the molecules COS and CS_2 for which an approximate analytical treatment has been given by one of us (A. O. R.).[*] The values of the mean collision area thus estimated are :—

COS	$1{\cdot}09 \times 10^{-15}$cm.^{2},
CS_2	$1{\cdot}31 \times 10^{-15}$cm.^{2}.

The former compares satisfactorily with that deduced from viscosity measurements, namely, $1{\cdot}06 \times 10^{-15}$cm.^{2}; the latter cannot yet be compared with experimental results.

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[*] A. O. Rankine, Phil. Mag., Vol. 44, p. 292 (1922).

DISCUSSION.

Dr. D. Owen inquired whether the results obtained could be interpreted as favouring either the static or the dynamic theory of the atom.

Prof. Rankine replied in the negative, no deductions to which the test could be applied having been made by the advocates of the dynamic theory. Prof. Bohr had told him that his theory led to a definite mechanism of combination for fluorine agreeing with the static mechanism of the salt type, in which atoms combine in virtue of their respective excess and defect of electrons as compared with the inert-gas atoms of their periodic group. The method described in the Paper could not be applied to such compounds as they are not volatile. Compounds comprising two non-metals combined by the sharing of electrons, as in the present case, had not yet been brought within the scope of Bohr's theory.

Dr. E. H. Rayner referred to the thermal expansion of crystals and inquired whether the ordinary coefficient of expansion applied to the distances between the centres of the atoms which had been assumed in Fig. 1 of the Paper. In that case, was it justifiable to apply these distances to atoms in the gaseous state ?

Prof. Rankine replied that X-ray measurements were supporting the view that the ordinary coefficients of expansion are applicable to atomic distances in crystals, but the Lewis-Langmuir theory implies that even in a gas the atoms which are sharing electrons are contiguous. This assumption leads to consistent results.

V. *The Thermal Effect of Vapours on Rubber.* By A. S. HOUGHTON, *B.Sc., A.I.C.*

RECEIVED JULY 1, 1922.

(COMMUNICATED BY SIDNEY SKINNER, M.A., F.INST.P.)

ABSTRACT.

This Paper relates to the heating of rubber when suddenly immersed in various vapours. It describes the method of measuring the thermal changes by means of thermo-electric junctions and gives the results obtained with benzene, pyridine, ammonia and water-vapour. The effect is compared to that of water-vapour on cellulose.

DURING the course of an investigation on the surface action of rubber the phenomenon of the passage of gas through rubber membranes was frequently encountered, and it seemed important to consider this matter in greater detail. It will be remembered that Graham found that the passage of gases through rubber was not according to the ordinary laws of porous plug diffusion, and that his explanation was that the gas was liquefied on entering the rubber, passed through the rubber as a liquid and was evaporated back to its normal state on leaving the rubber on the opposite side. The modern idea considers the first operation as some form of adsorption, and consequently we expect some thermal changes on the membrane during its exposure to the gas.

EXPERIMENTAL CONSIDERATIONS.

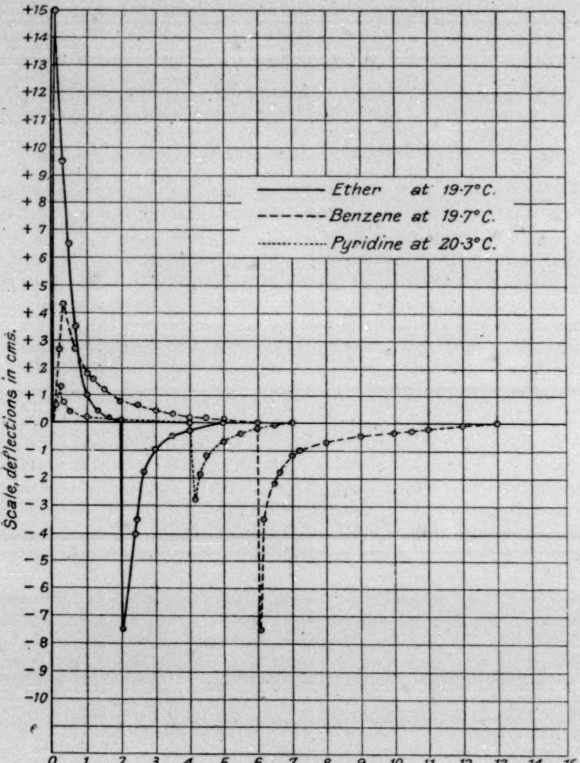
Preliminary experiments were first made with two exactly similar delicate thermometers, the bulb of one being covered with a thin film of rubber by means of rubber solution, whilst the other was left bare. When these were plunged into carbon dioxide it was noticed that the rubbered thermometer recorded a higher temperature than the other, thus indicating that some heating effect was going on in the rubber.

A more accurate means of recording this heating effect was afforded by thermo-electric junctions of copper and Eureka wire. Lengths of these wires were soldered together and the alternate junctions coated with rubber solution. The heating effect was measured by means of a delicate low-resistance mirror galvanometer to which these junctions were connected.

It was shown by experiments using this method that a definite heating effect was given by various vapours such as chloroform, benzene, toluene, pyridine and essential oils such as oil of lavender and oil of guaniol. It was also proved that if alternate unrubbered junctions were placed in such vapours no heating effect was given, so that the effect was due to some heating of the rubber film itself.

The final form of apparatus consisted of a brass jacket with a wooden lid, through a hole in which the junctions were placed and held in position by a packing of cotton-wool. In this way the junctions were protected from convection currents of air, thus ensuring a steady galvanometer deflection, since the whole could be lifted over and placed in the bottle containing the vapour.

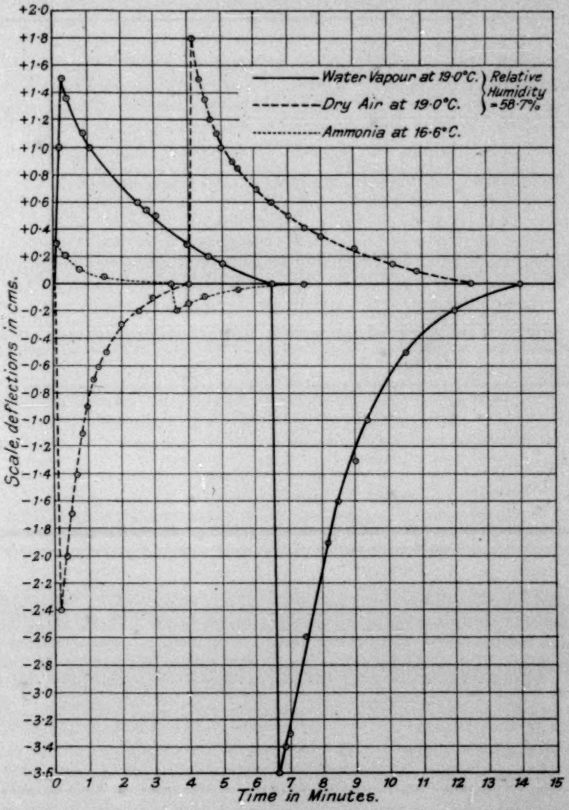
Experiments were carried out in the following manner: The liquid in the bottle was well shaken, to ensure that the space above it was saturated with vapour, and the jacket with the junctions placed over it. The movement of the beam of light from the mirror galvanometer was noted, times being taken with a stop-watch until the beam returned to zero. Then the jacket with the junctions was taken



GRAPH 1.

away from the bottle of vapour, and placed over nut-charcoal, so that any traces of vapour remaining in the jacket were rapidly absorbed. The deflections of the beam to the opposite side of the scale produced by this withdrawal were noted as before. These deflections were plotted against time and specimen curves for ether, benzene and pyridine are shown in Graph 1. These curves have been drawn to scale and consequently it has been thought unnecessary to give tables of observations.

As shown in Graph 1, the type of curve obtained in these experiments is quite a regular one. The chief points to be noted are the sudden heating on plunging the junctions into the vapour followed by a gradual return to zero, as the rubber becomes saturated with the vapour, and the sudden cooling on withdrawing the



GRAPH 2.

junctions followed by another gradual return to zero as the vapour evaporates from the surface of the vapour.

The next step in the investigation was to see if such gases as ammonia and carbon dioxide, which pass rapidly through rubber membranes, had a similar heating effect on the rubber-coated junctions. The pure dry gases were contained in wide-necked stoppered bottles and were allowed to stand for some time at room temperature. The experiments were carried out in the usual way, and although the deflections produced were much smaller they showed that the behaviour of carbon dioxide and ammonia was quite normal. In the case of ammonia junctions of platinum and iron were used since the gas was found to attack the copper junctions, thus giving faulty results. The curve for ammonia is given in Graph 2, and we can see it is identical in type with the previous curves.

It was thought interesting in view of recent work on hygrometry to try the effect of moist and dry atmospheres. A small wide-necked bottle was lined inside with three or four layers of wet filter-paper and the usual set of junctions were slipped into the moist air in the bottle. A heating effect was at once given and the behaviour was similar in every way to that of the vapours previously tried. The dry atmosphere was obtained by means of a little concentrated sulphuric acid at the bottom of a similar bottle, and in this case a cooling effect was noted on introducing the rubbered junctions and a heating effect on their withdrawal. The curves for these two experiments are shown in Graph 2.

Finally, a calibration of junctions was carried out so that some idea of the temperature value of the beam deflections could be obtained, and it was found that a deflection of 14 cm. approximately indicated 1°C. difference in temperature between the alternate junctions.

THEORETICAL CONSIDERATIONS.

The only previous work with any bearing on this research is that done on the heating of cellulose by water-vapour, and references to various work on this subject are given at the end of the Paper.

The heating of cellulose by water-vapour is generally attributed to the condensation of the aqueous vapour on the cellulose fibres with consequent liberation of heat of condensation. The results obtained with moist and dry atmospheres in the latter part of this work certainly point to some similarity in the behaviour of rubber and cellulose. Rubber differs from cellulose, however, in its non-fibrous structure and wide range of action with vapours. So it is probable that only part of the heat produced is due to the condensation of the vapour on the rubber, and the additional heat is supplied either by some form of chemical union, or by the formation of a solid solution between the vapour and rubber. The loose nature of this second phase is shown by the rapid return to the normal state on withdrawal of the rubber from the vapour, but the experimental facts undoubtedly prove the existence of this phase.

In conclusion, I desire to thank my supervisor, the Principal of the Chelsea Polytechnic, for suggesting the subject of this research, and also the Department of Scientific and Industrial Research for the grant that has enabled me to carry out this work.

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Sir W. Roberts, 1880, *Ibid.*, Vol. 23, p. 55.

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Dr. Cobbett, 1900, Proc. Camb. Phil. Soc., Vol. 10, p. 372.

Orme Masson, 1905, Proc. Roy. Soc., Vol. 74.

Trouton, 1906, *Ibid.*, Vol. 77.

Masson and Richards, 1906, *Ibid.*, Vol. 78.

Tyndall and Chattock, 1921, Proc. Phys. Soc., "Hygrometry," Nov. (1921).

DISCUSSION.

Principal S. Skinner said it was interesting to see to what a large number of vapours india-rubber responds, as if it could smell them! In addition to the vapours mentioned in the Paper, the author had tried a number of those which form the basis of artificial scents, with marked effect. It might be worth considering whether the sense of smell is not connected with the adsorption of vapours by membranes covering the olfactory nerve-endings. The apparatus constitutes a sort of hygrometer for vapours other than water, and the humidity of the atmosphere is indicated by measurements of the kind illustrated in Fig. 2. The investigation began from the phenomenon of the diffusion of vapours through a rubber membrane, and possibly some Fellows present had experience in this connexion during the war.

Mr. **J. S. Anderson** asked whether it would be feasible to weigh the adsorbed vapour and see whether there was any time lag between adsorption and rise of temperature. Such a measurement would help to decide between chemical action and adsorption as an explanation of the phenomenon. The latter explanation is favoured by the relative heating effects of ether and benzene, which are roughly in the ratio of 3 : 1, for in the case of charcoal the heating effects are in the opposite direction, but are in the same ratio.

Dr. **H. Borns** asked whether the author had experimented with water and other substances in the liquid state. It is a fact that rubber swells in water and most other liquids, sometimes apparently without any chemical action taking place. It would be interesting to know whether the same effect can be observed with gases.

Mr. **F. J. W. Whipple** said it had been proposed to saturate the fabric of pilot balloons with ammonia for 24 hours before inflation to enable it to expand without rupture or the development of "pin-holes." Could the author give any information bearing on this suggestion? If it is the case that water evaporating from rubber gives the same effect as from cotton, the former fabric might have advantages over the latter for the purposes of hygrometry.

Dr. **D. Owen** said that it was most interesting to see the well-known phenomenon connected with cellulose and water vapour extended to quite different substances; but quantitative measurements of the calories liberated would be necessary to decide between adsorption and chemical action as an explanation of what occurred. The observations of rise of temperature need to be supplemented by a knowledge of the heat capacity and emissivity of the rubber, &c. An attempt might be made to measure the gain in weight, and so relate the heat evolved to the latent heat of the vapour—if such connection exists. The rubber membrane has the great advantage over cellulose that its area can be measured, so that the thickness of the adsorbed film might be estimated.

Dr. **J. S. G. Thomas** said that during the war it was sometimes found that balloons which had been filled with coal-gas could not be re-inflated after deflation. This was apparently due to chemical action of the gas on the rubber, which made the latter sticky.

Author's reply (communicated):—Although I appreciate the helpful nature of the discussion on my Paper it is rather difficult to give definite answers to many of the questions raised. I should, however, like to emphasise one or two points with regard to the action described in my Paper. In the first place the action of the vapour on the rubber is practically instantaneous, the maximum heating effect being reached in many cases within 5 seconds. The return to the normal state, on withdrawing the rubber from the vapour, is equally rapid, so that any weighing of the amount of vapour absorbed would have to be done in an atmosphere saturated with that vapour. Another point is that the amount of rubber on the junctions is small, being less than

0·1 gms. The final decision between the chemical action and adsorption therein would, as Dr. Owen pointed out, depend entirely on a quantitative estimation of the amounts of vapour absorbed and heat evolved, and so some delicate work will have to be carried out before this point is settled.

With regard to the questions concerning balloon fabric it is already a well-known fact that rubber keeps much better in the presence of a slight trace of vapours such as carbon bisulphide and it is probable that ammonia acts in a similar way. It may be that rubber saturated with such a vapour is less prone to oxidation, which is one of the chief causes of the deterioration of rubber; in addition, the vapour slightly warms the rubber and thereby tends to increase its elastic properties. The case of balloons filled with coal-gas is an extreme one. Here we have the hydrocarbon vapours, which coal-gas contains, in close contact with the rubber for some considerable period. As my experiments show, these vapours have a much greater effect than such gases as ammonia, and consequently the action, whether chemical or one of adsorption, is intensified, and the surface of the rubber is attacked. In my own experiments it has been noticed that if the rubber is used for some time with the more powerful vapours it becomes sticky, but no marked change in volume has been observed.

A DEMONSTRATION of a Calling Device for Wireless Telegraphy was given by T. D. PARKIN, *of Marconi's Wireless Telegraph Co., Ltd.*

THE device is intended for ships employing a single wireless operator, and is adapted to ring an alarm bell on receipt of waves having a characteristic group frequency in addition to a characteristic high frequency. The transmitter comprises a brass balance-wheel carrying a diametrally arranged bar of soft iron and moving between fixed field magnets, the circuit of which is closed periodically by contacts carried by the wheel. The connections are such that the oscillation of the wheel is maintained, and a second contact moving therewith effects the emission of wave-groups of the required group-frequency. The receiver comprises a field-magnet to the coils of which regular pulses of current are imparted on receipt of the above-mentioned wave-groups, acting upon a balance-wheel the rim of which consists of a ring of steel so magnetised as to have poles at opposite ends of a diameter. The natural frequency of this balance-wheel is adjusted to that of the expected wave-groups, so that after about 11 of these have been received the oscillations of the wheel attain sufficient amplitude to effect the ringing of an electric alarm bell provided with a hold-on circuit. The apparatus is mounted on springs and gimbals.

DISCUSSION.

Capt. C. W. HUME inquired whether provision had not been made for an "S.O.S." signal.

Mr. R. L. SMITH-ROSE inquired as to the range of the apparatus.

Dr. A. O. RANKINE inquired whether the receiver might not be affected by a letter comprising several dots, such as "H," sent at the proper group frequency.

Mr. PARKIN, in reply, stated that an adaptation to "S.O.S." signals was being considered, but had not yet been perfected. The apparatus had been worked at a range of 110 miles. It was less sensitive than a human operator, but would be effective at all distances at which distress signals could be of any value. The receiver should be adjusted so as not to respond to fewer than 11 impulses, in order to be immune from disturbance by dot letters or by the "erase" sign.

A DEMONSTRATION of an Apparatus for Testing the Tensile Strength of Gas Mantles was given by Mr. J. T. ROBIN.

THE apparatus comprises a vertical cylindrical float which is surrounded by water in a cylindrical container. It is surmounted by a cup carrying wax, and the lower edge of the mantle to be tested is attached to the float by allowing the melted wax to solidify while the mantle dips into it, the latter being suspended in the usual way. The mantle is then put in tension by running off the water slowly, so that the mantle takes a gradually increasing proportion of the weight of the float. As soon as the mantle ruptures the float drops and automatically effects cessation of the flow of the water. The apparatus having been previously calibrated, the breaking weight can be read from the height of the remaining water, which is shown by an inspection tube connected to the container at its lower end, and backed by an adjustable scale. Special precautions are observed in handling the mantle and burning off the cellulose. The shear strength is taken to be proportional to the tensile strength. It is found that some mantles can support 500 times their own weight.

VI. *The Theory of the Singing Flame.*

By E. G. RICHARDSON, *B.Sc., East London College, University of London.*

RECEIVED OCTOBER 10, 1922.

ABSTRACT.

The various theories of the action of the Singing Flame put forward since its discovery are reviewed, and experiments to test the relative merits of the two most recent theories are described; Lord Rayleigh's theory is shown to fit the results most closely, in that (1) heat is given by the flame to the air in the tube at each condensation, and (2) stationary waves are formed in the gas as well as in the air-tube. But the lengths of gas-tube unfavourable to the "singing" cover, in fact, a much more restricted range than Lord Rayleigh surmised.

THE fact that a jet of hydrogen, burning in an open tube, would, under certain conditions, cause a musical note to be emitted, was first observed by HIGGENS* in 1777, and other observers studied various aspects of the phenomenon, without attempting to account for it, until De La Rive advanced his theory, *i.e.*, that the periodic condensation of water vapour by the burning hydrogen caused the emission of the note. That this cannot be the true cause of the "singing" was shown by Faraday, who was able to replace the hydrogen by carbonic oxide, a gas which produces no moisture in its combustion, without detriment to the effect. Faraday himself put forward an alternative theory, that the note was caused by successive explosions of the gas with the oxygen of the air, the flame then dying out until a further supply of air arrived at the jet, when it was re-ignited. For Wheatstone had shown in his revolving mirror that the flame was not steady, but vibrated up and down, so as to appear in the moving mirrors as a succession of images. As a result of his experiments, Sondhauss,† who of all the investigators has examined the phenomenon the most thoroughly, considered the cause of the "singing" to be the heating of the air in the neighbourhood of the jet, the subsequent change of density causing a compression to flow away from the jet, thus starting the air in the large outer tube (hereafter referred to as the air tube), to sound its natural tone. He also found that certain combinations of air tube and gas-supply tube would not sing, and concluded that the length of the gas tube must vary with different gases, in order that the oscillations of gas and air may be in step near the jet. If these oscillations were stopped by a plug of cotton-wool in the air tube near the jet, the singing ceased.

Zoch,‡ on the other hand, imagined that the current of air up the tube, caused by the combustion, was the origin of the singing. This idea was elaborated by Terquem§ as follows: the influx of air to the flame causes it to rise; when it has used up the air, it falls again until fresh air rises, when the process is repeated, until after a few periods the rise and fall of the flame, and the periodic rise of air adjust themselves to the natural frequency of the air in the larger tube.

Lord Rayleigh|| reverted to the heat transfer theory of Sondhauss. He showed theoretically that, unlike ordinary resonance, the impulses given to the

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* Nicholson's Journ. (1797).

† Ann. der Phys. 109, 1 and 426 (1860).

‡ Ann. der Phys. 127, 589 (1866).

§ Ann. der Phys. 134, 468 (1868).

|| Theory of Sound, II, 227.

resonator (*i.e.*, the air in the tube) by the impressing force (the heat) must occur at the maximum displacement or condensation, and not in the neutral position. Also that for the continuance of the vibrations the gas tube must be of such a length, and in such a position, that a condensation at the jet will travel down it and back again so as to arrive at the jet in phase with a condensation in the air tube (as Sondhauss had previously stated), in fact, that there should be stationary waves in the gas tube as well as in the air tube.

Finally, Gill,* apparently without having seen Lord Rayleigh's paper, has revived the older theory, wherein an accidental change of pressure in the air tube starts the latter faintly sounding, the successive condensations forcing the flame down into the jet, and each rarefaction letting it grow again.

It appears, then, that the modern theories divide on the question of the primum mobile of the action: does the air tube act as a resonator to the flame, or does the air in the tube force the movement of the flame into agreement with it? It was in an attempt to settle which theory was best supported by facts that the following work was undertaken.

PHASE OF HEAT SUPPLY.

It is noted above that Lord Rayleigh's theory requires the heat to be given to the air at or in the quarter-period preceding a condensation; so that, if the flame gives most heat when it is at its maximum size, the flame should be longest, and the gas should come out most freely a little before the maximum pressure in the air tube near the jet (if one allows for the time taken by the actual transfer of heat).

The other theory, in Gill's form of it, requires that each phase of maximum pressure should force the flame down, or temporarily stop the flow of gas, which should be most free at a rarefaction.

The vital point, then, is the relative phase of the vibrations of the singing flame and of the air near the jet. This I found in the following manner. A brass tube, 60 cm. by 4 cm. was taken and a gas tube with a jet† placed in it in such a position that the "singing" started spontaneously and vigorously. A hole, 1 cm. diameter, was bored in the brass tube at this point, also a larger hole, over which a piece of glass was fixed by sealing-wax, to serve as a window through which the singing flame could be observed. Over the small hole a König manometric capsule was placed, the burner of which was brought right round in front of, and close to, the window, so that when lit this manometric flame appeared vertically beneath the singing flame. The jets were placed as close as possible to each other, one inside and one outside the air tube; and all joints were carefully plugged, as a leakage of pressure is inimical to the "singing." Both flames were fed by coal gas, and when the inner flame was singing, the two were examined in a rotating mirror, and two sets of images were seen one above the other. This is an inconvenient method of observation, because (1) the images jump about, as the four mirrors do not succeed each other in the same vertical plane; (2) when the mirror and the flames are still, the latter appear correctly over each other only in one position of the mirror, owing to parallax. In order to compare the flames in a better way they were looked at directly through a stroboscope (a rotating card with equidistant slits whose speed could be varied). This apparatus, originally applied by Töpler* to the singing flame alone (without the König flame), allows one to lengthen the apparent period of the flames at will. By this means it was seen that *they rose and fell almost simultaneously and in the same phase.* This definitely rules out Gill's theory.†

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* Sill. Journ. 4, 177 (1897).

† With the gas at a pressure of 4 in. of water, I found that the diameter of the jet must lie between 0·4 and 2 mm. With smaller jets the flames will not sing, with larger they are immediately extinguished.

In order to have a record of this phase relation I next attempted a photograph.

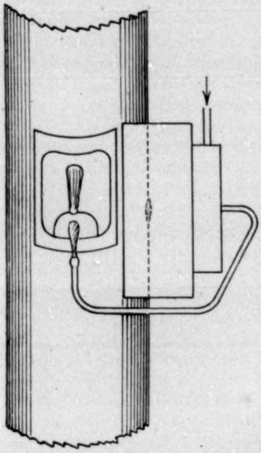

FIG. 1.

Manometric flames have been photographed several times, but not singing flames, nor, therefore, the two together. The following methods have been tried :—

(1) An instantaneous photograph of the revolving mirror.
(2) Sliding a sensitised plate across the back of the camera, while the flame is vibrating (Merritt‡).
(3) Keeping the plate fixed, but rotating the camera (Hallock§).

I tried all these three ways, but could get nothing from the first—indeed, I think no one has ever succeeded with this method ; nor from the second, and the third reproduced only the manometric flame, even when the gas was mixed with acetylene to render it more actinic, and with very rapid plates. The failure of the singing flame to register seems to be due to its inferior luminosity compared to that of the manometric flame, added to the high frequency (about 260). I thereupon devised a new method which was more successful. The apparent period of the flames was lengthened by the stroboscope and they were photographed through the slits, the camera being rotated as in Hallock's method, using extra-rapid panchromatic plates, and feeding the flames with coal gas plus as much acetylene as they would bear without becoming smoky. The stroboscope allows impressions of several images of slightly differing phases to coincide and gives more chance to the light to act on the plate, but, of course, the stroboscope must be of slightly longer period, not shorter, otherwise the vibrations appear in the reverse order. At first the images were subject to parallax (Print 6), which only shows the true relationship of phase at the point marked ; but later, at the suggestion of Prof. Lees, parallax was avoided by rotating round the optical centre of the lens (Prints 9 to 13). Besides the phase relationship these prints also show the difference in character of the flames. The manometric flame rises to a maximum gradually and falls suddenly to start again, whereas the other rises quite suddenly, as Töpler noted, so that the heat is given over a small fraction of the period just before the maximum pressure.

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* Ann. der Phys. 128, 126 (1866).

† The movement of the membrane is propagated with the speed of sound to the manometric flame ; the indications of the latter were consequently about 1/10,000 second "late" a time which can be neglected in comparison with the period of the oscillation (1/260 second).

‡ Phys. Rev., 1, 166 (1893).

§ Phys. Rev., 2 305 (1895).

In order to make this fit in rigorously with Lord Rayleigh's theory, we must suppose, either (1) that the heat is given instantaneously to the air when the flame is at its maximum, or (2) that the luminosity which accompanies the maximum is a sign of a fall in the temperature of the flame, which temperature is really greatest before the maximum height, and therefore a little before the compression occurs. In order to test this, I set up an optical pyrometer (Féry's) behind the stroboscope, and various phases were " held " and focused on the thermo-couple, but although the instrument showed differences in the amount of radiation it was not sufficiently sensitive to enable one to settle the question of which was the hottest ; perhaps a radio-micrometer would have done it. At any rate, the fact that, " with the same gas pressure, the temperature of a flame is greater as its surface is less " (Prof. Smithells, in the Dict. of Applied Chem.) is in favour of the second supposition.

With the stroboscope removed, it was found that a flame emitted much less energy when singing than the same flame when steady.

GAS.	GALVANOMETER DEFLECTION.	
	Steady flame.	Singing flame.
Acetylene and coal gas	9½	3
Ditto	7¼	2½
Ditto	6	2
Coal gas	0½	0

THE GAS SUPPLY TUBE.

Lord Rayleigh in his Paper refers to the work of Sondhauss, stating that the latter found that the phenomenon depended largely on the length of the gas tube, and then proceeds to show that the tube must be shorter than a ¼-wave-length (for the particular gas and note emitted) or longer than a ½ and shorter than ¾ of the wave-length in order that singing may start. Suitable data were therefore obtained, in order to test this question of the length of the gas tube. The results are best viewed on the accompanying graphs, each for a constant air tube, and showing the position in the air tube at which the flame due to each gas tube would sing. Sondhauss used a gas tube which terminated in one of the tubules of a Wolff's bottle. I found that the bottle was not large enough to prevent variations of pressure at the lower end of the tube, and therefore a large earthenware jar of about 20 litres capacity was used, fed by coal gas from the main. The graphs show three distinct cases. In the case when the jet is slowly pushed up the air tube, the violence of the oscillation extinguishes the flame before it reaches the node of the air tube ; on the other hand, if the flame be placed in the upper half of the tube, and then moved down, it is extinguished before it reaches the node. These lengths correspond roughly to the lengths (less than $\frac{1}{4}L$——, between $\frac{1}{2}L$ and $\frac{3}{4}L$——, between L and $1\frac{1}{4}L$——, &c.) of Lord Rayleigh's theory, after allowance is made for the antinode at the lower end of the gas tube being a little distance beyond its end ; and for the node at its upper end lying considerably above the jet (*vide infra*). There are other comparatively restricted regions of the graph within which I have

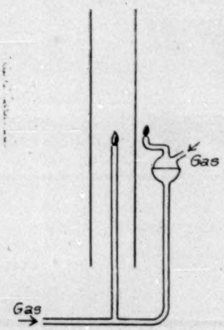

FIG. 2.

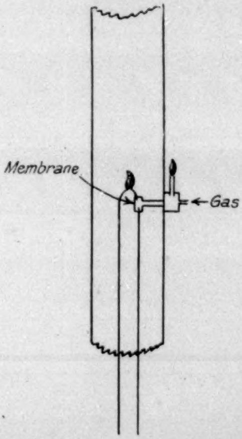

FIG. 3.

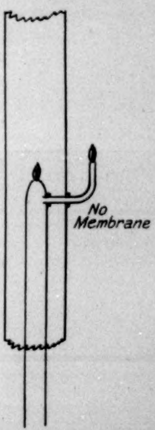

FIG. 4.

not been able to force the flame into vibration at all. These correspond roughly to lengths equal to or just greater than $\frac{1}{4}L$ and $\frac{3}{4}L$.

But it is not correct to say that the flame will not sing with gas tubes lying between these lengths. The singing is less intense, it is true, and often requires a larger flame encouraged by a tuning fork of the same pitch. Also the vibration is not intense enough to extinguish the flame at the node, where it goes on singing (Lord Rayleigh believed no singing possible at the node, and even beyond), until the changes of presssure in the air tube are too small to affect it, and it becomes steady.

It occurred to me that the explanation of these three different cases might be that in the first the gas tube was of such a length that it resounded in phase with the air tube, but that in the second the vibration in the air tube, once started, was able to react on the flame, and to force the gas column to vibrate in spite of its natural period being different, just as a tuning fork can force a column of air into vibration, and, if the latter is of the correct length, to resound and assist the vibration. This was confirmed in the following manner.

Lord Rayleigh says "stationary or approximately stationary waves are formed in the gas as well as in the air." To test this I fixed a manometric capsule on a piece of glass tubing, 10 cm. by 2 cm., and made the gas tube of pieces of the same glass, so that the capsule could be "built in" to any place of the gas tube. I noticed that when the jet remained unlit, a tuning fork at the top of the air tube caused perturbations of the manometric on the gas tube. Accordingly, selecting lengths of air tube and gas tube, which produced vigorous singing, I was able to find well marked nodes and antinodes in the longer gas tube, corresponding fairly well with the value of λ, found from $V=n\lambda$, V for coal gas being 490 m./sec. (Zoch)[*] (Fig. 16). It will be noticed that, as mentioned above, the jet itself is not usually a node; this would be some distance above, near the node of the air tube. In order to test whether the large jar in which the tube ended was a true antinode, another manometric capsule was placed with its lead near the end of the gas tube, inside the jar.

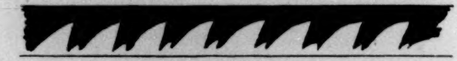

FIG. 5A.

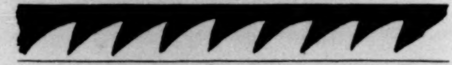

FIG. 5B.

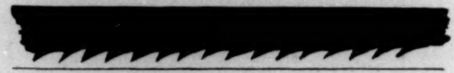

FIG. 5C.

When an example of the second case was tried—*i.e.*, air tube 63 cm., gas tube, 157 cm., no marked node or antinodes could be found, but the manometric flame was about equally indented, wherever it was placed in the gas tube, showing that here was an instance of forced vibration, but no resonance. Apropos of this state of affairs it may be noted that Rogers[†] succeeded in making an alcohol flame sing, even when the supply tube was plugged up by a wick to conduct the spirit; in this case there could be no resonant vibration of the tube.

In the third case mentioned above the gas tube was probably of such a length that a vibration sent down it and reflected back would arrive at the jet in the most inimical phase, and upset the vibration of the flame at once, so that "singing" once started would be immediately stopped (*vide* Lord Rayleigh, *loc. cit.*). How-

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[*] Ann. der Phys., 128, 497 (1866).

[†] Phil. Mag. (4), 15, 261 (1858). Sill. Journal (2), 26, 1 (1858).

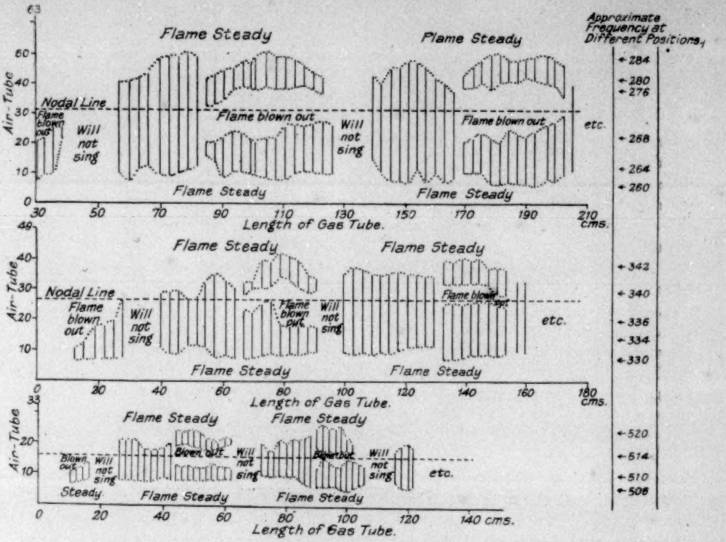


FIG. 15.

The vertical lines show the various positions in the air tube at which given gas tubes could be coaxed to sing—*e.g.*, the 5th line in the first graph shows that, with a gas tube 57 cm. long, the flame commenced to sing at a height of 9 cm. up the air tube, and continued so, when pushed further up, until it reached a height of 39 cm., when it ceased to sing.

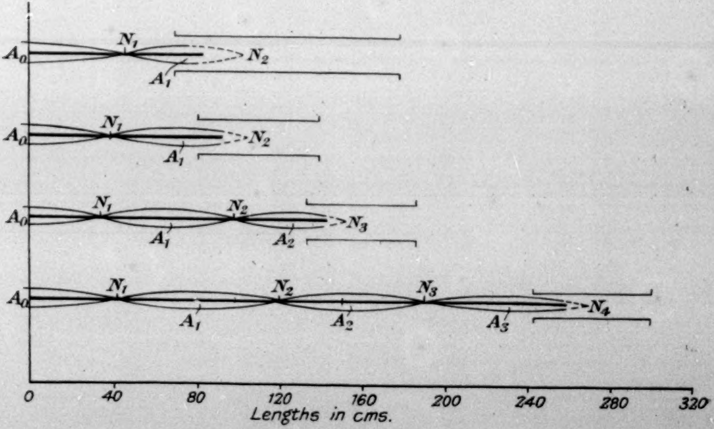

A_0
N_1
N_2
N_3
N_4
A_1
A_2
A_3
0
40
80
120
160
200
240
280
320
Lengths in cms.

FIG. 16.

ever, I do not attach much importance to this gap; in fact, Zoch says that every combination of tubes and gas which the ried could be made to act.

In order to find the relation between the pressure of the gas and the state of the flame, Gill attached a manometric flame to the base of the supply tube, and compared the flames in a revolving mirror (*see* Fig. 2). Before reading Gill's Paper I had tried to find the same relation by the apparatus shown below (Fig. 3), wherein a side tube led from the gas tube through the air tube, and so to a manometric capsule (the membrane may be dispensed with, and the gas lit directly in the side tube) (Fig. 4). I compared the flames in a mirror, but subsequently I repeated the observations with the more exact stroboscope.

All three forms of apparatus lead to the same result; *the flames are in opposite phases.* This, as Gill truly stated, is due to the gas being held back when the flame strikes back to the jet, so momentarily increasing its pressure, as when a water tap is turned off. Gill considered that this showed that the gas was forced in by the air pressure, but adversely to this possibility the singing flame and air pressure are in phase. It follows that the air outside and the gas inside the jet are in opposite phase in regard to pressure.

CHANGES OF PITCH WITH DIFFERENT POSITIONS OF THE FLAME.

Two final points which I do not think have been noted before: As the jet is pushed up the air tube, the pitch of the singing flame rises. Observers have stated that the pitch of the note is the same as that natural to the tube, due regard being had to the temperature. In order to see whether this was so I lowered a taper into an organ pipe. On blowing the pipe by the bellows and raising the taper, the pitch was found to rise similarly, within the region wherein a flame will usually sing.

If a Singing Flame resumes the steady state in the upper half of the tube, just before it ceases, the frequency reverts to what it was initially. In corroboration of the change of pitch noted here, H. Pflaum, in his work on Rijke's Gauze, found a rise of more than a semitone as the heated gauze was moved up the tube.

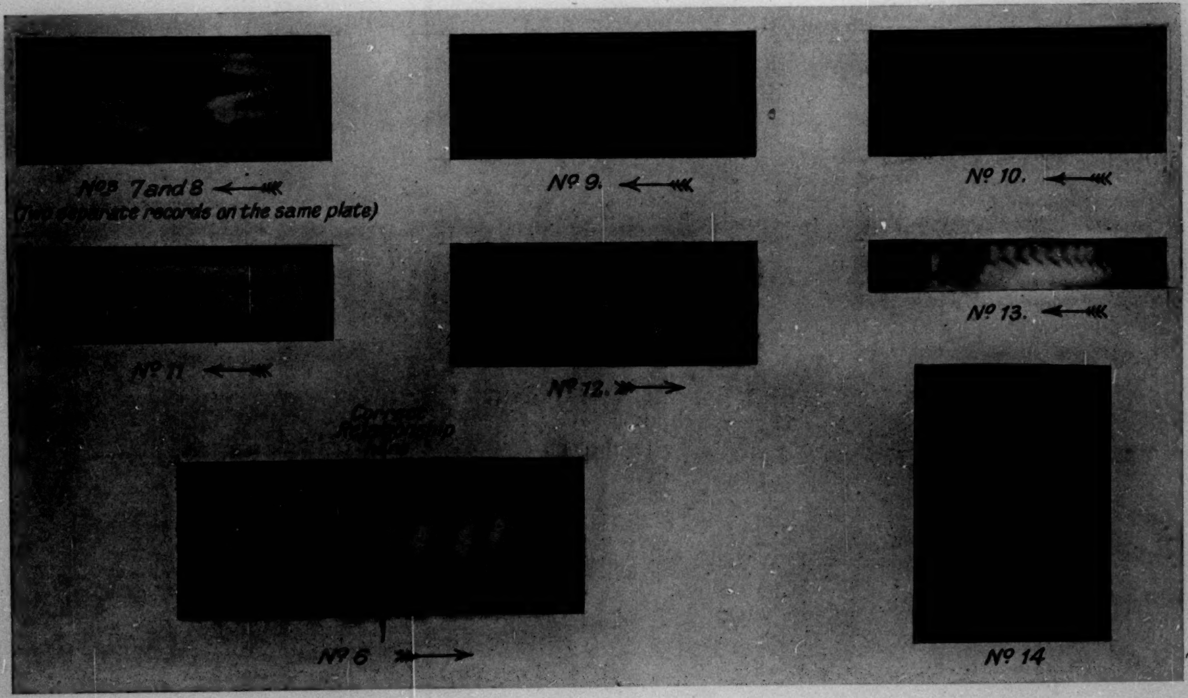
HARMONICS.

It was early found that, in certain positions, a flame would give the fundamental of the air tube and its first harmonic at the same time. Such a flame, examined in the revolving mirror, showed smaller flames between the large ones (Fig. 5A). When the jet was pushed higher, only the big flames remained and the fundamental tone (Fig. 5B); when lower twice as many little flames (and the octave alone) were given (Fig. 5C). This shows that (in Fig. 5A) the large flames produce the fundamental and the two sets combined produce the harmonic. A steady photograph of such vibrating flames (Fig. 14) shows one flame within the other plainly.

In conclusion I wish to acknowledge my indebtedness to Professor Lees, of East London College, for suggesting this work, and for help in carrying it out.

DISCUSSION.

Prof. C. H. LEES (communicated): While the Rayleigh theory of the flame covers most of the observed facts and may be taken as satisfactory in its broad outlines, there are other facts which Mr. Richardson has brought to light with which it is not in complete accord. To cover these some modification of the theory will be necessary, but so far as I can see these modifications are not of a serious nature.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No 6
No 14

FIGS. 6 TO 14.—THE SINGING FLAME IS ABOVE IN EACH CASE. THE ARROWS SHOW THE DIRECTION IN WHICH THE FLAMES MOVE.

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VII. *Unit Surfaces of Cooke and Tessar Photographic Lenses.* By ALICE EVERETT, *M.A., The National Physical Laboratory.*

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(COMMUNICATED BY T. SMITH, M.A., F.INST.P.)

ABSTRACT.

A number of rays in an axial plane (and a few general rays) are traced through the lens systems by exact methods, and on each ray the positions of the conjugate points for unit magnification are found by Mr. T. Smith's formulæ. The results show that for general rays the loci of these "unit points" are three-dimensional. The loci are surfaces only when the chief rays are bound by some conditions, such as passing through a fixed point of the object. In this case the unit surfaces are not fixed, but shift with the object point. The curvature changes with aperture, and with the distance of the object point from the optic axis. Hence the unit surfaces cannot coincide with wave-fronts. Within the region for which the lenses are designed, the curvature of both object and image unit-point loci is positive (convex to the light source) and the image locus is more curved than the object locus. These facts do not coincide with the theoretical conclusions obtained by Mr. Smith for either a thin system or an ideal thick system, but accord more nearly with the latter, especially as regards the sign of the difference of curvatures of the two unit surfaces.

In this work :—

The terms "unit points" and "unit surfaces" are used briefly to denote conjugate points for unit magnification, and their loci.

The secondary focus (*i.e.*, focus for rays in a transverse section) only is considered.

The suffix λ refers to the λ-th refracting surface, and medium following it.

Distances are measured positively in the direction light travels.

WHILE engaged recently on some investigations relating to photographic lenses, the writer was privileged to see an advance proof of an article by Mr. T. Smith ("Optical Calculations," Dictionary of Applied Physics, Vol. 4, p. 287, 1922), in which a passage occurs pointing out that the unit surfaces in a thin lens cannot both be planes, and giving expressions for their curvatures in the simple cases of (1) a single thin lens, when the rays are nearly parallel to the axis ; and (2) a system of lenses of zero thickness in contact, when the rays are incident near the vertex.

The idea struck her that it might be of practical interest to investigate the forms of the unit surfaces in an actual photographic lens system, and two well-known types, Cooke and Tessar, were selected for examination. They have comparatively few (six and seven) refracting surfaces, and some of the requisite data had already been calculated by the writer for another purpose.

The constructional data for the Tessar, which was designed by Rudolph and Wandersleb for Zeiss, are taken from "Zeitschrift für Instrumentenkunde," 1907, pp. 78, 79. It consists of four lenses—hence the name—of which the last two are cemented, and is said (*see* Turrière) to be intended for portraiture, instantaneous work, and general amateur purposes. The data for the Cooke, which refer to a patent of 1894, are taken from von Rohr's "Theorie des photographischen Objectivs," 1899, p. 240. (The first thickness is given wrongly as 2·59 in Gleichen's "Lehrbuch der geometrischen Optik," 1902, p. 490.) This lens, designed by H. Dennis Taylor, of T. Cooke & Sons, and manufactured by Taylor, Taylor & Hobson, Ltd., is remarkable for combining efficiency with simplicity, being composed of three single separated lenses, the first and third made of the same glass. The figures, it should be noted, are for an early form. Although the constructional data (also given in Turrière's "Optique Industrielle," Vol. 1), for both lenses are probably sufficiently accurate to illustrate their general features, it should not be assumed that they justify any deduction regarding relative merits of actual lenses of these two popular types.

Unfortunately, since at least one of the unit surfaces is always virtual, it is not possible to examine them *in situ* by direct physical means.

The process adopted for this investigation was to trace by exact methods the paths of a number of rays with different inclinations and apertures through the system examined, and then to find the positions of the unit points on each ray. For descriptions of modern methods of tracing rays reference may be made to Mr. T. Smith's articles in the Proceedings of the Physical Society (Vol. 25, 1913, p. 239; Vol. 27, 1915, p. 502; Vol. 30, 1918, p. 221; Vol. 33, 1921, p. 174). Von Seidel's methods described in the German textbooks were intended for logarithmic computation. In the Logarithm Age, when all expressions used had to be in the form of products, and a formula with a sum or difference was anathema, artificial devices had to be resorted to, but the introduction of the calculating machine has altered that, and the ordinary formulæ of co-ordinate geometry can be used, though, of course, experience is required to ensure the most accurate arrangement. Tables are only needed for the sines of angles. A "T. I. M." calculating machine, and seven-figure natural sines, were employed for the work here described.

Owing to the time and labour involved in exact methods of studying the unit surfaces, various approximate methods have been in use.

Outline of Present Theoretical Knowledge.—For the chief facts stated in this outline the writer is also indebted to Mr. T. Smith. The conclusions "A" and "B" are from his article mentioned at the beginning of this Paper, and the paragraphs in quotation marks are from a note which he very kindly communicated on receiving the writer's results, in order to facilitate comparison. An earlier Paper may also be referred to (T. Smith, "Principles and Methods of Calculating Telescope Objectives," Transactions of the Optical Society, Vol. 18, 1917, p. 160; and Collected Researches of the National Physical Laboratory, Vol. 14, 1920, p. 99), where in particular it is pointed out that in thin systems the curvatures of the unit surfaces afford a measure of the amount of first order coma present in the image.

The unit (or principal) surfaces play a prominent part in the design of optical instruments. In elementary treatment based on a collinear, or point to point theory, they are taken as fixed planes, and their positions with respect to the foci give a general idea of the course of the rays, the position and size of the image, and the scope of the instrument. The collinear theory, however, in which small quantities above the first order are neglected, only holds for paraxial rays, and is insufficient when dealing with real instruments, especially photographic lenses where large inclinations and apertures come into question. Consideration of a lens in the form of a solid sphere, in which any diameter may be taken as optic axis, will show the fallacy of the assumption that the principal surfaces are planes.

The next obvious step is to take into account some or all of the small quantities of the next order available—the so-called first order aberration terms. In some cases a good deal of information may thus be gained without taking the thickness into account, simply regarding all the refracting surfaces as cutting the axis in the same point, in which case the two axial unit points coincide with one another at this point.

For any such combination of thin lenses in air, it is known that with approximately unit magnification, images near the axis show no astigmatism, and are free from spherical aberration and coma. Consequently the image of a surface is here a single surface.

For thin lenses, when the rays are not greatly inclined to the axis:

A (1) The unit surfaces are in general spherical.
(2) The curvatures depend on the lens shapes.
(3) For a system of given power, the difference between the curvatures of the object unit surface and the image unit surface is a constant independent of the lens shapes.
(4) The difference under (3) is of opposite sign to the difference between the incident and emergent wave-fronts as they pass the axial unit points.
(5) The proper adjustment of the curvatures of the unit surfaces is essential to removal of aberrations.

The consideration of small quantities of the third order (first order aberrations) is insufficient to show whether the unit surfaces are fixed or not.

"For thick systems it is theoretically possible to obtain entirely free from all aberrations a plane image of a plane object for one particular magnification, and simultaneously abolish spherical aberration for the axial unit points. Then for beams with chief rays passing through these points the image will be free from all aberrations depending on skewness of rays, not only for one particular magnification, but for all magnifications. This is the most nearly perfect state of correction it is possible to achieve in a thick coaxial system for monochromatic light. It is not known how closely any real system can be constructed to agree with these properties, for in practice the surfaces must almost invariably be spherical, and the properties of stable transparent solids available as refracting media are exceedingly circumscribed. In addition, the necessity for fulfilling chromatic requirements, and considerations affecting the cost of manufacture, may necessitate deliberate departure from otherwise desirable conditions."

A thick system corrected in the theoretically most desirable manner has the following properties :—

B (1) There are no fixed unit surfaces, but a separate pair of surfaces for every object point in the aberrationless plane.
(2) These unit surfaces are spherical.

(3) The centres of curvature of the unit surfaces are the aberrationless object and image points to which they correspond, and consequently

(4) The unit surfaces coincide with the wave-fronts as they pass through the unit points.

"The contradiction under (4) in the properties of the thin and ideal thick lenses is of particular interest, and accounts for some of the imperfections in results based on consideration of thin systems, imperfections which are most manifest when the aperture and field of view are large. Moreover as photographic lenses aim at reaching the conditions laid down for the ideal lens, but are composed of elements which are not far removed from thin lenses, it is instructive to see which type they approach more closely. It is assumed that the lenses are corrected for infinity, or for an object so distant that correction for infinity may be assumed. In this case, both in the thin and the ideal thick systems, the second unit surface would be made to coincide with emergent wave position, and any failure to reach the proper spherical form for these image-space surfaces will correspond to errors which the designer has consciously attempted to remove."

"From the point of view of the present investigation, particular interest attaches to the shape of the first unit surfaces. In a perfect lens they would be plane. In a thin lens their curvatures would be greater than those of their images, and of the same sign. The designers in all probability have made no conscious attempt to control their shapes. A study of the shapes of the first unit surfaces should be of value to lens designers."

Coming now to the present problem, it is well known that a small pencil of rays from an object point will in general after refraction no longer converge to a point; a given ray will only meet two of the consecutive rays originally surrounding it. If the chief ray of the thin pencil lies in an axial plane, the chief ray will meet the consecutive rays in the meridional and sagittal planes, and will meet them in different points, the primary and secondary foci respectively. If we consider a family of chief rays obeying some law, for instance, rays passing through a fixed point, then as the object point describes a plane perpendicular to the axis, the primary and secondary foci will generally describe two different curved surfaces. In other words, the image of a plane perpendicular to the axis is not a plane, but a pair of curved surfaces. By suitably designing a lens this defect can be abolished for a particular position of object and image, or for a particular magnification, but not for all magnifications. In an ordinary photpgraphic lens the particular magnification selected for correction would naturally be that corresponding to the most usual position of the plate, and would be considerably less than unity. Hence for all other magnifications, including unity, curvature and astigmatism might be expected in the image.

The present Paper deals only with the secondary focus, consideration of the primary focus being deferred. The secondary focus is the simpler to deal with because it lies in the same axial plane as the object point, and, for a general ray, we may follow Mr. Smith in defining conjugate points as the points where a ray cuts an axial plane. The relation between object and image points is clearly a reciprocal one.

Taking the optic axis as axis of x, the first vertex as origin in the object space, the last vertex as origin in the image space, and the direction the light travels as positive,

let u be the distance of object point P from first surface measured along the ray.
v distance of secondary focus Q from last surface measured along the ray.
x_0, y_0, z_0, co-ordinates of P.
x_n, y_n, z_n co-ordinates of Q.
X_1, Y_1, Z_1 co-ordinates of point of incidence.
X_n, Y_n, Z_n co-ordinates of point of exit.
L_0, M_0, N_0 direction-cosines of incident ray.
L_n, M_n, N_n direction-cosines of refracted ray,

The expression of the fact that P and Q are conjugate gives one relation between u and v. If another relation is given, such as the magnification for small transverse lengths through Q and P (measured by the ratio of the distances of Q and P from the optic axis), then u and v can both be found. It is to be noted that the solution is unique.

Extremely neat and convenient formulæ for finding u and v in terms of the magnification m for a known ray have been given by Mr. T. Smith elsewhere ("On Tracing Rays through an Optical System," Proc. Phys. Soc., Vol. 27, 1915, p. 508), namely

$$u = U + \frac{\mu_0}{mK} \quad . \quad . \quad . \quad (1)\,; \qquad v = V - \frac{\mu_n \, . \, m}{K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(the lettering is altered). The constants U, V are evidently the values of u, v for $m = \pm\infty$, $m = \pm 0$ respectively, and the corresponding points may be termed the principal oblique foci for the ray. K is a quantity which may be called the oblique power for the ray. If ξ, ξ' be the distances of any pair of conjugate points from these foci, then $\xi \xi' = -\frac{\mu_0 \mu_n}{K^2}$, just as on the axis.

U, V, K are calculated in the same way as the corresponding quantities in the analogous paraxial formulæ, except that, in the expression for the power k_λ of a single surface, $\mu_\lambda \cos \varphi'_\lambda - \mu_{\lambda-1} \cos \varphi_\lambda$ must be written instead of $\mu_\lambda - \mu_{\lambda-1}$, and the thickness t_λ must be measured obliquely along the ray instead of along the axis.

If the values u'', v'' of u, v for any magnification m'' have been found, the values for any other magnification m can be deduced in a moment from the formulæ (1, 2), since evidently

$$u - u'' = \frac{\mu_0}{K}\left(\frac{1}{m} - \frac{1}{m''}\right) \quad . \quad . \quad . \quad (3)\,; \qquad v - v'' = -\frac{\mu_n}{K}(m - m'') \quad . \quad . \quad . \quad . \quad . \quad (4)$$

A similar remark applies to the rectangular co-ordinates (x, y, z) (x'', y'', z'') of the conjugate points for the two magnifications. For the incident ray

$$x_0 - x_0'' = \frac{\mu_0 L_0}{K}\left(\frac{1}{m} - \frac{1}{m''}\right)\,; \quad y_0 - y_0'' = \frac{\mu_0 M_0}{K}\left(\frac{1}{m} - \frac{1}{m''}\right)\,; \quad \text{\&c.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For the emergent ray

$$x_n - x_n'' = -\frac{\mu_n L_n}{K}(m - m'')\,; \quad y_n - y_n'' = -\frac{\mu_n M_n}{K}(m - m'') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Suppose P, Q to be a pair of conjugate points for which there is accurate point-to-point representation, so that all rays from P are refracted through Q, and let m

be the corresponding magnification. Then, if ρ be the distance of the 1st unit point from P, and σ the distance of the 2nd unit point from Q, on any such ray, (3) (4) give

$$\rho = \frac{\mu_0}{K}\left(\frac{1}{m}-1\right);\ \sigma = -\frac{\mu}{K}(m-1)\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \quad (7)$$

m is obviously constant for all rays through P and Q. If K is also constant for all such rays, then the 1st unit point lies on a circle round P with radius $\frac{\mu_0}{K}\left(\frac{1}{m}-1\right)$, and the 2nd unit point lies on a circle round Q, with radius $-\frac{\mu_n}{K}(m-1)$. If P lies at infinite distance, then

$$\rho = \infty\ ;\ \sigma = -\frac{\mu_n}{K}.\ \ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \quad (8)$$

It can be proved that K is constant for rays through any pair of object and image points on the aberrationless planes in the ideal thick lens. Hence the results in "B" above.

After the rays required for the investigation had been traced by exact methods through the lens systems, the values of u and v corresponding to unit magnification were then calculated by the Smithian formulæ (1) (2), thus fixing the positions of the unit points. For convenience in plotting, the rectangular co-ordinates of the unit points were then determined from these data.

The main investigation is confined to rays in an axial plane, since these afford an insight into the performance of the system, while involving much less time and labour than three-dimensional rays. In the case of Cooke, however, a few rays in three dimensions are included, and are tabulated separately in Table VII. They are incident at 20° to the axis in planes parallel to the plane ($z=0$) of the axial rays at

(I) a point on rim of 1st surface 45° from bottom.
(II) ,, ,, ,, 90° ,,
(III) the point of intersection of the planes $y=0$ and $z=2/3$ radius of rim.

Also in the case of the Cooke some axial rays have been included which would ordinarily be stopped by mechanical means, and, as might be expected, the loci show extraordinary forms in these regions, for which the lens is not designed. In the Tessar the case is different; its sixth and seventh surfaces, forming the last convex lens, intersect at a distance of only 10·221 mm. from the axis, and obviously rays incident on the surfaces beyond this limit must not be considered, since even if the space between the produced surfaces were filled with glass the rays would strike them in the wrong order. The ray for $\psi_0=5°$, full aperture, has however been included because it passes so near the limit, though just outside it, that any error caused thereby must be trifling.

The results of the investigation are given in the following tables.

It will be noticed that the paraxial unit points, or principal points, for the Tessar are in the middle of the lens, and only 1·0525 mm. apart, while for the Cooke they fall a little way behind the first surface, and are only 0.6755 mm. apart. The points are not "crossed" in either case, the second principal point being on the image side of the first principal point.

While the main object of the investigation was to find the loci of the unit points (or conjugate points for unit magnification) for oblique rays and large apertures, the positions of conjugate points for magnifications —0, $-\infty$, and (for the Tessar) —1, have also been calculated. These however are not given here, as the printing would be costly, and they can be computed from the data for the unit points by formulæ (5) (6) with the aid of the ray-constants which are tabulated below.

In the following tables, the axis of x is the optic axis, the origin being taken at the first vertex for the object, and at the last vertex for the image. The incident ray is defined by (1st) the inclination ψ_0 to the optic axis, and (2nd) the aperture. For the present purpose aperture is measured by the length of the perpendicular on the ray from the first vertex, the description "full aperture" signifying that this perpendicular is equal to the semi-diameter of the front lens.

TESSAR PHOTOGRAPHIC LENS, 1907.

$f_D = 100$. Aperture ratio $= \frac{f}{4\cdot5}$

	Radius.	Thickness.	μ
1	+ 26·3		
		3·7	1·61342
2	∞		
		4·0	1
3	− 58·1		
		1·7	1·57391
4	+ 23·9		
		5·3	1
5	−146·7		
		1·7	1·53
6	+ 22·3		
		4·6	1·61451
7	− 36·3		
		Total 21·0	

TABLE I.—*Co-ordinates of Conjugate Points for Unit Magnification.*
(Unit Points.)

Ray.		Object.		Image.	
ψ_0	Aperture.	x_0	y_0	x_n	y_n
0°	0	+8·79608	0	−11·15139	0
	$+\frac{1}{3}$	8·80748	$+3\cdot\dot{7}0\dot{3}$	−11·08460	+ 3·70362
	$+\frac{2}{3}$	8·85050	$7\cdot\dot{4}0\dot{7}$	−10·87075	+ 7·40738
	+Full	8·95401	$11\cdot\dot{1}$	−10·46590	$+11\cdot\dot{1}$
5°	−Full	9·87600	−10·28952	− 9·63045	−10·28952
	$-\frac{2}{3}$	9·47512	− 6·60674	−10·31733	− 6·60672
	$-\frac{1}{3}$	9·10210	− 2·92152	−10·83964	− 2·92137
	0	8·75133	+ 0·76565	−11·22177	+ 0·76565
	$+\frac{1}{3}$	8·42827	+ 4·45523	−11·46357	+ 4·45523
	$+\frac{2}{3}$	8·16195	+ 8·14978	−11·53380	+ 8·14979
	+Full	8·05315	+11·85811	−11·32532	+11·85809
20°	−Full	12·29516	− 7·34912	− 7·86657	− 7·34909
	$-\frac{2}{3}$	10·88695	− 3·92027	− 9·54039	− 3·92028
	$-\frac{1}{3}$	9·53630	− 0·47047	−10·98399	− 0·47046
	0	8·29120	+ 3·01776	−12·19216	+ 3·01775
	$+\frac{1}{3}$	7·36882	+ 6·62343	−12·99609	+ 6·62346

TABLE II.—*Ray Constants.*

Oblique power.	Object.		Image.	
$\frac{1}{K}$	$\frac{\text{Cos } \psi_0}{K}$	$\frac{\text{Sin } \psi_0}{K}$	$\frac{\text{Cos } \psi_n}{K}$	$\frac{\text{Sin } \psi_n}{K}$
99·80777	+99·80777	0	+99·80777	0
99·61340	99·6134	0	99·5445	− 3·70376
99·2357	99·2357	0	98·9589	− 7·40739
99·6087	99·6087	0	98·9870	−11·1111
99·8553	99·4753	+ 8·70295	98·0324	+18·9925
99·6036	99·2246	8·68102	98·4234	+15·2877
99·9713	99·5909	8·71307	99·2919	+11·6349
100·1162	99·7352	8·72570	99·7992	+ 7·96007
99·8629	99·4829	8·70362	99·7725	+ 4·24841
99·4283	99·0499	8·66574	99·4269	+ 0·51591
99·7304	99·3509	8·69208	99·6802	− 3·16600
105·6271	99·2570	36·1266	96·2649	+43·4757
105·5281	99·1640	36·0927	97·6480	+40·0130
105·7371	99·3604	36·1642	99·1879	+36·6347
105·4313	99·0730	36·0596	100·1199	+33·0418
96·5888	97·5359	35·5002	99·6978	+28·8767

TABLE III.—*Curvature of Unit Surfaces of Tessar Lens.*

Radii and Centres of Circles through Unit Points on Certain Rays (ψ_0=Inclination to Axis of Incident Ray).

Rays.				Object.			Image.		
ψ_0	Apertures.			Radius.	Co-ordinates of centre.		Radius.	Co-ordinates of centre.	
					x	y		x	y
0°	$-\frac{1}{3}$	0	$+\frac{1}{3}$	601·65	610·44	0	102·72	91·573	0
	$-\frac{2}{3}$	0	$+\frac{1}{3}$	504·16	512·95	0	97·898	86·747	0
	−Full	0	+Full	390·94	399·73	0	90·392	79·241	0
5°	$-\frac{1}{3}$	0	$+\frac{1}{3}$	492·93	499·63	45·605	98·012	86·302	9·0098
	$-\frac{2}{3}$	0	$+\frac{2}{3}$	406·71	413·86	36·816	92·827	81·293	8·3737
	−Full	0	+Full	288·49	296·27	24·431	83·362	71·900	7·0883
20°	$-\frac{2}{3}$	$-\frac{1}{3}$	0	139·25	123·95	42·336	59·089	44·220	20·603*
	−Full	$-\frac{1}{3}$	$+\frac{1}{3}$	87·36	91·92	28·580	50·087	36·065	16·707

* A circle with same centre, but radius 58·90, would pass through unit point on ray at $+\frac{1}{3}$ Ap., and a circle with same centre, but radius 59·11, would pass through unit point on ray at −Full Ap.

COOKE PHOTOGRAPHIC LENS, 1894.

$f_D = 100$. Aperture ratio $= f/7{\cdot}7$.

	r	t	μ
1	+14·6		
		2·99	1·6114
2	−101·3		
		0·38 air	1
3	−55·9		
		0·46	1·5482
4	+13·3		
		8·95 air	1
5	+1012·0		
		1·83	1·6114
6	−69·8		
	Total	14·61	

TABLE IV.—*Co-ordinates of Unit Points.*

Ray.		Object.		Image.	
ψ_0	Ap.	x_0	y_0	x_n	y_n
0°	0	+1·318334	0	−12·61616	0
	+$\frac{2}{3}$	1·34903	+4·329004	−12·5070	+4·32885
	+Full	1·43258	6·493506	−12·3296	+6·49350
20°	−Full	3·67797	−5·57158	−10·33238	−5·57155
	−$\frac{2}{3}$	2·87204	−3·56149	−11·2484	−3·56146
	0	1·29869	+0·47268	−12·8903	+0·47274
	+$\frac{2}{3}$	−0·83603	+4·30254	−14·7926	+4·30246
	+Full	−2·522	+5·99	−15·94	+6·046
30°	−Full	+4·90668	−4·66518	− 9·52814	−4·66521
	−$\frac{2}{3}$	3·84048	−2·78140	−10·7561	−2·78143
	0	1·39853	+0·8072	−13·335	+0·8082

TABLE V.—*Ray Constants.*

$\frac{1}{K}$	Object.		Image.		
	$\frac{\cos\psi_0}{K}$	$\frac{\sin\psi_n}{K}$	$\frac{\cos\psi_n}{K}$	$\frac{\sin\psi_n}{K}$	
100·36558	100·36558	0	100·3656	0	
100·26068	100·2607	0	100·1672	4·3289	
100·8236	100·8236	0	100·6143	6·4935	
108·0561	101·5395	36·9574	99·3349	42·5288	
105·7992	99·4187	36·1854	98·0492	39·7470	
106·3902	99·9741	36·3876	100·1450	35·9149	
109·4811	102·8785	37·4447	104·3464	33·1422	Stopped
113·39	106·55	38·78	108·55	32·76	Stopped
119·1260	103·1661	59·5630	100·3292	64·2281	
115·1086	99·6870	57·5543	98·0285	60·3357	
120·4529	104·316	60·2269	104·7781	59·4187	Stopped

TABLE VI.—*Curvature of Unit Surfaces of Cooke Lens.*

Radii and Centres of Circles through Unit Points on Certain Rays.

Rays.				Object.			Image.		
ψ_0	Apertures.			Radius.	Co-ordinates of centre.		Radius.	Co-ordinates of centre.	
0°	$-\frac{2}{3}$	0	$+\frac{2}{3}$	305·23	306·55	0*	85·874	73·258	0
	−Full	0	+Full	184·59	185·91	0	73·709	61·092	0
	−Full	$-\frac{2}{3}$	0	103·14	104·45	+1·4333	57·489	44·868	0·7161
20°	−Full	$-\frac{2}{3}$	0	346·2	322·0	123·2	79·92	61·93	28·57

* The unit points on the rays through full aperture lie only 0·104 mm. within this circle.

COOKE LENS.—RAYS NOT IN AN AXIAL PLANE.

$L_0=\cos 20°=0·9396926, \quad M_0=\sin 20°=0·3420201, \quad N_0=0°.$

Ray.	Co-ordinates of Point of Incidence.		
I. at 45° Pt.	+1·5235183	−4·5916025	+4·5916025
II. ,, 90° ,,	+1·5235183	0	+6·4935065
III. ,, Pt. $\frac{2}{3}$ horizontally	+0·6565528	0	+4·329004

TABLE VII.—*Co-ordinates of Unit Points.*

Ray.	Object.			Image.		
	x_0	y_0	z_0	x_n	y_n	z_n
I.	+3·08357	−4·02379	+4·59160	−10·90850	−4·02379	+4·59160
II.	+1·33969	−0·0669	+6·49351	−12·60245	−0·06631	+6·49345
III.	+1·30300	+0·23529	+4·329004	−12·78241	+0·23527	+4·32900

TABLE VIII.—*Ray Constants.*

Ray	$\frac{1}{K}$	Object.			Image.		
		$\frac{L_0}{K}$	$\frac{M_0}{K}$	$\frac{N_0}{K}$	$\frac{L_n}{K}$	$\frac{M_n}{K}$	$\frac{N_n}{K}$
I.	108·1397	101·6180	36·9859	0	99·9569	41·0097	−4·59160
II.	108·4210	101·8824	37·0822	0	101·6509	37·1488	−6·49345
III.	106·8006	100·3598	36·5280	0	100·3529	36·2927	−4·32900

The co-ordinates of conjugate points for any magnification *m* can quickly be calculated from the tabulated co-ordinates of the unit points by (5) (6). For rays parallel to $z=0$ we have for the incident side

$$x=x_0-\frac{\cos\psi_0}{K}\left(1-\frac{1}{m}\right) \qquad y=y_0-\frac{\sin\psi_0}{K}\left(1-\frac{1}{m}\right)$$

and for the emergent side

$$x=x_n+\frac{\cos\psi_n}{K}(1-m) \qquad y=y_n+\frac{\sin\psi_n}{K}(1-m)$$

Example.

The figures are for rays incident at 20° to the axis at—full aperture.

TABLE IX.—*Co-ordinates of other Conjugate Points.*

Ray $\psi_0 = 20°$. Ap.—Full.

Magnification.	Object.		Image.	
	x	y	x	y
-0	$-\infty$	$-\infty$	$x_n + \frac{\cos \psi_n}{K}$	$y_n + \frac{\sin \psi_n}{K}$
			(Second Prin. Oblique Focus)	
Tessar	$-\infty$	$-\infty$	$+88 \cdot 3984$	$+36 \cdot 1266$
Cooke	$-\infty$	$-\infty$	$+89 \cdot 0026$	$+36 \cdot 9573$
$-\infty$	$x_0 - \frac{\cos \psi_0}{K}$	$y_0 - \frac{\sin \psi_0}{K}$	$+\infty$	$+\infty$
	(First Prin. Oblique Focus)			
Tessar	$-86 \cdot 9618$	$-43 \cdot 4757$	$+\infty$	$+\infty$
Cooke	$-97 \cdot 8616$	$-42 \cdot 5289$	$+\infty$	$+\infty$
-1	$x_0 - \frac{2 \cos \psi_0}{K}$	$y_0 - \frac{2 \sin \psi_0}{K}$	$x_n + \frac{2 \cos \psi_n}{K}$	$y_n + \frac{2 \sin \psi_n}{K}$
(Inverted equal image)				
Tessar	$-186 \cdot 219$	$-79 \cdot 6023$	$+184 \cdot 663$	$+79 \cdot 6024$
Cooke	$-199 \cdot 401$	$-79 \cdot 4864$	$+188 \cdot 337$	$+79 \cdot 4861$

As a matter of fact the positions of the oblique principal foci are found in the process of determining the unit points.

CONCLUSIONS.

(As stated above, only secondary or transverse foci are here considered.)

General.

It is at once evident from the results that for chief rays in an axial plane the unit points on either side lie scattered over a considerable area, and hence for general rays in space the loci are three-dimensional. The loci of the unit points will only be surfaces when the chief rays are bound by some condition, such as passing through a fixed point.

Consider, for example, rays from a given object point at infinity, or parallel rays of given inclination. The surface locus is represented in the plane of the paper by a line section. The results show that

C (1) The unit surfaces are not fixed, but shift with the inclination of the parallel rays, *i.e.*, withthe position of the infinitely distant object point.

(2) The unit surfaces are not spherical, the curvature of the plane section changes with aperture.

(3) The curvatures alter also with the inclination of the rays, *i.e.*, with the distance from the axis of the infinitely distant object point.

(4) In all cases examined, within the region for which the lens is designed, the curvature of both first and second unit loci for parallel chief rays is positive (convex to the light source), and the second locus is more curved than the first.

(5) The unit surfaces not being plane or spherical cannot coincide with wave-fronts.

Comparing these results with "A" and "B" in the "outline" above, on the assumption that both lenses are corrected for infinity, or, at least, for a very distant object, it is seen that the facts do not coincide with the theoretical conclusions either for a thin or an ideal thick system, but that they accord more nearly with the latter, especially as regards (4) the sign of the difference of curvatures of the two unit surfaces.

If instead of a pencil of rays from a given object point, we take rays satisfying some other condition, such as passing through the vertex of the lens, or having a given aperture, *i.e.*, having H, the perpendicular on the ray from the vertex, given, then quite different forms are obtained for the unit surfaces.

So much for general deductions. Some individual characteristics of the lens may now be considered.

Tessar.

For parallel rays, the curvature of the unit surfaces, which, as might be expected, is very slight for zero inclination near the axis, appears to increase fairly regularly with inclination, and with aperture.

For rays through the vertex of the lens, or aperture $=0$, the plane section of the unit locus is a rather small arc curving rapidly to the object side with increasing inclination, the curvature being more marked in the second locus. With increasing apertures the arcs lengthen. For aperture $-2/3$, the arcs slope down to the right on the diagrams; for $-$full aperture the unit points for inclinations 0° and 20° are nearly level. The aggregate second locus for rays in the plane of the paper appears to narrow almost to a point on the optic axis in the region of the unit point for paraxial rays, and then to widen rapidly as the distance from the axis increases. The first locus, however, appears to have a narrow neck about 1·5 mm. above the axis, near where the unit point for inclination 10° would lie, and to widen rapidly above and below this neck.

Cooke.

For parallel rays the 20° locus appears to be flatter than the corresponding portion of the 0° locus, but it is difficult to compare pencils of widely different inclination, where rays of the same aperture play different roles. For rays through the vertex, of inclinations 0° to 30°, the diagram shows that the first unit surface, or, rather, its section, curves only slightly, and to the right or image side, while the second surface curves quickly to the left. The loci for apertures $-2/3$, and $-$full, slope up to the right or image side.

The writer's best thanks are due to Mr. T. Smith for encouragement and facilities granted when starting the work, and for valuable criticism and information when discussing the completed results.

VIII. *Vibration Galvanometers with Asymmetric Moving Systems. By* R. LL. JONES.

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ABSTRACT.

In the theory of the vibration galvanometer with one degree of freedom certain specified conditions are assumed to hold. If these conditions are not satisfied in a galvanometer, the instrument will show multiple resonance. This indicates the need of a more general theory. The theory of vibrations of a system with two degrees of freedom is briefly given, expressions for the amplitudes of the forced vibrations are deduced, and the conditions for resonance ascertained. The results are applied to a galvanometer in which the moving system is asymmetrically hung on a laterally yielding axis, and it is shown that the formula for the amplitude is capable of reproducing with fair accuracy the sensitivity curve of the galvanometer, which shows multiple resonance. Asymmetry always lowers the sensitivity of the resonance, and the loss can be ascertained by the help of the sensitivity curve. The analogy to the vibrations in coupled circuits is pointed out.

I wish to express my gratitude to A. Campbell for helpful criticism, advice and references.

CONTENTS.

I. Theory of vibration galvanometer with one degree of freedom.
II. Vibration galvanometer showing multiple resonance.
III. Resonance in vibrating systems with two degrees of freedom.
IV. Vibrations of Asymmetric systems.
  (*a*) The equations of motion.
  (*b*) The forced vibrations.
  (*c*) The free vibrations.
  (*d*) Application.
V. Effect of asymmetry on sensitivity.
IV. Mechanical analogue of coupled circuits.
VII. Conclusion ; Summary.

I. THEORY OF THE VIBRATION GALVANOMETER WITH ONE DEGREE OF FREEDOM.

THE theory of the vibration galvanometer has been considered by Wenner,* A. Campbell,† Butterworth,‡ and others.§ In most of these investigations the moving system is supposed to have only one degree of freedom—namely, freedom to rotate about the axis of suspension and control. The theory so developed is applicable to galvanometers in which

(*a*) The axis is rigid and does not yield to forces tending to produce lateral displacements ;

(*b*) The axis, although not rigid, passes through the centre of mass of the moving system and is a principal axis at that point.

For, in the first case, any reactions on the axis called into play by the angular motion of the system can produce no displacement ; while, in the second case, no reactions arise and no constraint is necessary in order to secure that the displacement of the system shall continue to be one of angular rotation alone round the axis, provided that the external forces which may be impressed on it reduce to a couple round this axis. The impressed forces will reduce to a couple if the field and the moving system have this axis as an axis of symmetry.

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* Wenner ; Bulletin Bureau of Standards, Vol. 6, pp. 347 (1910).

† A. Campbell ; Proc. Phys. Soc., Vol. 26, Part 2, pp. 120 (1914).

‡ Butterworth ; Proc. Phys. Soc., Vol. 24, Part 2, pp. 75 (1912).

§ Full references will be found in Dictionary of Applied Physics, Vol. 2, pp. 979-980.

In a vibration galvanometer the axis is usually not rigid, and it may happen through errors of construction that it does not pass through the centre of mass of the moving system. The system may be mechanically and electrically unsymmetrical with regard to the axis, and the field may also be unsymmetrical. In these circumstances the motion of the system will not be restricted to simple angular displacements round the axis and a more general theory is required.

II. VIBRATION GALVANOMETER SHOWING MULTIPLE RESONANCE.

It is well known that if the free period of a vibration galvanometer with only one degree of freedom be kept constant while the frequency of the current passing through it is changed, the galvanometer shows high sensitivity at and near the resonating frequency. At all other frequencies differing by more than a few per cent. from the resonating frequency the sensitivity is small. Rosa and Grover,[*] however, found that a vibration galvanometer may show high sensitivity at two different frequencies. For instance, one of the galvanometers they used gave a maximum sensitivity at a frequency of 110·6 and also at 120 periods per second, while at 114 the sensitivity was a minimum. The diagram in Fig. 1 shows the sensitivity of their galvanometer over the range 106 to 126 per second. They draw special attention to one remarkable feature of this diagram—namely, that at frequencies between 110·6 and 120 the sensitivity is much lower than it is immediately outside these values. The ordinary theory is incapable of accounting for these observations.

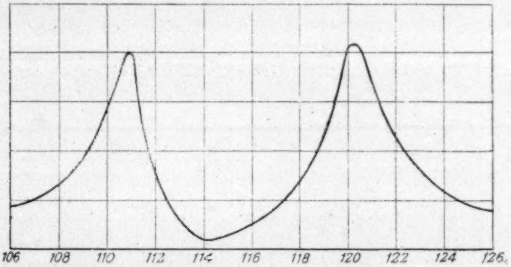

FIG. 1.

The moving system of this galvanometer was a magnet. It was attached to a wire clamped at two points, and the control could be varied by altering the distance between these points and the tension of the wire. Wenner[†] states that this axis did not pass through the centre of mass of the moving system, and that this lack of mechanical symmetry was introduced on the repair of a broken suspension. The system had evidently more than one degree of freedom, owing to the lateral yielding of the wire during the motion of the unbalanced system round it, and in attempting to account for its motion and mechanical behaviour we will apply the theory of the vibrations of a system with two degrees of freedom. A brief outline of the theory and some deductions from it applicable to the vibration galvanometer, more especially the conditions for resonance, are given in § III. below.* It will be shown that the theory gives a formula for the amplitude of the forced vibration which is capable of reproducing with considerable accuracy the curve of sensitivity of Rosa and Grover's galvanometer over the range of frequencies given in Fig. 1, and enables us to deduce from the sensitivity curve the mechanical caracteristics of the moving system.

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[*] Rosa and Grover; Bulletin Bureau of Standards, No. 3, p. 298 (1905).

[†] *Loc. cit.*

III. Resonance in Vibrating Systems with Two Degrees of Freedom.

Consider a system with two degrees of freedom making small vibrations about a position of stable equilibrium. Let its position at any instant be defined by two independent co-ordinates q_1, q_2, and let $q_1=0$, $q_2=0$ in the position of equilibrium. Then if q_1 and q_2 are suitably chosen, we can express the kinetic energy T, the dissipation function F, and the potential energy V of the system in the forms

$$\left.\begin{aligned} T&=\tfrac{1}{2}(a_1\dot{q}_1^2\times 2a_{12}\dot{q}_1\dot{q}_2+a_2\dot{q}_2^2)\\ F&=\tfrac{1}{2}(b_1\dot{q}_1^2\qquad\qquad +b_2\dot{q}_2^2)\\ V&=\tfrac{1}{2}(c_1q_1^2\qquad\qquad +c_2q_2^2)\end{aligned}\right\}\tag{1}$$

in which $a_1 \ldots b_1 \ldots\ldots c_2$ are constants, as we are considering small oscillations.

By Lagrange's method, the equations of motion of the system are

$$\left.\begin{aligned} a_1\ddot{q}_1+b_1\dot{q}_1+c_1q_1+a_{12}\ddot{q}_2&=Q_1\\ a_{12}\ddot{q}_1+a_2\ddot{q}_2+b_2\dot{q}_2+c_2q_2&=Q_2\end{aligned}\right\}\tag{2}$$

where $Q_1\,Q_2$ are the applied forces of corresponding types.

Let $Q_1=e^{ipt}$ and $Q_2=0$ in (2), where $i=\sqrt{-1}$. Writing D for $\frac{d}{dt}$ we get

$$q_1=\left[\frac{a_2D^2+b_2D+c_2}{(a_1D^2+b_1D+c_1)(a_2D^2+b_2D+c_2)-a_{12}^2D^4}\right]e^{ipt}$$

$$=\frac{(c_2-a_2p^2)+i\,.\,b_2p}{\{(c_1-a_1p^2)+i\,.\,b_1p\}\{(c_2-a_2p^2)+i\,.\,b_2p\}-a_{12}^2\,.\,p^2}\,.\,e^{ipt}$$

$$\left.\begin{aligned} &=\frac{S_2}{R}\,e^{i(pt-\chi\cdot\psi_2)}\\ \text{and}\qquad q_2&=\frac{a_{12}\cdot p^2}{R}\,e^{i(pt-\chi)}\end{aligned}\right\}\tag{3}$$

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* A fuller treatment of the theory is given in Rayleigh's Theory of Sound, Vol. 1, Chap. 4.

Similarly, when $Q_1=0$, $Q_2=e^{ipt}$, we have

$$\left.\begin{aligned} q_1&=\frac{a_{12}\cdot p_2}{R}\,e^{i(pt-\chi)} \\ q_2&=\frac{S_1}{R}\,e^{i(pt-\chi+\psi_1)} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

In (3) and (4)

$$\left.\begin{aligned} S_1{}^2&=a_1{}^2p^4\left[x_1{}^2+\frac{\lambda_1{}^2}{p^2}\right] \\ S_2{}^2&=a_2{}^2p^4\left[x_2{}^2+\frac{\lambda_2{}^2}{p^2}\right] \\ R^2&=a_1{}^2a_2{}^2p^8\left[\left(x_1x_2-\delta-\frac{\lambda_1\lambda_2}{p^2}\right)^2+\left(\frac{\lambda_2x_1+\lambda_1x_2}{p}\right)^2\right] \\ 1_{\tan}\psi_1&=\lambda_1/px_1 \\ 1_{\tan}\psi_2&=\lambda_2/px_2 \\ 1_{\tan}\chi&=\left(\frac{\lambda_2x_1+\lambda_1x_2}{p}\right)\Big/\left(x_1x_2-\delta-\frac{\lambda_1\lambda_2}{p}\right) \end{aligned}\right\} \quad . \; . \; . \quad (5)$$

where

$$\left.\begin{aligned} X_1&=\frac{\omega_1{}^2-p^2}{p^2} & X_2&=\frac{\omega_2{}^2-p^2}{p^2} \\ \omega_1{}^2&=\frac{c_1}{a_1} & \omega_2{}^2&=\frac{c_2}{a_2} \\ \lambda_1&=\frac{b_1}{a_1} & \lambda_2&=\frac{b_2}{a_2} \\ & & \delta=\frac{a_{12}{}^2}{a_1a_2} \end{aligned}\right\} \quad . \; . \; . \quad (6)$$

The real parts of (3) give the vibrations in q_1 and q_2 when $Q_1=\cos pt$ and $Q_2=0$.

When $Q_1=0$ and $Q_2=\cos pt$ the vibrations in q_1 and q_2 are given by the real parts of (4).

From these it follows that when $Q_1=\cos pt$ and $Q_2=r\cos(pt+\varphi)$ the vibration in q_2 is given by

$$q_2=\frac{1}{R}\left[rS_1\cos(pt+\varphi-\chi+\psi_1)+a_{12}p^2\cos(pt-\chi)\right] \quad . \; . \; . \; . \; . \quad (7)$$

This vanishes provided

$$\left.\begin{aligned} \varphi+\psi_1&=\pi \\ r&=a_{12}\cdot p^2/S_1 \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

From (3) . . . (8) we get the following :—

(i) If $a_{12}=0$, the forced oscillation in q_2 due to Q_1 given in (3) vanishes. In this case the co-ordinates q_1, q_2 are normal co-ordinates, and the oscillation in q_1

is the same as if the system had only one degree of freedom. Since $\delta=0$ when $a_{12}=0$, R will be given by

$$R^2=a_1^2a_2^2p^8\left[\left(X_1^2+\frac{\lambda_1^2}{p^2}\right)\left(X_2^2+\frac{\lambda_2^2}{p^2}\right)\right]$$

and

$$\frac{S_2}{R}=\frac{1}{a_1\{(\omega_1^2-p^2)^2+\lambda_1^2p^2\}^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which is identical with the expression for the amplitude in a system with only one degree of freedom.

(ii) The values of ω_2 and p being given, let us determine what value ω_1 must have in order that S_2/R in (3) may be a maximum. The amplitude S_2/R will be a maximum when

$$\omega_1^2=p^2\left[1+\delta X_2\Big/\left(X_2^2+\frac{\lambda_2^2}{p^2}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

When $\delta=0$ relation (10) reduce to

$$\omega_1=p$$

which is the condition for resonance in a system with one degree of freedom when resonance is obtained by tuning the system.

When ω_1 has the value given in (10)

$$S_2/R=\frac{1}{a_1p}\left(X_2^2+\frac{\lambda_2^2}{p^2}\right)\Big/\left[\lambda_1\left(X_1^2+\frac{\lambda_1^2}{p^2}\right)+\lambda_2\delta\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

When $\delta=0$ relation (11) reduces to

$$S_2/R=\frac{1}{a_1\lambda_1p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

which is that (9) reduces to when $\omega_1=p$ and gives the amplitude at resonance in a system with one degree of freedom.

Denoting the amplitude in (11) by $A\delta$ and that in (12) by A_0, we have

$$A\delta : A_0=\left(X_2^2+\frac{\lambda_2^2}{p^2}\right) : \left(X_2^2+\frac{\lambda_2^2}{p^2}+\frac{\lambda_2}{\lambda_1}\delta\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

As all the quantities in this ratio are positive, it will be seen that the presence of a term involving the product (q_1q_2) in the expression for the kinetic energy means that the amplitude at resonance is always less than it would be were $a_{12}=0$, *provided other conditions remain unchanged.* A fuller discussion is given in § V. below.

(iii) The values ω_1 and p being given, S_2/R the amplitude of q_1 in (3) will be a maximum or minimum when ω_2 has a value which satisfies the equation.

$$X_2^2-\frac{1}{X_1}\left(\delta+\frac{2\lambda_1\lambda_2}{p^2}\right)X_2-\frac{\lambda_2^2}{p^2}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

When $\lambda_1=\lambda_2=0$ this equation reduces to

$$\left.\begin{array}{l} X_2=0 \\ \text{or} \quad X_1X_2=\delta \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

These conditions are discussed in the following sub-section (iv) :—

(iv) When ω_1 and ω_2 are fixed the values of p which make the amplitude of q_1 given in (3) a maximum or a minimum are readily obtained if the damping be neglected. The values so obtained will not differ materially from the values making S_2/R a maximum or a minimum when the damping is taken into account, unless the damping be excessive.* Thus the amplitude will be

$$\begin{array}{ll} \text{a minimum when} & X_2=0 \text{ or } p=\omega_2 \\ \text{a maximum when} & X_1X_2=\delta \end{array}\Bigg\} \quad \ldots\ldots \quad (16)$$

Hence two values of p make the amplitude a maximum, one value being greater and the other smaller than either ω_1 and ω_2. When p has either of these values the amplitude, which will be denoted by A, is given very approximately by

$$A=\frac{X_2}{a_1p(\lambda_2X_1+\lambda_1X_2)} \quad \ldots\ldots \quad (17)$$

Let p_1 p_2 denote the values of p which satisfy the relation $X_1X_2=\delta$ in (16). Let $p_1>\omega_1>\omega_2>p_2$ and suppose that ω_1 and ω_2 are far apart. Then when $p=p_1$ X_2 will be numerically much greater than X_1 at this frequency and the amplitude A_1 given by (17) becomes

$$A_1=\frac{1}{a_1\lambda_1p_1} \quad \ldots\ldots \quad (18)$$

which is identical with (12). Hence in such a case the amplitude at resonance is nearly the same as if the system had only one degree of freedom. If A_2 denotes the amplitude at the frequency corresponding to $p=p_2$, then, in general, $A_2<A_1$. When $p_2>\omega_2>\omega_1>p_1$, A_2 is always less than A_1.

(v) The amplitude of q_2 in (3) is a maximum when

$$\omega_1^2=p^2\left[1+\delta X_2/\left(X_2^2+\frac{\lambda_2^2}{p^2}\right)\right] \quad \ldots\ldots \quad (19)$$

ω_2 and p being kept unchanged, and also when

$$\omega_2^2=p^2\left[1+\delta X_1/\left(X_1^2+\frac{\lambda_1^2}{p^2}\right)\right]. \quad \ldots\ldots \quad (20)$$

ω_1 and p being kept unchanged.

(vi) When $Q_1=\cos pt$ and a constraint is imposed so that $q_2=0$, the magnitude and phase of the force exerted by the constraint are given by (8), and $\frac{a_{12}\cdot p^2}{S_1}\cos(pt-\psi_1)$ gives the reaction. The reaction vanishes when $a_{12}=0$. The system in this case vibrates under Q_1 as if it had only one degree of freedom and no constraint is necessary in order to secure that $q_2=0$ for all values of t (§ I. *b*).

When a constraint is imposed so that $q_2=0$, the amplitude of q_1 is given by (9) when $Q_1=\cos pt$.

In the following section the results given above are applied to the vibration galvanometer with an asymmetrically mounted moving system. The moving system may be a magnet forced to vibrate by periodic variations in the field produced by the current or a coil through which the alternating current passes suspended in a constant field.

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* In a vibration galvanometer the damping should not be large, in order that the instrument may have the necessary selectivity and sensitivity.

IV. VIBRATIONS OF ASYMMETRIC SYSTEMS.

(*a*) *Equations of Motion.*

Let nS (Fig. 2) be the magnet attached to the middle point O of the wire AB. The wire is fixed at A and B and is at right angles to the direction Oy of the uniform magnetic field H. Take Ox at right angles to Oy and OA as axis of x. In the equilibrium position ns is at right angles to H. Let G be the centre of mass of the moving system, and let its co-ordinates in the position of rest be $(a, 0)$. We will suppose that the system is symmetrical with respect to the plane xy. During the motion, O may be displaced. Let $E(xy)$ be its position at a time t and θ the angular displacement of ns round AB. Let m be the mass of the moving system and mk^2 its moment of inertia about an axis through G parallel to AB. Let m^1 be the effective mass* of the wire AB. Since $OG=a$, we have for the kinetic energy T of the moving parts

$$T=\tfrac{1}{2}(m+m^1)(\dot{x}^2+\dot{y}^2)+\tfrac{1}{2}m(a^2+k^2)\dot{\theta}^2-ma\sin\theta\ .\ \dot{x}\dot{\theta}+ma\cos\theta\ .\ \dot{y}\dot{\theta}.$$

The only co-ordinate explicitly contained in T is θ. As we are considering small oscillations about the position $x=0$, $y=0$, $\theta=0$, we may give to θ where it occurs in T its equilibrium value $\theta=0$. Further, we shall suppose that m^1 is small compared with m and shall write m for $(m+m^1)$ in the coefficient of $(\dot{x}^2+\dot{y}^2)$. Thus we get

$$T=\tfrac{1}{2}m[\dot{x}^2+\dot{y}^2+(a^2+k^2)\dot{\theta}^2+2a\dot{\theta}\dot{y}] \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (21)$$

This is also the expression for the kinetic energy of the moving coil system a^1b^1 shown in Fig. 3. The potential energy V is given by

$$V=\tfrac{1}{2}v(x^2+y^2)+\tfrac{1}{2}\mu\theta^2$$

Let

$$F=\tfrac{1}{2}(k_x\ .\ \dot{x}^2+k_y\ .\ \dot{y}^2+k\ .\ \dot{\theta}^2)$$

The equations of motion of the system under an impressed couple (H) are

$$\left.\begin{aligned} m(a^2+K^2)\ddot{\theta}+K_\theta\ .\ \dot{\theta}+\mu\theta+ma\ .\ \ddot{y}&=(\mathsf{H})\\ ma\ .\ \ddot{\theta}+m\ddot{y}+K_y\ .\ \dot{y}+vy&=0\\ m\ddot{x}+K_x\ .\ \dot{x}+v\ .\ x&=0 \end{aligned}\right\} \quad . \ . \ . \ . \ . \ . \ . \quad (22)$$

(*b*) *The Forced Vibrations.*

As our object is in the first instance to get the forced vibration in θ, the first and second equations in (22) suffice. The third equation shows that the vibration in x is a free vibration which subsides after the initial stage and ultimately disappears. Thus the equations for the determination of θ are identical in form with (2) when $Q_2=0$.

Let $(\mathsf{H})=e^{ipt}$. The vibrations in θ and y are given by q_1 and q_2 respectively in (3) where

$$\left.\begin{aligned} \omega_1^2&=\frac{\mu}{m(a^2+K^2)} & \lambda_1&=\frac{K_\theta}{m(a^2+K^2)}.\\ \omega_2^2&=\frac{v}{m} & \lambda_2&=\frac{K_y}{m}\\ &\delta=\frac{m^2a^2}{m\ .\ m(a^2+K^2)} \end{aligned}\right| \quad . \ . \ . \ . \quad (23)$$

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* Rayleigh's Theory of Sound, Vol. 1, p. 205.

Thus when (H) $=\cos pt$ in (22) we have

$$\left.\begin{aligned}\theta&=\frac{S_2}{R}\cos(pt-X+\psi_2)\\ y&=\frac{ma\,.\,p^2}{R}\cos(pt-X)\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (24)$$

Let $\omega_1=2\pi\,.\,n_1$ and $\omega_2=2\pi\,.\,n_2$. Then n_1 is the frequency of the undamped free vibration of the moving system round an unyielding and flexurally rigid axis AB (Figs. 2 and 3), under the control in AB, and n_2 is the frequency of the undamped free lateral vibration of the wire AB loaded at its middle point O with a mass m equal to the mass of the moving system. The damping factors of these vibrations are $e^{-\frac{1}{2}\lambda_1 t}$ and $e^{-\frac{1}{2}\lambda_2 t}$ respectively.

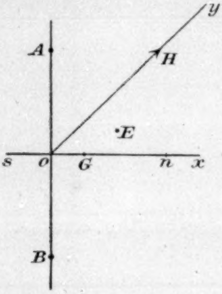

FIG. 2.

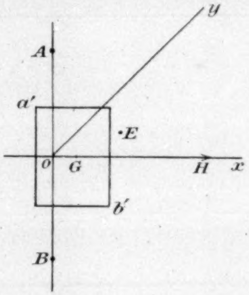

FIG. 3.

Let $a=s\,.\,k$ where s is a measure of the asymmetry of the moving system. Then

$$\delta=\frac{a^2}{a^2+K^2}=\frac{s^2}{s^2+1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (25)$$

(*c*) *The Free Vibrations.*

The vibration in x is given by the third of equations (22). The vibrations in which θ and y are concerned are determined by the first two of (22) with (H) $=0$ which lead to the following:—

$$\{(\omega_1^2-p^2)^2+\lambda_1^2p^2\}\ \{(\omega_2^2-p^2)^2+\lambda_2^2p^2\}-2\delta p^4\ \{(\omega_1^2-p^2)(\omega_2^2-p^2)-\lambda_1\lambda_2p^2\}+\delta^2p^8=0 \quad . \; . \; . \quad (26)$$

giving $p/2\pi$ the frequencies of the free vibrations.

(*d*) *Application to the Observations represented in Fig.* 1.

The sensitivity at different frequencies from 106 to 126 per second of the galvanometer used by Rosa and Grover is given in Fig. 1. The ordinates of this curve were measured and the results are entered in column (3) of Table I. with the corresponding frequencies in column (1). The curve shows that the amplitude is a minimum at the frequency 114·3 per second. Thus from (16) we have $x_2=0$ when $p=2\pi\times114\cdot3=\omega_2$. The amplitude is a maximum at the frequencies 110·6 and 120·3

Let $p_1=2\pi\times120\cdot3$ and $p_2=2\pi\times110\cdot6$. By (16) we get

$$\left(\frac{\omega_1^2}{p_1^2}-1\right)\left(\frac{\omega_2^2}{p_1^2}-1\right)=\left(\frac{\omega_1^2}{p_2^2}-1\right)\left(\frac{\omega_2^2}{p_2^2}-1\right)=\delta$$

From this and (25) since $\omega_2=2\pi\times114\cdot3$ we get ω_1, δ and s.

Thus

$$\left.\begin{aligned}\omega_1&=2\pi\times116\\ \omega_2&=2\pi\times114\cdot3\\ \delta&=6\cdot82\times10^{-3}\\ s&=\frac{1}{12}\text{ very nearly}\end{aligned}\right\}\quad.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \qquad (A)$$

The ratio of the amplitude at the frequency 120·3 to the amplitude at 110·6 is by Table I. $=\frac{46\cdot8}{44\cdot0}=1\cdot064$. At these resonance frequencies the formula for the amplitude is (17). Equating the ratio given by (17) to 1·064 and evaluating X_1 and X_2 at these frequencies, we get an equation involving λ_1 and λ_2 which yields

$$\lambda_1=4\cdot05\lambda_2$$

This enables us to express the ratio of the amplitudes given by the formula S_2/R in (24) at two selected frequencies in a form involving only one of the λ's—say λ_2. Hence equating the ratio thus expressed to the "observed ratio" derived from Fig. 1 or Table I., we obtain an equation giving λ_2.* Thus, from the amplitudes

at 110·6 and 106, $\lambda_2^2=16\cdot9$
and at 120·3 and 126, $\lambda_2^2=17\cdot9$
$\therefore$ mean $\lambda_2^2=17\cdot4$.

Hence we obtain

$$\left.\begin{aligned}\lambda_1&=16\cdot90\\ \lambda_2&=4\cdot17\end{aligned}\right\}\quad.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \qquad (B)$$

With the constants given in (A) and (B) the amplitude of the vibration in θ has been calculated from the following relation

$$\text{amplitude}=\text{constant}\times S_2/R,$$

the constant being such a value as to make the calculated amplitude at 120·3 equal to 46·8, the "observed" amplitude. The values thus calculated are given in column (2) of Table I.

---

* In selecting two frequencies it is advisable to choose a resonating frequency as one. At such a frequency the amplitude is largely determined by the corresponding λ. At frequencies well removed from resonance the amplitude is only slightly affected by the values of the λ's, the term $(x_1x_2-\delta)^2$ in (5) being much greater than the others.

TABLE I.

Frequency $=n$. (1)	Amplitude. Calculated. (2)	Amplitude. Observed. (3)	Amplitude. Calc-obs. (4)	X (5)	ψ_2 (6)	$X-\psi_2$ (7)
				°	°	°
106	10·0	9·9	0·1	12·0	2·2	9·8
108	15·2	15·3	−0·1	18·9	2·9	16·0
110	35·9	35·0	0·9	53·1	4·3	48·8
110·6	44·0	44·0	0·0	94·0	5·0	89·0
112	14·2	16·0	−1·8	160·1	8·1	152·0
114	1·6	2·0	−0·4	178·2	47·9	130·3
114·3	—	—	—	178·6	90·0	88·6
116	5·6	5·6	0·0	185·5	168·9	16·6
118	15·5	15·5	0·0	198·2	174·8	23·4
120	44·5	45·6	−1·1	250·6	176·6	74·0
120·3	46·8	46·8	—	267·2	176·8	90·4
122	25·4	24·3	1·1	326·1	177·4	148·7
124	14·0	13·5	0·5	341·6	178·0	163·6
126	9·7	9·8	−0·1	347·1	178·3	168·8

The agreement with the observed values is satisfactory and gives weight to the deductions enumerated below to which we have been led by application of the theory to the observations represented in Fig. 1.

(i) The frequency of the free undamped lateral vibrations of the suspension *AB* (Fig. 2) in the direction *Oy* is 114·3 per second, the wire being loaded with a mass equal to that of the moving system. The damping factor of this vibration is $e^{-2·09t}$, which means that the amplitude diminishes during each second to 13 per cent. of its value at the beginning of the second.

(ii) The frequency of the free undamped angular vibrations of the system about a flexurally rigid axis coinciding with the axis of suspension and under the control in this axis is 116 per second. The damping factor of this vibration is $e^{-8·45t}$. The meaning of this can best be realised by stating that the amplitude diminishes by 7 per cent. of its value in each period.

(iii) The centre of mass *G* of the magnet *ns* (Fig. 2) is at a distance from the axis of suspension *AB* equal to 1/12th of the radius of gyration of *ns* about an axis through *G* parallel to *AB*. In arriving at this deduction it should be remembered that we have assumed that the effective mass of the wire *AB* is negligible compared with the mass of the magnet.

The frequencies of the free vibrations of the system involving θ and y are obtained by solving* equation (26) in which the constants $\omega_1 \ldots \delta$ have the values given in (A) and (B). The frequencies obtained are given under (2) in Table II. In column (3) of the same table will be found the values of a in the damping factor e^{-at} of the corresponding vibration.

TABLE II.

(1) $\omega/2\pi=n$	(2) Free Frequency	(3) a
$n_2=114·3$	110·6	4·32
$n_1=116$	120·3	6·33

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* For method solution *see* G. W. Pierce's "Electric Waves and Oscillations," Ch. 8 & 9.

If we put $\lambda_1=\lambda_2=0$ in (26) and solve, we obtain 110·6 and 120·3 for the free frequencies in this case also. This might have been expected because if the dissipative forces be comparatively small the periods of the normal vibrations are the same as if the dissipative forces were absent.

V. EFFECT OF ASYMMETRY IN MOVING SYSTEM ON SENSITIVITY.

If q_1 denotes the co-ordinate of the moving system of a vibration galvanometer whose amplitude is observed, it will be evident from what has been given above that asymmetry introduces a term of the form $\dot{q}_1\,\dot{q}$ into the expression for the kinetic energy of the system. Asymmetry may arise from errors of construction, and it is of practical importance to estimate the effect it has on sensitivity. The point has been partially considered in § III. (ii); a fuller discussion is given below.

We will take the case where ω_2 and p are kept constant, and resonance is obtained by altering ω_1, this being the case of most frequent occurrence in practice. The conditions for and the amplitude at resonance for a system with two degrees of freedom are given in (10) and (11). But when by a change in the constitution of the system we make $\delta=0$, then in general the "inertia constants" a_1 and a_2 will also be altered, and (13) is not directly applicable. Let their values become ${}_0a_1$ and ${}_0a_2$. Then relation (12) becomes

$$A_0=\frac{1}{{}_0a_1\,\lambda_1\,p}$$

and instead of (13) we shall have

$$A\delta : A_0=\frac{{}_0a_1}{a_1}\left(X_2^{\,2}+\frac{\lambda_2^{\,2}}{p^2}\right) \quad : \quad \left(X_2^{\,2}+\frac{\lambda_2^{\,2}}{p^2}+\frac{\lambda_2}{\lambda_1}\delta\right). \quad . \quad . \quad . \qquad (27)$$

In the case considered in § IV., symmetry can be restored by making $OG=0$ (Fig. 2), m and k remaining unchanged.

Then
$$\frac{{}_0a_1}{a_1}=\frac{1}{1+\ ^{2}}=1-\delta$$

which is always less than 1.

In general the effect of asymmetry is to reduce the sensitivity in accordance with the expression given in (27). The following special cases may be noticed :—

(i) λ_2 very small compared with λ_1. Then

$$A\delta : A_0={}_0a_1 : a_1.$$

(ii) ω_2 and p are far apart. Then X_2 will be large compared with the other quantities in (27), and again

$$A\delta : A_0={}_0a_1 : a_1.$$

In the following table the ratio $A\delta/A_0$ is given for the galvanometer considered in § IV. for various values of δ.

(i) In column (2) when the galvanometer is tuned to an impressed frequency of 110·6.

(ii) In column (3) when it is tuned to an impressed frequency of 120·3.

TABLE III.

	1	2	3
s Approx.	δ	$A\delta/A_0$	
		$p/2\pi=110{\cdot}6$	$p/2\pi=120{\cdot}3$
1/32	$0{\cdot}1\times10^{-2}$	0·95	0·97
1/25	$0{\cdot}2\times10^{-2}$	0·90	0·95
	$0{\cdot}3\times10^{-2}$	0·86	0·93
	$0{\cdot}4\times10^{-2}$	0·82	0·91
	$0{\cdot}5\times10^{-2}$	0·79	0·89
	$0{\cdot}6\times10^{-2}$	0·76	0·87
	$0{\cdot}7\times10^{-2}$	0·72	0·84
	$0{\cdot}8\times10^{-2}$	0·69	0·82
	$0{\cdot}9\times10^{-2}$	0·67	0·80
1/10	$1{\cdot}0\times10^{-2}$	0·64	0·78
	$2{\cdot}0\times10^{-2}$	0·47	0·65
	$3{\cdot}0\times10^{-2}$	0·38	0·54
	$4{\cdot}0\times10^{-2}$	0·31	0·47
	$5{\cdot}0\times10^{-2}$	0·26	0·42
	$6{\cdot}0\times10^{-2}$	0·23	0·37
	$7{\cdot}0\times10^{-2}$	0·20	0·33
	$8{\cdot}0\times10^{-2}$	0·17	0·30
	$9{\cdot}0\times10^{-2}$	0·15	0·27
1/3	$10{\cdot}0\times10^{-2}$	0·14	0·25

As already stated, tuning is supposed to be effected by altering ω_1 alone. Hence in these cases the loss in sensitivity is not serious, provided s does not exceed about 1/32, say. This gives an idea of the accuracy in construction necessary to ensure that no material diminution in sensitivity occurs owing to asymmetry. The authors state they used the galvanometer when it was tuned to about 110 generally. Since δ was found to be $0{\cdot}68\times10^{-2}$ it will be seen from the table that the sensitivity was about 72 per cent. of what it would have been had the moving system been mounted symmetrically.

It is therefore of practical importance in the construction and mounting of the moving system in a vibration galvanometer, to make certain that the system is symmetrical in these respects with regard to the axis of rotation. A galvanometer can be tested for asymmetry by observing its sensitivity at different frequencies when an alternating current, preferably of sine wave-form and constant amplitude, is passed through it, and drawing the sensitivity curve as in Fig. 1 from the observations. Asymmetry, if present, will be shown by two peaks in the curve, and a region of very low sensitivity somewhere between the two peaks. Its amount can be determined as in § IV. from the curve, and the loss of sensitivity resulting therefrom can be evaluated by the help of (27). It may be advisable to obtain more than one sensitivity curve for a galvanometer after tuning it to another and a different frequency.

When a constraint is applied so as to keep $q_2=0$ at all values of t resonance will occur when $\omega_1=p$ and the amplitude is given by

$$A\delta=\frac{1}{a_1\lambda_1 p}$$

With $\delta=0$ this would become

$$A_0=\frac{1}{{}_0a_1\lambda_1 p}$$

and no constraint would be necessary. Thus $A\delta$ is always less than A_0.

Equations (7) and (8) enable us to determine the force exerted by the constraint, and show that the necessary constraint cannot be obtained by mounting the moving system in a non-uniform or unsymmetrical field on account of the condition $\varphi+\psi_1=\pi$.

VI. Mechanical Analogue of Coupled Circuits.

Equations (22) show that in a vibration galvanometer with an asymmetric moving system, such as that considered in §IV., we have a mechanical system capable of motions closely analogous to the electrical vibrations in a pair of coupled circuits. For the mechanical system the equations of motion, when there is no external force impressed on it, may be written

$$\left.\begin{aligned} &\alpha\frac{d^2\theta}{dt^2}+\beta\frac{d\theta}{dt}+\gamma\theta+\mu\frac{d^2y}{dt^2}=0 \\ &\mu\frac{d^2\theta}{dt^2}+\alpha'\frac{d^2y}{dt^2}+\beta'\frac{dy}{dt}+\gamma'y=0 \end{aligned}\right\} \quad . \quad . \quad . \quad (28)$$

Let $R_1L_1C_1$ be the resistance, inductance and capacity arranged in series in a circuit in which the current is i_1, and let $R_2L_2C_2$ be resistance, &c., in a second circuit in which the current is i_2. Let M be the mutual inductance between the circuits. Then we have for the currents i_1 and i_2 the following equations :—

$$\left.\begin{aligned} &L_1\frac{d^2i_1}{dt^2}+R_1\frac{di_1}{dt}+\frac{1}{C_1}i_1+M\frac{d^2i_2}{dt^2}=0 \\ &M\frac{d^2i_1}{dt^2}+L_2\frac{d^2i_2}{dt^2}+R_2\frac{di_2}{dt^2}+\frac{1}{C_2}\cdot i_2=0 \end{aligned}\right\} \quad . \quad . \quad . \quad (29)$$

The correspondence between the quantities involved in (28) and (29) is shown in the following table where the mechanical quantity is placed immediately underneath the corresponding electrical quantity :—

L_1	R_1	$\frac{1}{C_1}$	M	L_2	R_2	$\frac{1}{C_2}$	electrical.
α	β	γ	μ	α'	β'	γ'	mechanical.

By attaching the moving system to a bifilar suspension, the tension and the distance apart of whose fibres are separately adjustable, and by making s variable, it would be possible to alter the periods ω_1 and ω_2 and the asymmetry separately, and thus effect changes in the mechanical system corresponding closely to changes in L, C and M in the electrical case.

VII. Conclusion—Summary.

After the work detailed above had been completed, my attention was called to a Paper by Kennelly and Taylor entitled: "Some Properties of Vibrating Telephone Diaphragms" (Proc. American Phil. Soc., Vol. 55, pp. 415-46, 1916).

In this Paper the authors describe phenomena observed by them in irregularly-clamped telephone diaphragms exactly analogous to those observed by Rosa and Grover in their vibration galvanometer, represented in Fig. 1. They trace the observed abnormal behaviour of such diaphragms to " the presence of local and loosely-clamped diaphragm boundary areas having a natural frequency independent of the main diaphragm," and draw the inference that " it would only be necessary to attach a local vibrational system . . . to a properly-clamped diaphragm in order to simulate the . . . behaviour of an irregularly-clamped diaphragm." The description of an experimental test which confirms this inference follows. In an appendix to their Paper they give a " provisional theory which was arrived at by searching for a quantitative expression that would satisfy " the observations, and they remark that " it bears a close analogy to the theory of alternating-current coupled circuits."

It will be seen that the authors have anticipated the original purpose of the present Paper. The theory is, however, presented in a form very different from that adopted here. It is formulated with reference to, and its applicatioins directed to the subject of the Paper. It seems, therefore, that the mode of presentation and the application of the theory made in the present Paper, as well as the more searching test of its indications given above, may be of service. The interpretation of, and the derivation of the mechanical characteristics of the system from, the sensitivity curve given in § IV. and the effect of asymmetry on the sensitivity discussed in § V. do not appear to have hitherto received much attention, and may be of value.

IX. *The Relation between Molecular and Crystal Symmetry as shown by X-Ray Crystal Analysis.* By G. SHEARER, *M.A., B.Sc.*

RECEIVED OCTOBER 28, 1922.

(COMMUNICATED BY SIR W. H. BRAGG, F.R.S.)

ABSTRACT.

The methods of X-ray analysis enable the number of molecules associated with the unit cell to be determined. With the help of this information an attempt is made to connect the symmetry properties of the crystal with this number and with the symmetry properties of the molecules from which the crystal is formed.

The symmetry number for each of the 32 crystal classes is given, and is shown to mean the minimum number of asymmetric molecules necessary in the unit cell to satisfy the symmetry conditions. The relative orientations and positions of these molecules in the cell are discussed.

It is suggested that this symmetry number is the actual number of molecules in the cell when the molecule is asymmetric; further that, if the molecule possesses symmetry, this symmetry appears also in the crystal, and the number of molecules in the unit cell is obtained by dividing the symmetry number of the crystal by the symmetry number of the molecule.

Evidence is produced in support of these hypotheses and examples are given of their application to inorganic and organic crystals.

IT has long been known that the various crystals can be divided into 32 classes distinguished from one another by the symmetry which they possess. Much geometrical work has been done on these crystal forms* but, so far, little progress has been made towards a physical interpretation of these geometrical properties. Recent work by the methods of X-ray analysis has thrown much light on the nature of the elementary lattice and the structure of molecules and has provided a basis for an advance in this direction.

The elementary lattice is the smallest which, repeated through space, will build up the crystal. As each lattice is identical with all others, it must show in itself all the symmetry properties of the crystal. These symmetry properties belong to three classes, (1) those associated with planes of symmetry, (2) those associated with axes of symmetry, (3) those associated as an axis of compound symmetry or a centre of symmetry.

Bravais† in 1850 showed that there are 14 space lattices which in themselves show the full symmetry of the various crystal classes. All of these lattices do not, from an X-ray point of view, appear to be independent. Thus, Bravais differentiates between the simple cube, the centred cube and the face-centred cube. These may all be regarded as one or more interpenetrating simple cubes. The same applies to the centred lattices in the orthorhombic and tetragonal classes. If these are eliminated, there are left nine different lattices, one each associated with the triclinic, hexagonal, tetragonal and cubic classes, two each with the monoclinic, orthorhombic, and trigonal systems. The hexagonal lattice is identical with one of the two trigonal lattices. There have thus been eliminated from consideration all lattices which show more than one molecule associated with the unit cell. The view here adopted of an elementary lattice is that of a parallelopiped which, with perfectly symmetrical molecules placed at its vertices, shows the full symmetry of the class of crystal with which it is associated. It will be shown that by a combination of a definite number of similar interpenetrating lattices the symmetry of any class can be obtained even if the molecule possesses no symmetry of its own.

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* For a full account of the mathematical theory of the space lattice and its symmetry properties, *see* H. Hilton's "Mathematical Crystallography," Oxford (1903).

† Bravais, Paris Ecole Polytechn. Journal, 19, 1-128 (1850).

First of all, it is necessary to define what is meant by a plane or axis of symmetry in the elementary lattice. Take the parallelopiped which is the elementary lattice of the crystal class (Fig. 1) and consider molecules placed at its vertices. As the lattice has to repeat through space all these molecules must be orientated in exactly the same way relative to the lattice. Consider, for example, the vertex *A* of the lattice. At *A* eight lattices meet and the molecule at *A* belongs equally to all of them. The molecule* *A* of this cell corresponds to the molecules *B*, *C*, *D*, *E*, *F*, *G*, *H* of the other cells which meet at *A*. It follows that all the molecules at the vertices must be orientated relative to the lattice in exactly the same way.

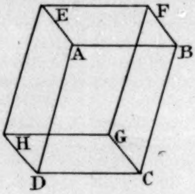

FIG. 1.

Suppose the crystal has a plane of symmetry parallel to the face *A*, *B*, *C*, *D*. There must exist a set of molecules—" *B* " molecules—which are the mirror images of the molecules at the vertices—the " *A* " molecules. As these cannot coincide with the " *A* " molecules unless the molecule mirror images into itself there must be a separate lattice precisely similar in shape to the first but displaced in some way relative to the first lattice. It is clear that, to pass from an " *A* " molecule to a " *B* " molecule, we must take the mirror image of the "*A* " molecule in some plane parallel to the plane of symmetry and then give it a translation to some point in the cell. A reflection in the same plane and an equal translation applied to any other " *A* " molecule must give rise to another " *B* " molecule. Fig. 2 shows these two interpenetrating lattices, the arrow-heads denoting the orientation of the molecules.

Exactly the same process applied to the " *B* " molecules must give rise to " *A* " molecules. This reciprocal relation puts certain restrictions on the relative positions of the " *A* " and " *B* " molecules in the cell. These will be considered later.

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* The term "Molecule" is used in this Paper to denote "Crystal Molecule." It does not follow that the arrangement of atoms in it is identical with the arrangement in the molecule in solution as dealt with in chemistry. There appears, however, to be a strong resemblance between the two in many cases. It has, of course, the same component atoms and molecular weight.

In the same way, the existence of a digonal axis gives rise to a second set of molecules whose orientations relative to the "*A*" molecules are obtained by rotating these about the axis through 180°. Once more, to pass from the one set to the other, in addition to the rotation there must be a translation and this translation must be such that the operation which, applied to the first set, gives the second, will, applied to the second set, give the first. Similar relations hold for axes other than digonal.

Having now defined what we mean by the elementary cell and planes and axes of symmetry, we may now proceed to determine the number of interpenetrating simple lattices, or, in other words, the number of molecules in the elementary cell which are necessary to satisfy the symmetry properties of the 32 crystal classes. First of all, let it be assumed that the molecule from which the crystal is constructed possesses no symmetry of its own. It is clear that the existence of a digonal axis must give rise to a second molecule in the cell whose orientation relative to the first is obtained by a rotation of 180° about the axis. A trigonal axis will give rise to two other molecules in the cell whose orientations are obtained by rotations of the first through angles of 120° and 240° about the axis. In the same way a plane of symmetry will give rise to a second which is the mirror image of the first in the plane of symmetry. A combination of a plane of symmetry and a digonal axis at right angles to it will give an elementary cell with four differently orientated molecules—"*A*," "*B*," "*C*" and "*D*" molecules. The orientation of the "*B*" molecules is obtained by reflection in the plane of symmetry, that of the "*C*" molecules by rotation of the "*A*'s" about the axis, while that of the "*D*" molecules may be obtained either by a rotation of the "*B*" molecules or by a reflection of the "*C*" molecules.

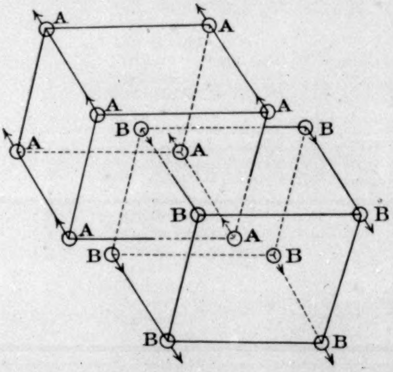

FIG. 2.

In this way it is possible to construct a table of the 32 classes and tabulate the minimum number of asymmetric molecules which must necessarily appear in the elementary cell in order to satisfy the symmetry properties of the crystal. This number is most readily obtained by taking the stereographic projection of the crystal and finding the number of points on the diagram corresponding to the general face. This number may conveniently be called the "Symmetry Number" of the crystal class.

TABLE.

Crystal class.		System.	Class.	Symmetry number or number of asymmetric molecules per cell.
No.	Schönfliess symbol.			
1	C_1	Triclinic ...	Triclinic Asymmetric	1
2	S_2		,, Pinakoidal	2
3	S	Monoclinic	Monoclinic Domal	2
4	C_2		,, Sphenoidal	2
5	C_2^h		,, Prismatic	4
6	V	Orthorhombic	Rhombic Bisphenoidal	4
7	C_2		,, Pyramidal	4
8	V^h		,, Bipyramidal	8
9	C_4	Tetragonal ...	Tetragonal Pyramidal	4
10	S_4		,, Bisphenoidal	4
11	D_4		,, Trapezohedral	8
12	C_4^h		,, Bipyramidal	8
13	C_4^v		Ditetragonal Pyramidal	8
14	S_4^u		Tetragonal Scalenohedral	8
15	D_4^h		Ditetragonal Bipyramidal	16
16	C_3	Trigonal ...	Trigonal Pyramidal	3
17	C_3^i		,, Rhombohedral	6
18	D_3		,, Trapezohedral	6
19	C_3^h		,, Bipyramidal	6
20	C_3^v		Ditrigonal Pyramidal	6
21	D_3^d		,, Scalenohedral	12
22	D_3^h		,, Bipyramidal	12
23	C_6	Hexagonal ...	Hexagonal Pyramidal	6
24	D_6		,, Trapezohedral	12
25	C_6^h		,, Bipyramidal	12
26	C_6^v		Dihexagonal Pyramidal	12
27	D_6^h		,, Bipyramidal	24
28	T	Cubic ...	Tetragonal Pentagonal Dodecahedral	12
29	O		Pentagonal Icositetrahedral ...	24
30	T^h		Dyakis Dodecahedral	24
31	T^d		Hexakis Tetrahedral	24
32	O^h		Hexakis Octahedral	48

It has already been mentioned that it is possible to lay down certain restrictions as to the relative positions of the variously orientated molecules in the lattice. Take, for example, a crystal with a plane of symmetry perpendicular to the "*b*" axis of the crystal. Place "*A*" type molecules of the verticles of a parallelopipedon of sides *a*, *b*, *c*, with angles 90°, β, 90°—one of the monoclinic lattices. Suppose that the molecule at the origin may be represented by the set of points

$$x_1, y_1, z_1$$

$$z_2, y_2, z_2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$z_n, y_n, z_n$$

All the molecules of type " A " in the crystal may then be represented by the set

$$\begin{array}{l} x_1+la,\ y_1+mb,\ z_1+nc \\ x_2+la,\ y_2+mb,\ z_2+nc \\ \ldots\ldots\ldots \\ x_n+la,\ y_n+mb,\ z_n+nc \end{array}$$

where l, m, n are any positive or negative integers.

A reflection of these molecules in the plane $y=O$ combined with a translation αa, βb, γc parallel to the three axes gives the set of " B " molecules,

$$\begin{array}{l} x_1+la+\alpha a,\ -y_1-mb+\beta b,\ z_1+nc+\gamma c \\ x_2+la+\alpha a,\ -y_2-mb+\beta b,\ z_2+nc+\gamma c \\ \ldots\ldots\ldots\ldots\ldots \\ x_n+la+\alpha a,\ -y_n-mb+\beta b,\ z_n+nc+\gamma c \end{array}$$

The same operation applied to this set must bring it into coincidence with the " A " set. If we carry out this operation we obtain the set,

$$\begin{array}{l} x_1+la+2\alpha a,\ y_1+mb,\ z_1+nc+2\gamma c \\ x_2+la+2\alpha a,\ y_2+mb,\ z_2+nc+2\gamma c \\ \ldots\ldots\ldots\ldots\ldots \\ x_n+la+2\alpha a,\ y_n+mb,\ z_n+nc+2\gamma c \end{array}$$

For these to represent the " A " set, we must have

$$2\alpha a=O \text{ or } a \text{ or } 2a, \text{ \&c., and } 2\gamma c=O \text{ or } c \text{ or } 2c, \text{ \&c.}$$

Hence $\alpha=O$ or $\frac{1}{2}$, and $\gamma=O$ or $\frac{1}{2}$

Hence the " B " molecules must lie on lines parallel to the " b " axis either coinciding with the edges of the lattice or passing through the middle points of the sides or faces.

Consider now a digonal axis of symmetry parallel to the " b " axis. A rotation of the " A " molecules through 180° combined with a translation $\alpha'a$, $\beta'b$, $\gamma'c$ parallel to the axes gives the set of " C " molecules

$$\begin{array}{l} -x_1-la+\alpha'a,\ y_1+mb+\beta'b,\ -z_1-nc+\gamma'c \\ -x_2-la+\alpha'a,\ y_2+mb+\beta'b,\ -z_2-nc+\gamma'c \\ \ldots\ldots\ldots\ldots\ldots \\ -x_n-la+\alpha'a,\ y_n+mb+\beta'b,\ -z_n-nc+\gamma'c \end{array}$$

The same operation applied to this set must bring it into coincidence with the " A " set. This gives the set

$$\begin{array}{l} x_1+la,\ y_1+mb+2\beta'b,\ z_1+nc \\ x_2+la,\ y_2+mb+2\beta'b,\ z_2+nc \\ \ldots\ldots\ldots\ldots \\ x_n+la,\ y_n+mb+2\beta'b,\ z_n+nc \end{array}$$

Hence, we must have $\beta'=O$ or $\frac{1}{2}$

Hence, the " C " molecules must lie either in the $a-c$ planes or in planes halfway between successive $a-c$ planes.

Similar relations may be worked out for the other symmetries which the crystal may possess. For example, the existence of a trigonal, tetragonal or hexagonal axis involves the existence of three, four or six molecules, derived from each other by successive rotations of 120°, 90°, or 60° about the axis. The projections of corresponding points of these on a plane perpendicular to the axis form an equilateral triangle, square or regular hexagon according to the nature of the axis. Their distribution in a direction parallel to the axis is such that either they all lie in the basal planes or are equally distributed between successive basal planes. In the latter case the successive molecules form a regular spiral which may be right or left-handed. The simultaneous existence of a plane of symmetry passing through the axis or perpendicular to it gives rise to a second spiral which is in the opposite sense to the first. The position of this spiral relative to the first one obeys the conditions already worked out as we may consider the whole of the first spiral as a more complicated molecule.

In this connection it is interesting to note that the rotation of the plane of polarisation of light by a crystal is intimately associated with the existence of this spiral arrangement and the dextro or lævo nature of the rotation depends on the type of spiral. If any spiral has this property it is clear that the existence of a plane of symmetry introduces a spiral which will give an equal and opposite rotation and the crystal will appear inactive. This is, of course, shown by the fact that the only crystals which show this rotation are those whose only symmetry is associated with axes of symmetry. A digonal axis is, of course, inactive as it is impossible to distinguish between a rotation of 180° in a right or left-handed direction. Any biaxial crystal which rotates the plane of polarisation must do so owing to the existence of some spiral arrangement within the molecule itself.

In this way we can go a very considerable distance towards the solution of the crystal structure of any crystal using only the ordinary crystallographic data and, in the case of a symmetrical molecule, the number of molecules in the unit cell. Certain quantities remain to be determined. These are such quantities as β, α', γ' in the above discussion and the sides of the equilateral triangle, square or hexagon in the case of uniaxal crystals.

So far it has been assumed that no symmetry exists in the molecule. As long as an asymmetric molecule is in question the number of molecules in the unit cell must be that given in the table or an integral multiple of it. If it were less, the crystal could not possibly have its full symmetry.

If, however, the molecule possesses a certain amount of symmetry of its own, a number less than that given in the table may suffice. Thus a molecule with a plane of symmetry may form a crystal of, say, class 5 with only two molecules per cell, the second being derived from the first by a rotation about the digonal axis. This can only happen, however, if the plane of symmetry of the molecule is set in the crystal parallel to the plane of symmetry of the crystal, in this case a plane perpendicular to the " b " axis.

The converse of this propostion is of importance. If it is found that the number of molecules in the unit cell is less than the symmetry number for the crystal, it follows that the molecule must possess some symmetry and that this symmetry is reproduced in the crystal. As the number of molecules in the elementary cell must be an integer, it follows that the symmetry which the molecule shares with the crystal must be represented by an integer which is obtained by dividing the number given in the table corresponding to this class of crystal by the number of molecules in the unit cell. Thus, if n is the number characteristic of the class and m the number per unit cell, then n/m is an integer and the crystal shares n/m fold symmetry with the molecule. This number may be called the Symmetry Number of the molecule.

So far it has not been claimed that the number of molecules given in the above table is the actual number present when the molecule is asymmetric, but only that the number in the cell must be either this number or an integral multiple of it. It naturally suggests itself that this number is the actual number. This means that the number of molecules in the elementary cell is the minimum necessary to satisfy the symmetry conditions. It seems natural to suppose that a crystal will be constructed with the minimum material. If this is true, it implies that each unit cell contains one and one only of each differently orientated molecule.

A very considerable amount of experimental evidence exists in support of this hypothesis. In testing it, attention must be confined to asymmetric molecules. W. H. Bragg* has recently investigated a number of aromatic organic compounds, and many of these have molecules which can possess little or no symmetry. A large percentage of these form monoclinic prismatic crystals and show four molecules in the elementary cell. This is what is to be expected if the assumption made above is true. So far no example has been found where the number is greater than the symmetry number of the crystal. Certain cases have been found where the number is less than the symmetry number. This must mean that the molecules themselves possess some symmetry which they share with the crystal. In all the cases where this occurs there is good reason to suppose that the molecule does possess symmetry. For example, both naphthalene and anthracene form monoclinic crystals with two instead of four molecules in the unit cell. This shows that these molecules both possess a twofold symmetry. The use of such information will be discussed later. Both α- and β-naphthol belong to the same class, but have four molecules in the elementary cell. It is only to be expected that the addition of an OH group to one side of the naphthalene double ring will destroy any symmetry which the naphthalene ring possessed. In the same way the addition of the COOH, &c., groups in such substances as benzoic, salicylic and phthalic acids apparently destroys the symmetry of the benzene molecule, as these compounds all form monoclinic prismatic crystals with four molecules per cell. Tartaric acid is a monoclinic sphenoidal crystal and, as is to be expected, has two molecules in the unit cell.

As no case has occurred which contradicts the hypothesis, and as there is a considerable mass of evidence in its favour, it seems reasonable to conclude that it is generally true that the number of molecules in the elementary cell is always the minimum number necessary to impart to the crystal the necessary symmetry properties.

The question of molecules which do possess symmetry must now be considered. Here again a natural hypothesis suggests itself, viz., that all the symmetry of the molecule is reproduced in the crystal. It seems reasonable to assume that nature in forming a crystal will make use of all the symmetry already existing in the molecule

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* Bragg. Proc. Phys. Soc., Vol. 34, p. 33 (1921).

Indeed, this is really the same assumption as has already been made in the discussion of asymmetric molecules. It simply amounts to the statement that the minimum amount of material is used in the building up of a crystal. If this hypothesis is true it follows that the crystal must always show at least as much symmetry as the molecule from which it is formed. Indeed, there is evidence in favour of the fact that in general the symmetry of the crystal is higher than that of the molecule.

In the present state of our knowledge of molecular structure it is more difficult to test this assumption. There is no doubt that much of the symmetry is certainly made use of, but it is impossible to say at present exactly what symmetry any particular molecule possesses. There seems, however, to be considerable evidence that all the symmetry of the molecule is used in the formation of the crystal. The fact that most of the elements form crystals of a high order of symmetry with relatively few molecules in the unit cell shows that a considerable amount, if not all, of the symmetry of the atoms is reproduced in the crystal. The same applies to many simple inorganic salts. For example, $CaCO_3$ and Al_2O_3 are both likely to possess a trigonal axis, the oxygen atoms being arranged trigonally round the rest of the molecule. A calculation of the symmetry of the molecule from the number of molecules in the unit cell and the nature of the crystal shows that the molecules must possess a trigonal axis. Quartz forms a crystal of class 18 with three molecules in the elementary cell. This implies that the SiO_2 molecule itself possesses a digonal axis and that the molecules are so arranged in the crystal that this digonal axis of the crystal coincides with one of the digonal axes of this type of crystal. So far no case has been investigated which does not seem to agree with the assumption that all the symmetry of the molecule is reproduced in the crystal.

If this hypothesis is true, it should be of great service in determining the structure of molecules. If, in addition to being able to say that a molecule possesses such and such symmetry we can also say that it possesses no other symmetry, we are in a strong position to decide on the nature of the molecule and also on the question of what symmetry in a molecule really means. In dealing with an atom it is clear that the outer electrons must be the important factors in deciding the crystal structure and the information to be derived from such arguments as those given above should throw considerable light on the distribution of these outer electrons. It is immaterial whether the electrons are considered as stationary or as moving in orbits. In the latter case the distribution of the orbits or of the normals to the orbits must show the symmetry properties.

Our knowledge of the distribution of the outer electrons is still too vague to permit of a complete interpretation of the results which the information derived from crystallographic measurements gives, but certain steps may be made in this direction. For example, many of the elements crystallise in the highest form of cubic symmetry and possess two or four atoms in the unit cell. This means that the atoms in question possess either a 24-fold or a 12-fold symmetry. An examination of the table suggests that the symmetry of the atoms is that of one of the lower forms of cubic symmetry. This leads to the conclusion that there is some form of cubic arrangement of the electrons or of the electron orbits. Similar conclusions as to the existence of some such cubic arrangement have been reached as a result of the consideration of the chemical properties of various atoms. Carbon in the form of the diamond is a crystal of class 31 with eight atoms in the unit cell. This fact ascribes to the carbon atom a symmetry number 3—that is to say, a trigonal axis.

Bohr has arrived at the conclusion that the four outer electrons in the carbon atom move in four 2_1 orbits whose normals are arranged tetrahedrally. Such an arrangement provides a ready explanation of the existence of a trigonal axis in the atom, but would also suggest the existence of other trigonal axes of symmetry. It may, of course, be necessary also to consider the two inner electrons, or the arrangement may not be perfectly tetrahedral. The structure of graphite is still somewhat uncertain. Debye and Scherrer* found that their observations agreed with the assumption of a rhombohedral lattice with eight atoms in the unit cell. On the other hand, Hull† suggests four interpenetrating hexagonal lattices. A consideration of the numbers given in the table for the various classes of trigonal crystals shows that if the above assumptions are true it is impossible to have eight atoms in the unit cell. The same objection does not apply to Hull's suggested structure. Hull's structure supports the idea that the carbon atom in graphite possesses a trigonal axis just as it does in the case of diamond.

A few typical examples of the application of these principles may now be considered in more detail. Take first the case of ruby—Al_2O_3. This forms a crystal of class 21, possessing a trigonal axis, three digonal axes inclined at angles of 120 deg. to each other and perpendicular to the trigonal axis, and three planes of symmetry passing through the trigonal axis and bisecting the angles between successive digonal axes. The stereographic projection of the crystal class is shown in Fig. 3.

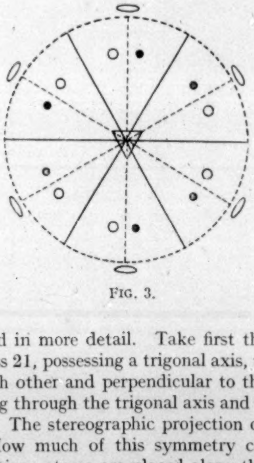

FIG. 3.

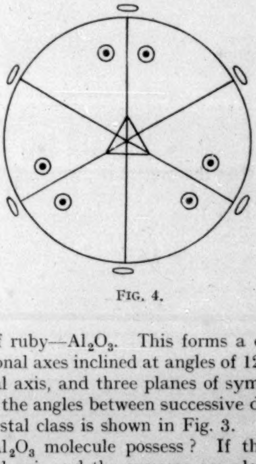

FIG. 4.

How much of this symmetry can the Al_2O_3 molecule possess? If the two aluminium atoms are placed along the trigonal axis and the oxygens are placed at the corners of an equilateral triangle in the equatorial plane of the two aluminiums, a highly symmetrical molecule is formed. If the component atoms are considered as spheres, the symmetry of the molecule is that shown in Fig. 4. It has a trigonal axis, three digonal axes, three planes of symmetry each passing through the trigonal axis and one of the digonal axes and an equatorial plane of symmetry. Although this molecule possesses, like the crystal, both planes and axes of symmetry, the distribution of these is not that of the crystal. In fact, Fig. 4 corresponds to a crystal of class 22. The ruby crystal has planes of symmetry bisecting the angles between the digonal axes. In order to form such a crystal from the above molecule it would be necessary to introduce a second molecule into the lattice, whose orientation is such that it is the mirror image of the first in a plane of symmetry bisecting the angle between two of the digonal axes of the molecule. If this were done the symmetry of the resulting molecule would be that shown in Fig. 5. The crystal would then belong to the dihexagonal bipyramidal class. As observations show that the crystal is ditrigonal scalenohedral and not dihexagonal bipyramidal, it follows that the molecule cannot have all the symmetry attributed to it above. It is clear that the maximum symmetry which the molecule can possess is (1) a trigonal axis, (2) either three digonal axes or three planes of symmetry. It cannot possess both the planes and the axes. It is difficult to decide whether it is the planet or the axes which actually exist in the molecule. It is obvious, however, that it is not possible to consider the various atoms as spheres. The deciding factor in the determination of the symmetry must be the distribution of the outer electrons in the component atoms.

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* Debye and Sherrer, Phys. Zeit., 13, 297 (1917).

† Hull, Phys. Rev., 10, 692 (1917).

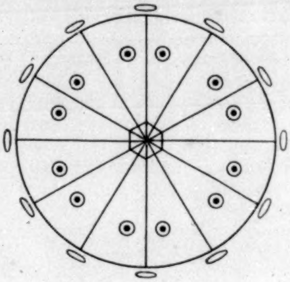
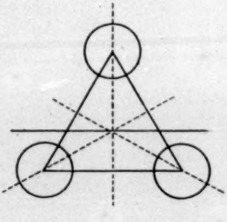


FIG. 5.



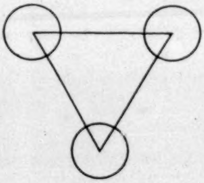


FIG. 6.

As the moleule cannot possess the full symmetry of the crystal there must be at least two molecules in the unit cell. The second of these is derived from the first by a reflection in a plane of symmetry or a rotation about a digonal axis according as the molecule possesses in itself the axes or the planes of symmetry. The result will be two sets of Al_2O_3 molecules whose oxygen atoms are arranged as shown in Fig. 6. This is in entire agreement with the results obtained by Bragg* from measurements by X-ray methods.

Quartz (SiO_2) provides another interesting example. It forms a crystal of the trigonal trapezohedral class and has three molecules in the unit cell. If the molecule were asymmetric six molecules would be necessary. Hence the crystal must share with the molecule a two-fold symmetry. Since the crystal possesses no planes of symmetry this common element of symmetry must be a digonal axis. Hence the SiO_2 molecule must be capable of reproducing itself when rotated through 180 deg. about some axis. It must therefore be of some such form as is shown in Fig. 7. Here again, if the atoms are regarded as spheres, the molecule would possess in addition two planes of symmetry. As the crystal does not show these planes it is clear that the outer electrons in the oxygen atoms are so distributed that the molecule will not reflect into itself across a plane bisecting the angle between the two Si—O directions or across a plane passing through the centres of the three atoms. This fact throws further light on the symmetry of the ruby molecule. In that case it was not found possible to decide whether the molecule possesses the planes or the digonal axes of symmetry. The fact that the oxygens in the quartz molecule can rotate into each other but not reflect into each other makes it extremely probable that the same is true in the case of ruby, and that therefore the Al_2O_3 molecule possesses three axes but no planes of symmetry.

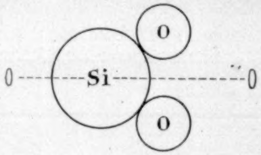


Fig. 7.

By placing three SiO_2 molecules in the unit cell, obtained from each other by successive rotations of 120 deg. about the trigonal axis and equally spaced along this axis, and such that the projections of corresponding points of these three molecules on a plane perpendicular to the trigonal axis form an equilateral triangle, all the necessary symmetry is given to the crystal. It is, of course, obvious that the trigonal axis of the crystal must be perpendicular to the digonal axis of the molecule. W. H. Bragg's measurements by X-ray methods result in a model which supports these arguments.

An examination of the crystal models of many simple inorganic compounds fails to reveal the presence of a definite molecule in the crystal. Thus in NaCl each sodium atom is surrounded by six chlorine atoms, and each chlorine atom by six equidistant sodium atoms. The same applies to KCl. In Ice each hydrogen is surrounded by two oxygens, and each oxygen by four hydrogens giving the necessary proportion of hydrogen to oxygen. In fact Bragg* has used this fact to construct a crystal model of ice, and has shown that the result agrees with the X-ray measurements made by Dennison. In more complex compounds the molecule certainly appears in the crystal, and it seems reasonable to assume that even in the above cases the molecule really exists and that, in the case of NaCl for example, each chlorine atom has its own particular sodium. It will be shown that if the molecule does actually exist, the sodium to which any particular chlorine belongs is not one of the six surrounding it at equal distances, but one of the sodiums arranged diagonally relative to the chlorine.

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* See W. H. and W. L. Bragg, X-Rays and Crystal Structure, 2nd edition.

If it is assumed that the molecule does actually exist, even in polar compounds, some interesting results as to the symmetry of various molecules follow.

NaCl and KCl form cubic crystals of classes 32 and 29 respectively. Both have four molecules in the unit cube. The symmetries which the molecules share with the crystals are therefore twelve-fold and six-fold respectively.

Consider the case of KCl. This six-fold symmetry must consist of (1) a trigonal axis, and (2) a digonal axis perpendicular to the trigonal axis. Such a symmetry involves the presence of two other digonal axes perpendicular to the trigonal axis, and inclined to each other and to the first digonal axis at angles of 120 deg.

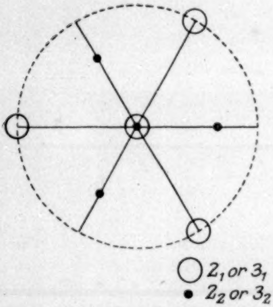

FIG. 8.

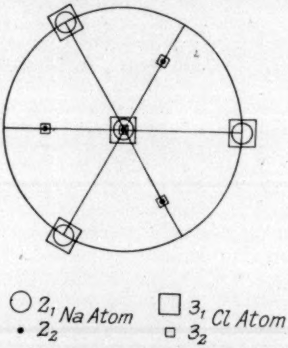

FIG. 9.

The question arises—how can the KCl molecule possess this symmetry ? If the K and Cl atoms are placed on the trigonal axis then there must be three digonal axes perpendicular to the K—Cl line. A rotation of the molecule through 180 deg. about one of these axes brings the K atom to the position of the Cl atom. It follows that the K and Cl atoms must be interchangeable as far as symmetry considerations go. This is not so unreasonable as it appears at first sight when it is remembered that what we have to deal with is a combination of a positively charged K atom with a negatively charged Cl atom. The outermost electron of K atom has passed over into and been absorbed by the Cl atom. Each atom has then the electron

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[*] W. H. Bragg, Proc. Phys. Soc., Vol. 34, p. 98 (1922).

arrangement of the inert gas Argon. They differ only in the existence of positive and negative charges on the two atoms. It is quite conceivable that this difference of charges has no bearing on the question of crystal symmetry, and we may therefore regard the K atom as being able to rotate into the Cl atom without making any difference in the molecule from a crystal point of view.

Similar arguments apply to the case of NaCl. The Na atom becomes a positively charged Neon atom, while the Cl atom becomes a negatively charged Argon atom. It appears that for crystallographic purposes a positively charged Na atom is identical with a negatively charged Cl atom. Such arguments point to the fact that in crystallographic questions the governing factor is not the nature of the nucleus, but the electron arrangement around the nucleus and more especially the arrangement of the outer electrons.

Consider the electron arrangement in these atoms. The following table is that given by Bohr* for the numbers of electrons in the various types of orbit :—

TABLE.

Element.	Number of electron orbits of various types.							
	1_1	2_1	2_2	3_1	3_2	3_3	4_1	4_2
Ne ...	2	4	4	—	—	—	—	—
Na ...	2	4	4	1	—	—	—	—
Cl ...	2	4	4	4	3	—	—	—
A ...	2	4	4	4	4	—	—	—
K ...	2	4	4	4	4	—	1	—

Both Neon and Argon possess two sets of outer electrons, four in each set. In Neon, the four 2_1 orbits are such that the normals to their planes are arranged tetrahedrally in space. The four 2_2 orbits are circular. Böhr does not state how these are arranged. The most symmetrical arrangement appears to be obtained by arranging three of the normals in a plane normal to one of 2_1 normals at angles of 120 deg. to each other, and the fourth normal in a direction perpendicular to this plane. The direction cosines of these planes may be represented by

$$\left.\begin{matrix}111\\ \bar{1}11\\ \bar{1}1\bar{1}\\ 1\bar{1}\bar{1}\end{matrix}\right\} \text{for the } 2_1 \text{ orbits and } \left.\begin{matrix}\bar{1}\bar{1}\bar{1}\\ 01\bar{1}\\ 1\bar{1}0\\ \bar{1}01\end{matrix}\right\} \text{for the } 2_2 \text{ orbits.}$$

The stereographic projection of these orbits is shown in Fig. 8.

The atom possesses one trigonal axis and three planes of symmetry, but no digonal axes of symmetry. If we consider two of these atoms placed end to end in opposite directions we obtain a molecule which possesses one trigonal axis, three digonal axes and four planes of symmetry as shown in the stereographic projection, Fig. 9.

This is the symmetry of the NaCl molecule. Fig. 10 shows the suggested distribution of electron orbit normals. If we rotate the lower atom of the molecule through some angle about the trigonal axis so as to destroy the planes of symmetry, we obtain a molecule which possesses a trigonal axis and three digonal axes—the necessary symmetry of the KCl molecule. The stereographic projection of such a molecule is shown in the figure given below.

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* Bohr, Zeitschr. fur Physik, 9, 1 (1922).

We thus see that by considering the distribution of the electrons round the nuclei we can construct molecular models which have the desired symmetry. It is, of course, not claimed that these figures represent the actual arrangement of the electrons ; many other arrangements could no doubt be constructed. The important thing is that it can be demonstrated that it is the electron distribution around the nuclei which matters and not the nuclei themselves.

Ice forms another interesting example. X-ray data indicate that the crystal is hexagonal with four molecules in the unit cell. An examination of Table I. shows that the molecule must possess at least a trigonal axis. This the molecule cannot do unless the three atoms of the molecule are arranged along the trigonal axis. X-ray data show that such is not the case. The hydrogen atom consists of a nucleus with one outer electron ; the oxygen atom has six outer electrons. If the two electrons from the hydrogens join with the six oxygen electrons to form the electron arrangement of neon, what has to be considered is a doubly charged neon atom surrounded by two hydrogen nuclei. It is possible that these nuclei play no part in the determination of the symmetry of the molecule from a crystallographic point of view, and that all that has to be considered is a doubly charged neon atom. In the discussion of the NaCl crystal it has been shown that this atom has probably a six-fold symmetry. As there are four ice molecules in the unit cell it follows that

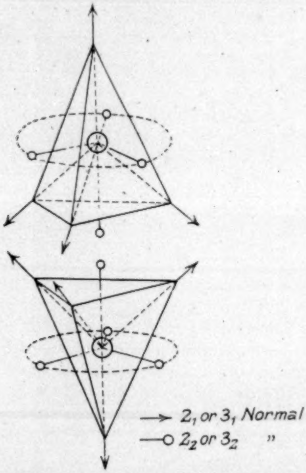

FIG. 10.

the crystal should possess 24-fold symmetry. This is in agreement with the assumption of a hexagonal form for the crystal.

It is also of interest that in the case of ice the distance between the centres of the oxygen and hydrogen atoms is 2·76 A.U., a distance considerably greater than the commonly accepted value. This fact may be explained if it is assumed that the hydrogens are playing no part except in determining the residual electric field round the molecule and that what has been measured is the distance between the centres of two negatively charged neon atoms.

In the same way the symmetry of ZnS should be similar to that of KCl. The crystal is cubic (hexakis tetrahedral) and possesses four molecules in the unit cell. The symmetry of the molecule must therefore be six-fold, that is to say, the same as that of the KCl molecule.

There are, of course, many inorganic polar compounds which cannot be considered as an aggregate of inert gas atoms. Such molecules as $CaCO_3$, $NaNO_3$, $NaClO_3$, &c., cannot be so regarded. The ClO_3 ion has 18 outer electrons which is the number possessed by Argon. The distribution of these electrons is, however, very different from that of the Argon atom, and we cannot consider the two as equivalent in any respect. As is to be expected, crystallographic measurements show that the molecule possesses a trigonal axis but no planes or digonal axes. In general, where two or more different atoms occur in one of the ions, this ion will not become identical in electron arrangement, with an inert gas atom. Hydrogen is anomalous in so far as it possesses only one electron, and when it has lost this electron only the positively charged nucleus remains, and it may be permissible in some cases to ignore the effect of this nucleus in dealing with the symmetry of the molecule from a crystallographic point of view. Ice appears to be an example of this.

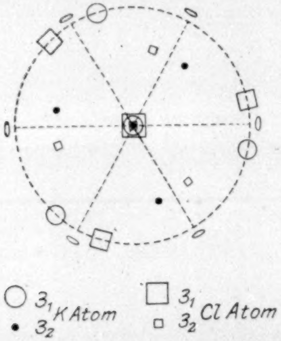

Fig. 11.

The application of these principles to organic crystals is somewhat more limited as the molecules of such compounds themselves possess little or no symmetry. A very large number of the crystals so far investigated are monoclinic prismatic and possess four molecules in the elementary cell. This would indicate a complete absence of symmetry in the molecules. There are, however, some notable exceptions. For example, both naphthalene and anthracene belong to this class, but have only two molecules in the unit cell. This must mean that these molecules share a two-fold symmetry with the crystal. Benzene forms a rhombic bipyramidal crystal and has two molecules per cell. This gives to the benzene molecule a four-fold symmetry, possibly a plane of symmetry and an axis of symmetry perpendicular to the plane. In view of the fact that the structure of the benzene ring has been the subject of much debate, this new information may be of considerable importance. There have been many suggestions as to the arrangements of the fourth valency bonds, but most of these would appear to give to the molecule a trigonal axis. This is not supported by crystallographic work. Whatever the arrangement of the bonds, the molecule, in its crystalline state at least, must show a plane of symmetry with a digonal axis perpendicular to it or some equivalent symmetry.

Neither the Kekulé nor the Claus formula conforms with this condition. On the other hand, the Ladenburg formula does, and so also does the formula suggested by Dewar. Ingold* has recently shown that this Dewar formula has much chemical evidence to support it. The arrangement of bonds is that shown in Fig. 12. If the puckered hexagon suggested by Bragg† is assumed and the bonds are arranged in this way, it is found that the molecule has the required symmetry. Fig. 12 shows the symmetry of the molecule. The points marked α are supposed to lie above the plane mean plane of the ring and those marked β below the mean plane.

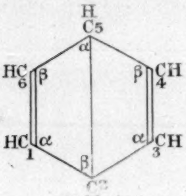

FIG. 12.

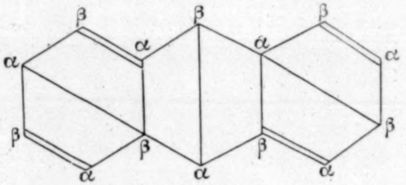

FIG. 13.

If this is extended to the case of anthracene the molecule has a centre of symmetry, and this is represented in Fig. 13. Naphthalene presents greater difficulties. The X-ray measurements show that the molecule has a twofold symmetry—probably a centre of symmetry. Such formulæ as those of Fig. 14 satisfy this condition. All the formulæ usually given for naphthalene appear to possess too great symmetry. Bragg's measurements of the naphthalene crystal show that there are two molecules in the unit cell and indicate further that the second is derived from the first by a rotation of 180° about the "*b*" axis. The presence of these two molecules, together with the existence of a centre of symmetry in the molecule, gives to the crystal the full symmetry of the monoclinic prismatic class.

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* Ingold, Journal Chem. Soc., June (1922).

† Bragg, Proc. Phys. Soc., Vols. 33, 34 (1921).

An examination of the measurements made by crystallographers on benzene substitution products would appear to indicate that when two of the hydrogens are replaced by the same groups the 1-3 product is a crystal of higher symmetry than the 1-2 or the 1-4 product. This probably implies that the 1-3 compound possesses in itself some symmetry, while the 1-2 and 1-4 compounds are in general asymmetric. If the 1, 2, 3, 4 positions are as shown in Fig. 12, it is clear that the 1-3 compound possesses a plane of symmetry, while the others are asymmetric. The objection to this point of view is that it is to be expected that there would exist more than one 1-3 compound and that these various compounds would show differences in crystalline and probably in chemical properties. No such differences have been observed. Until X-ray data are available as to the structure of some of these substances it is of little use pushing the question further.

Another interesting problem in crystallography is the determination of the conditions governing the choice by any molecule of some particular crystal form. If the molecule is asymmetric it is theoretically possible for it to crystallise in any of the 32 classes. On the other hand, it seems extremely improbable that 48 molecules will arrange themselves in such a way as to form a crystal of class 32. In fact, it appears that eight is about the maximum number of molecules probable in the elementary cell. It is to be expected that asymmetric molecules will crystallise in classes of low symmetry, and this is amply supported by the measurements made by crystallographers. For example, a statistical investigation of about 1,000 of the aromatic organic crystals described by Groth shows that approximately 60 per cent. form crystals which would require four asymmetric molecules, about 20 per cent. would require eight, and about 20 per cent. four. Some of these molecules may have some symmetry of their own, and the 20 per cent. containing eight is probably an over estimate. It seems that there is a decided tendency for four asymmetric molecules to come together, grouping themselves about an axis and plane of symmetry.

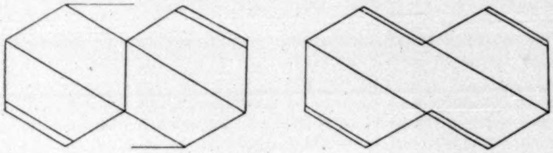

Fig. 14.

If the molecule possesses symmetry and our second hypothesis is true, that the symmetry of the molecule is reproduced in the crystal, then it is clear that the selection will fall on more highly symmetrical classes. The symmetry of the molecule imposes a lower limit to the symmetry of the crystal. It appears to be generally true that there is more than one molecule in the elementary cell. This means that the crystal is of higher symmetry than the component molecule. The only cases where we should expect only one molecule in the cell are in classes 1 and 32, the latter with a molecule or atom which in itself possesses perfect symmetry if such exists.

While these conditions impose certain restrictions on the choice of a crystal form by any particular molecule they still leave a considerable margin of choice. The remaining conditions must depend on the molecule and it seems fairly certain that an important factor is the distribution of the residual forces round the molecule. That such forces exist is shown by the fact that the crystals actually form. These forces are presumably of an electrical nature and it is on the distribution of these charges over the molecule that the final choice of a crystal form must depend. It is possible to imagine distributions of positive and negative charges over the molecular surface such that when two molecules came within each other's sphere of action they would naturally orientate themselves about a digonal or other axis of symmetry.

In the case of simple polar compounds, the two atoms of the molecule are oppositely charged and there will be a strong tendency for the negatively charged atom of any molecule to become surrounded by the positively charged atoms of other atoms and *vice-versa*. This is what happens in the case of NaCl and similar molecules.

Some of the considerations stated above have recently been used in attempts to solve the structure of somewhat complicated crystals and have proved of considerable value in the interpretation of the data derived from X-ray observations.

SUMMARY.

1. The nature of the elementary lattice and the meaning of axes and planes of symmetry in the lattice are discussed.

2. A table is given showing the number of asymmetric molecules necessary per unit lattice to produce the symmetry of the various crystal classes. The existence of fewer molecules in the unit cell is shown to imply the existence of symmetry in the molecule.

3. The relative positions of the variously orientated molecules in the lattice are considered, and it is shown that only certain positions are possible.

4. It is suggested that the number of molecules in the elementary cell is always the minimum necessary to satisfy the symmetry conditions ; also that any symmetry of the molecule is reproduced in the crystal.

5. Applications of these principles to particular cases of inorganic and organic crystals are given.

6. The factors underlying the selection by a molecule of some particular crystal class are briefly discussed.

In conclusion, the author has much pleasure in acknowledging his indebtedness to Prof. Sir W. H. Bragg, F.R.S., for much help during the development of these ideas.

This Paper originated in connection with work being carried out at University College, London, under the Department of Scientific and Industrial Research.

DISCUSSION.

The **PRESIDENT**, before calling on Sir William Bragg, congratulated him in the name of the Society on the honour recently conferred upon him by the Paris Academy of Sciences.

Sir **WILLIAM BRAGG**, after acknowledging the congratulations tendered to him, said that the Paper before the meeting was an important one, throwing a new light on the possibilities of investigating crystal structure. The Author's intricate but lucid argument affords a great simplification of the problem. Three unitary structures have to be considered : (1) The chemical molecule, as it exists in solution; (2) the elementary crystal lattice or cell, consisting of groups of (3) crystal molecules, in which the atoms and electrons are not necessarily arranged in the same manner as in chemical molecules. The Author's method enables one to determine the symmetry of these groups at once from the geometry of the crystal, with a very little help from X-rays. A considerable vista of research is thus opened up, which may serve to settle such questions as that of the static versus the dynamic model of the atom.

Dr. J. W. EVANS thought that the methods employed by Mr. Shearer gave promise of proving of great assistance in working out the structure of crystals. There were, however, some considerations that must not be neglected:

(1) It is by no means certain that molecules always maintain their identity in the crystalline state.

(2) Even where they exist in the crystal, they may not retain the same form and symmetry as in a free state.

(3) Although in an ideal crystal all the cells are identical, and have identical orientation, this is by no means always actually true. We know that cells of different composition may be employed indiscriminately if they are approximately the same in form and volume, as in the case of the plagioclase felspars.

In the same way, if the outer forms of cells of the same substance approximate to a higher symmetry than the cell itself, these cells may, even when differently orientated, be employed in building up a crystal structure. The same may be true with cells that are enantiomorphically related. In these cases the whole structure may have a higher symmetry than the individual cells as the result of either (*a*) a symmetrical arrangement of the cells in the nature of molecular or ultramicroscopic twinning—twinning is usually an attempt to attain higher symmetry—thus forming lattice cells of greater size; or (*b*) indiscriminate occurrence of cells differing in orientation or in enantiomorphic character. In the former case it might be very difficult to recognise by means of X-ray analysis the primary cells. In the latter case it would, I should think, be impossible.

The explanation of the crystalline structure of potassium chloride is not convincing. The symmetry of the molecules shown in Fig. 11 is that of the quartz or trigonal trapezohedral class which is represented by the symbol IIIUh (J. W. Evans, Min. Mag., Vol. 15, pp. 398-400, 1910), with three digonal axes at right angles to a trigonal axis, and making angles of 120° with each other; whereas that of the crystal structure is stated to belong to the cuprite or pentagonal eikositetrahedral, CUh, class, in which there are four trigonal axes corresponding to the diagonals of the cube and three digonal axes at right angles to one another and parallel to the edges of the cube. The cell contains four molecules, and the trigonal axes of these are supposed by the author to be parallel to the diagonals of the cube, but it is not explained how the 4×3 digonal axes at right angles to the diagonals are transformed into three digonal axes at right angles to one another.

From the examination of the material prepared at the Imperial College and a comparison of the work of other observers, Miss Knaggs, of the Imperial and Birkbeck Colleges, has shown that the symmetry of the crystal structures of the simpler carbon compounds frequently bears close relation to that of the molecule. Substances of the type CX_4, where X is an element, are usually cubic. Those of the types CX_3Y are trigonal or hexagonal, unless X is hydrogen, when the symmetry is lower. Those of the form $C(CX_3)_4$ are as a rule cubic, and those with $C(CX_2Y)_4$ are tetragonal. In all cases the symmetry is apt to be that of a higher or different class than would have been expected, though one belonging to the same system. In the case of the $C(CX_2Y)_4$ compounds the symmetry of the molecule would be that of chalcopyrite or tetragonal scalenohedral, IVBk, class, whereas that of the crystal usually belongs to the IVBc, or in one case IVBu class. It can be shown that symmetry of this character would be obtained by appropriate molecular twinning. Some of the compounds considered are dimorphic, crystals with less symmetry forming at lower temperatures.

The Speaker threw out the suggestion that crystal structure might possibly be due to the repulsion of the electrons of different atoms combined with an attraction of the atoms as a whole. If it be supposed that, contrary to the usual view, there are six outer electrons in chlorine having their mean position arranged like the points of an octahedron, and that in potassium there are eight electrons arranged like the points of a cube, it will be evident that a position of stability would be obtained when the points of each chlorine octahedron were opposite the centres of the faces of the surrounding potassium cubes, and the points of each potassium cube were opposite the centres of faces of chlorine octahedra, and that the potassiums and chlorines would then be arranged alternately in a cubic lattice just as we know to be the case.

Mr. **Shearer**, in reply, thanked Sir William Bragg for his remarks on the Paper He agreed with Dr. Evans as to the possibility of a difference between the molecule in a crystal and the molecule in solution. The two were not necessarily identical, although there was evidence that in many cases there was a strong resemblance.

Dr. Evans appeared to doubt the possibility of the determination of the true unit cell. What was done by X-rays was to measure the distance between two successive *identical* planes; in this way, the method of X-rays was much more powerful than any other method at present at the disposal of the crystallographer.

Dr Evans was apparently under a misapprehension as regards the author's suggestions as to the KCl molecule. Although the KCl molecule was given the symmetry of the quartz class, there were present in the unit cell four such molecules. By giving these the proper orientations in the cell it was possible to give the crystal the symmetry of the pentagonal icositetrahedral class. The trigonal axis of the molecule lay along—not perpendicular to—the diagonal of the cube. The three digonal axes were parallel to three diagonals of the faces of the cube. Thus, if 111 were proportional to the direction cosines of the cube diagonal, or trigonal axis, the direction cosines of the three digonal axes of the molecule were proportional to $1\bar{1}0$, $\bar{1}01$ and $01\bar{1}$. These were parallel to three of the diagonals of the faces of the cube, and made angles of 120° with each other and were perpendicular to the trigonal axis. The directions of these axes coincided therefore with those of the cubic class to which KCl belonged. The rest of the symmetry of the crystal arose from the presence of four of these molecules in the unit cell.

The results obtained by Miss Knaggs were interesting, and appeared to support the suggestions put forward in the Paper. The crystal in general possessed more symmetry than the molecule owing to the presence of more than one molecule in the unit cell. Until the number of molecules in the cell had been determined it was unsatisfactory to discuss the symmetry of the molecule.

As far as external evidence went there seemed nothing in support of the suggestions of Dr. Evans as to the electron arrangements in the K and Cl atoms. Our knowledge of atomic and molecular forces was at present extremely limited.

X. *Modification of the Powder Method of Determining the Structure of Metal Crystals.* *By* E. A. Owen, *M.A., D.Sc., and* G. D. Preston, *B.A., The National Physical Laboratory.*

Received Oct. 25, 1922.

ABSTRACT.

Plates of aluminium, iron, copper, lead and magnesium have been examined by means of the Bragg X-ray spectrometer employing radiation direct from a molybdenum anticathode. The maxima observed in the spectra are sufficiently intense to measure with accuracy, and the crystalline structure of the materials examined are readily determined. A few of the results obtained by the method are included in the Paper as typical examples.

I. Introductory.

The powder method of determining the structure of crystals, independently developed by Hull* and by Debye and Scherrer,† has had a wide application and has been extensively employed by Hull in particular for the examination of the structure of metal crystals. For this method the material under examination is reduced to a fine powder, a small quantity of which is placed in a tiny glass tube situated in the path of a beam of X-rays which has been rendered monochromatic by passing it through a suitable filter. The rays reflected at the different faces of the small crystals in the tube appear on a photographic film upon which they fall as a series of irregularly spaced lines, the positions of which suffice to solve the structure of the material.

W. H. Bragg‡ has recently shown that the structure of powdered materials can also be solved by the aid of the X-ray spectrometer. The powder in this case is pasted on a flat sheet of cardboard and the whole mounted on the table of the spectrometer in the position usually occupied by the face of a single crystal. Unfiltered radiation direct from a rhodium anticathode falls on the powder, both the α and the β lines of the K series spectrum of rhodium being reflected in turn into the ionisation chamber. If the structure is not too complicated, the presence of the two lines is an advantage rather than a disadvantage in that the reflection of the one line serves as a check upon the other. The advantage of this method over the photographic method is twofold—the time of exposure is very much shortened and the relative intensity of the lines in the spectrum is directly observed. In order to obtain a readable impression of the spectrum upon a photographic plate an exposure of about 300 milliampere-hours is required, whereas with the X-ray spectrometer the spectrum can be plotted in an hour or two, using only about 1 milliampere through the bulb.

The present Paper is an account of a further development of the method of determining the structure of metal crystals. Instead of reducing the metal, the crystalline structure of which is required, to a fine powder, a plate of the metal is taken and examined by means of the X-ray spectrometer in a manner similar to that adopted by Bragg. The few results given below show that the structure can be solved by this means in a comparatively short time with only small currents passing through the bulb. The examples chosen serve to illustrate the application of the method to the solution of the structure of crystals belonging respectively to cubic and hexagonal systems.

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* Hull, Phys. Rev., 10, p. 661 (1917).

† Debye and Scherrer, Phys. Zeitschr., 17, p. 277 (1916); 18, p. 291 (1917).

‡ W. H. Bragg, Proc. Phys. Soc., Vol 33 Part 4, p. 222 (1921).

II. Method of Procedure.

A plate of metal, either cast or rolled and measuring about 5 cm. by 2·5 cm., is subjected to heat treatment necessary to produce throughout its volume a mass of small crystals—crystals about 1 mm. or less across are suitable for the purpose. The plate is etched to remove the deformed surface layer of crystals or any oxide that may have formed, and is then smeared over with vaseline to prevent further oxidation. It is mounted on the spectrometer with one of its large surfaces adjusted so as to fall just over the centre of the crystal table.

The spectrometer is provided with an 8 in. circle graduated to read directly to 5 seconds of arc. The width of the chamber slit is kept at 2 mm., which corresponds to about one degree of arc, whilst the bulb and crystal slits are opened wide enough to allow a beam of rays to fall upon the plate just sufficient to cover it from side to side.

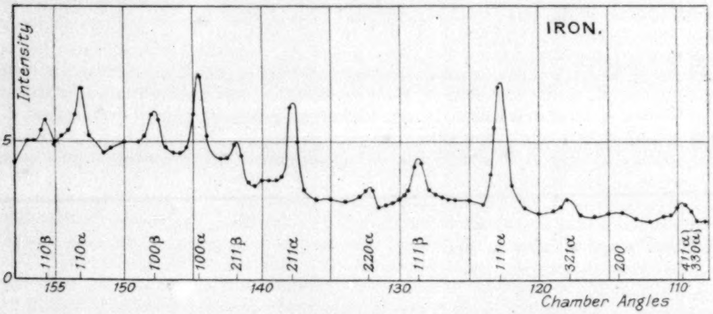

Fig. 1.

During a few preliminary observations the metal plate is fixed at any angle with the X-ray beam and readings are taken of the ionisation for different positions of the chamber. One or two of the main maxima are located in this way. The chamber is then fixed in the position to receive one of these maxima, whilst the plate is rotated through a range of angle and the position of the plate determined at which maximum ionisation occurs in the chamber. Having determined this position, the plate is set at that angle and both chamber and crystal table are rotated together, the former at twice the rate of the latter.

These preliminary observations determine the position of the plate at which there is the smallest amount of absorption of the radiation for the face corresponding to the particular maximum chosen. We are assuming that this condition obtains also approximately for all the other faces of the crystals, since the number of crystals in the area of the plate irradiated is very large and there will be as many crystals on the average whose 100 planes, for example, are in the surface of the plate as there are crystals presenting 111, 110 or any other plane. That this assumption is justified is proved by the relative intensity of a line reflected at a large angle when the plate is kept fixed and when it is rotated in the manner stated, the intensity in the latter case being found to be much greater than that in the former. In the present investigation no particular attention has been directed to the relative intensities of the lines in the spectra, the main object being to locate the lines over a wide range of crystal planes.

III. Results.

The five metals, magnesium, aluminium, iron, copper and lead, have been examined in detail. The curve obtained by the above method for iron is shown in Fig. 1, the radiation employed being that from a molybdenum anticathode for which $\lambda_a = 0{\cdot}711$Å and $\lambda_\beta = 0{\cdot}629$Å. The curves show the actual readings of intensity plotted against chamber angle, readings being taken at half degree intervals. The exposure for each reading was about 8 seconds. The Coolidge bulb employed was operated at 80,000 volts and had a current of 1 milliampere passing through it.

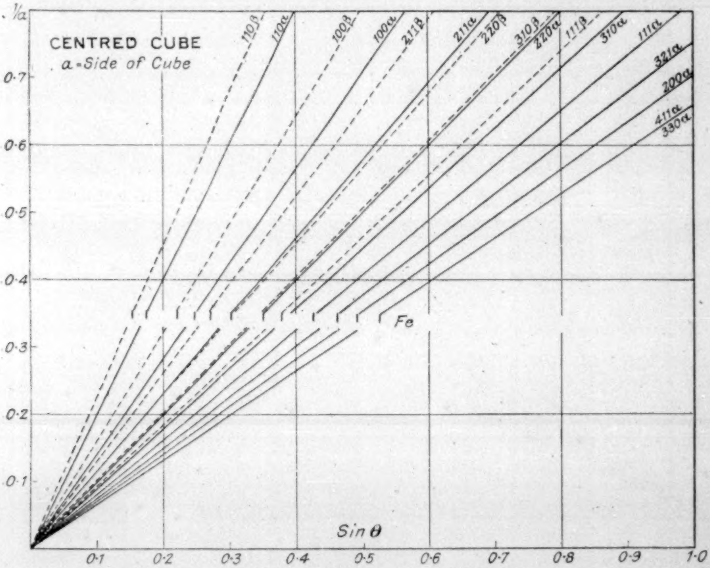

FIG. 2.

When reflection takes place at a crystal face the relation $n\lambda = 2d \sin \theta$ must be satisfied, where λ is the wave-length of the radiation reflected, d the distance between successive planes parallel to the face, and θ the angle of reflection. In the case of crystals of cubic structure, d for all faces is proportional to the length of the side of the elementary cube. Thus we have $n\lambda = 2ka \cdot \sin\theta$, where k is a constant characteristic of the set of planes at which reflection occurs. The values of k for different sets of planes in any one of the three possible cubic structures are fixed so that if we plot the reciprocal of a against $\sin\theta$ we get a series of straight lines passing through the origin, each straight line corresponding to one definite face of the crystal. The graphs for the face centred cube and for the centred cube

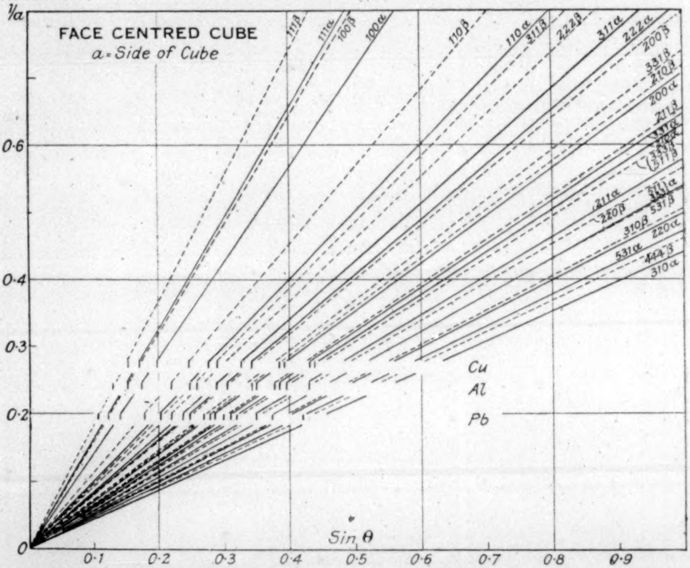

Fig. 3.

are shown in Figs. 2 and 3 respectively. In these graphs both the α and the β lines are included, the former being the full and the latter the broken lines. If the structure of the crystal is cubic, the lines in its spectrum should fit somewhere on one of these graphs. To test this the sines of half the chamber angles of the lines in the spectrum were set out on a slip of paper, which was passed over the graphs, the zero line of the one coinciding with the zero line of the other. We have shown in Figs. 2 and 3 where the best fit occurs in each case. Aluminium, copper and lead fit on the graph for the face centred cube and iron on that for the centred cube. The magnesium spectrum does not fit on any of these graphs and will have to be considered separately. In the case of each of the other four metals the fit is good and, with one or two exceptions, *all* the lines are accounted for.

From the position at which each spectrum fits the graph, we deduce at once the length of the side of the elementary cube. To obtain a more accurate determination of this quantity, its value was calculated independently for each line in the spectrum. The values calculated for iron are tabulated below and serve as an example of the order of agreement obtained.

TABLE I.—*Electrolytic Iron.*

Reading of Maximum.	θ	$\sin\theta$	Form.	$k \equiv d/a$	Side of. Elementary Cube. (a)
155·6°	8·85	0·1539	110β	0·707	2·867$\mathring{A}$
153·1°	10·1	0·1754	110α	0·707	2·867
147·8°	12·75	0·2207	100β	0·500	2·828
144·6°	14·35	0·2478	100α	0·500	2·869
141·8°	15·75	0·2714	211β	0·408	2·870
137·8°	17·75	0·3048	211α	0·408	2·909
132·3°	20·5	0·3502	220α	0·3535	2·872
128·7°	22·3	0·3795	111β	0·2885	2·849
122·8°	25·25	0·4266	111α	0·2885	2·888
117·9°	27·7	0·4648	321α	0·2672	2·863
114·2°	29·55	0·4932	200α	0·250	2·882
109·7°	31·8	0·5270	(411α) (330α)	0·2358	2·863
				Mean value of $a = 2{\cdot}869\mathring{A}$.	

The results obtained with the four metals aluminium, iron, copper and lead are summarised in Table II. Included in the table are the values of the number of atoms (n) associated with each elementary cube calculated from the observed mean values of a. In the last column are inserted for comparison the values of a found by Hull with the powder method. The greatest difference between these values and those obtained by the method described here is about 1 per cent. The calculated values of n in the next to the last column of the table are in accordance with the lattice assigned to the different elements, namely that for a centred cube, two atoms, and for a face centred cube, four atoms, are associated with each elementary cube.

TABLE II.

Element.	Structure.	Length of side of elementary cube (a).	$n = \frac{\rho a^3}{m}$	Values of a as measured by Hull.
Al	Face Centred Cube	4·041$\mathring{A}$	3·98	4·05A
Fe	Centred Cube	2·869	2·015	2·86
Cu	Face Centred Cube	3·628	4·06	3·60
Pb	Face Centred Cube	4·983	4·11	4·92

IV. MAGNESIUM.

The procedure in the case of magnesium had to be somewhat modified in order to include crystals of different axial ratios. Considering, for instance, crystals of the close packed hexagonal type, a graph was drawn for each set of planes in the crystals giving the relation between the logarithm of the reciprocal of the axial ratio (c) and the logarithm of the reciprocal of the distance between consecutive planes, assuming the side of the hexagon to be unity. Such a graph will be independent of the wave-length of the radiation reflected, and can, therefore, be used both for the α and for the β lines. The graph, shown in Fig. 4, is that for a closed packed hexagonal lattice. It can be used for crystals over a wide range of axial ratio.*

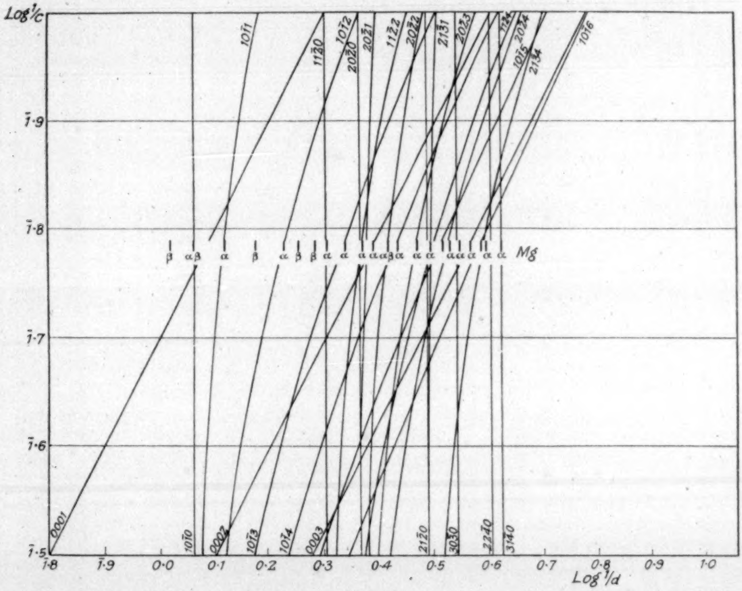

FIG. 4.

It should be mentioned that the magnesium plate examined was cut from a rod of the metal about 2·5 cm. wide. The specimen was polished and etched before the X-ray examination was conducted, but in this case it was not especially heat treated previously as in the other cases already considered.

The lines observed in the spectrum of this metal were very prominent, and as many as 22 lines were measured, including both α and β lines. The logarithm of the

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* A graph similar to this has been employed by Hull, the only difference being in the choice of the ordinates (*see* Hull, Phys. Rev., 17, p. 549, 1921).

TABLE III.—*Magnesium.*

Reading of maximum.	θ	Log. sin θ	Form (Miller-Bravais).	d	d/a ($c=1\cdot631$)	Length of side of hexagon. (a)
160·2°	6·65	$\bar{1}$·0633	$10\bar{1}0\ \beta$	$a\frac{\sqrt{3}}{2}$	0·866	3·116Å
158·5°	7·4	·1099	$10\bar{1}0\ \alpha$	$a\frac{\sqrt{3}}{2}$	0·866	3·187
			$10\bar{1}1\ \beta$	$a/\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	0·7649	3·162
156·4°	8·45	·1673	$10\bar{1}1\ \alpha$	$a/\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	0·7649	3·161
154·1°	9·6	·2222	$10\bar{1}2\ \beta$	$a/\sqrt{\frac{4}{3}+\frac{4}{c^2}}$	0·5937	3·151
151·7°	10·8	·2727	$10\bar{1}2\ \alpha$	$a/\sqrt{\frac{4}{3}+\frac{4}{c^2}}$	0·5937	3·186
150·4°	11·45	·2980	$11\bar{2}0\ \beta$	$a/2$	0·500	3·142
148·5°	12·4	·3318	$10\bar{1}3\ \beta$	$a/\sqrt{\frac{4}{3}+\frac{9}{c^2}}$	0·4605	3·156
147·4°	12·95	·3506	$11\bar{2}0\ \alpha$	$a/2$	0·500	3·171
145·2°	14·05	·3850	$10\bar{1}3\ \alpha$	$a/\sqrt{\frac{4}{3}+\frac{9}{c^2}}$	0·4605	3·181
142·7°	15·3	·4215	$1122\ \alpha$	$a/\sqrt{4+\frac{4}{c^2}}$	0·4263	3·159
141·5°	15·9	·4378	$0002\ \alpha$	$ac/4$	0·4077	3·181
139·3°	17·0	$\bar{1}$·4660	$20\bar{2}2\ \alpha$	$a/2\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	0·3824	3·178
			$20\bar{2}3\ \beta$	$a/\sqrt{\frac{16}{3}+\frac{9}{c^2}}$	0·3388	3·150
137·8°	17·75	·4840	$10\bar{1}4\ \alpha$	$a/\sqrt{\frac{4}{3}+\frac{16}{c^2}}$	0·3689	3·161
			$21\bar{3}1\ \beta$	$a/\sqrt{\frac{28}{3}+\frac{1}{c^2}}$	0 3209	3·190
134·8°	19·25	·5181	$20\bar{2}3\ \alpha$	$a/\sqrt{\frac{16}{3}+\frac{9}{c^2}}$	0·3388	3·183
132·5°	20·4	·5424	$21\bar{3}1\ \alpha$	$a/\sqrt{\frac{28}{3}+\frac{1}{c^2}}$	0·3209	3·177
130·0°	21·65	·5669	$10\bar{1}5\ \alpha$	$a/\sqrt{\frac{4}{3}+\frac{25}{c^2}}$	0·3053	3·156
128·7°	22·3	·5794	$20\bar{2}4\ \alpha$	$a/2\sqrt{\frac{4}{3}+\frac{4}{c^2}}$	0·2968	3·154
127·0°	23·15	·5945	$30\bar{3}0\ \alpha$	$a\sqrt{3}/6$	0·2886	3·133
124·9°	24·2	·6127	$0003\ \alpha$	$ac/6$	0·2718	3·190
122·7°	25·3	·6308	$10\bar{1}6\ \alpha$	$a/\sqrt{\frac{4}{3}+\frac{36}{c^2}}$	0·2593	3·207
121·5°	25·9	·6403	$21\bar{3}4\ \alpha$	$a/\sqrt{\frac{28}{3}+\frac{16}{c^2}}$	0·2553	3·188
117·5°	27·9	·6701	$31\bar{4}0\ \alpha$	$a\sqrt{\frac{3}{52}}$	0·2402	3·168
					Mean value of	$a=3\cdot17$Å

sine of the angle of reflection of each line in the spectrum was plotted on a slip of paper, and passed over the graph (Fig. 4) as previously until a position was found at which the lines fitted. In this case it is difficult to decide which are the α and which are the β line reflections. In order to ascertain this, a filter of zirconium was placed in front of the chamber slit, and a small portion of the spectrum again examined. In this way the β lines were almost entirely cut out, leaving only the α lines.

Of the 22 lines in the spectrum, 18 were found to be α lines and four β lines. The value of the axial ratio taken was that obtained from the α lines alone, and was found to be 1·631. The length of the side of the hexagon was calculated for each set of planes in the crystal both for the α and for the β lines. These values are included in Table III., the mean value being 3·17 Å.

As a further test on the structure, we may calculate the number of atoms associated with each elementary cell. The volume of the elementary triangular prism is $\frac{\sqrt{3}}{4}a^3c$, where a is the side of the hexagon and c the axial ratio. Then n, the number of atoms associated with each elementary triangular prism $= \frac{\sqrt{3}}{4} \cdot \frac{a^3c\rho}{m}$, where m is the mass of the atom and ρ the density of the material.

Hence,
$$n = \frac{\sqrt{3}}{4} \times \frac{(3{\cdot}17)^3 \times 1{\cdot}631 \times 1{\cdot}74}{24{\cdot}13 \times 1{\cdot}663} = 0{\cdot}98.$$

There is, therefore, one atom associated with each elementary cell, which conforms to the requirements of a hexagonal lattice of the close packed type.

A point which needs further explanation is the absence of the 0001 line in the observed spectrum curve. This fact was also noted by Hull, but on fuller investigation of a number of single crystals he found the line to be present. In the present instance the region of the spectrum where the line should appear was examined in greater detail, observations being taken at 10′ intervals instead of 30′ intervals. In this case the line was clearly indicated, although its intensity was small.

The above measurements suffice to show the utility of the method. It promises to be a valuable asset to the metallurgist and has an advantage over the powder method in that it allows the specimen, after being polished and etched for microscopical examination, to be used directly on the X-ray spectrometer. In addition, there is in this case no possible distortion of the crystals with consequent increased absorption of the radiation.

We desire to express our indebtedness to the Metallurgy Department of the Laboratory for their assistance in the preparation of the specimens.

DISCUSSION.

Sir **William Bragg** congratulated the Authors on their work, which indicates that the technique of crystal analysis is improving and that its methods are coming into general use. A consideration to be borne in mind is that one of the crystal planes may become unduly accentuated by the polishing process. Further, in the case of aluminium and other metals it is possible for individual crystals to be very large.

Dr. G. D. **West**, after dwelling on the advantages of focussing the X-ray beam by inclining the plane in which the crystals lie, inquired as to the width of the slits employed as the conditions varied, and as to the allowance to be made for variations in the output of the bulb.

Mr. **Preston**, in reply, stated that the Authors had been careful to use crystals less than 1 mm. in size. The chamber slit was kept at 2 mm., and the bulb slit varied from 2 mm. to 4 mm. for different parts of the spectrum. The output of the bulb could be kept constant by regulating the filament current, but for moderate accuracy little regulation was required.

XI. *The Cathode Ray Oscillograph.* *By* A. B. Wood, *D.Sc., F.Inst.P.*

Received November 10, 1922.

ABSTRACT.

The Paper deals with a new form of cathode-ray oscillograph adapted for commercial production and laboratory use. The instrument described is of the low-voltage type, in which a hot cathode is employed as a source of the electron current. This low-voltage type of oscillograph is much more sensitive than the high-voltage cold cathode type designed by M. Dufour. Various methods are described for focusing the cathode-ray stream, and a proposal has been made for an oscillograph with external (*i.e.*, outside the vacuum) photographic film. Experiments have been made to determine the most suitable photographic film or plate. Ordinary gelatine coated roll films or plates are unsuitable, owing to the marked absorption of the cathode-rays by the gelatine. The best results have been obtained with Schumann plates containing a proportion of calcium tungstate. This material phosphoresces with a light rich in ultra-violet, and consequently the secondary luminous effect on the Schumann plate is very great.

Mechanical, electrostatic and electromagnetic methods are described for generating a time-axis on the records. For certain purposes this time axis is sinusoidal, whilst for others it is linear. Numerous records of high-frequency A.C. wave forms and of impulsive electrical phenomena have been obtained, and a few of these are reproduced in the Paper. Brief reference is made in conclusion to the applications of the oscillograph to research and electrical engineering problems, where other well-known forms of oscillograph (Duddell, Einthoven, &c., types) cannot be employed on account of the inertia of the moving element.

I. Historical.

With the development of wireless telegraphy and telephony in which electrical oscillations of very high frequency are in everyday use, the need for a sensitive and efficient high-frequency oscillograph is becoming more acute. In the study of impulsive electrical phenomena also the need for such an oscillograph has long been felt.

Oscillographs at present in use, *e.g.*, the Duddell and the Einthoven types, although excellent at comparatively low frequencies, fail to give reliable records at frequencies much in excess of 1,000 ~ per second, for the true wave-form of the oscillation to be recorded is masked by the natural vibration inherent in the instrument itself. The upper limit of frequency which such an oscillograph can record faithfully is determined by the mechanical inertia and elastic limits of the moving parts and controls. At very high frequencies these oscillographs become very insensitive.

The first serious attempt to design an oscillograph in which the moving element was so light that it could respond to extremely high frequencies is due to Braun. He made use of the facts that a pencil of cathode-rays is easily deflected by magnetic and electrostatic fields, and that the rays produce visible and photographic actions under suitable conditions. It is well known that the mechanical inertia of the cathode rays is quite negligible even at extremely high frequencies (of the order 10^6 per second). The Braun tube has been described in many text-books of electricity, and requires little more than a brief reference here. In principle it consists of a vacuum tube in which a stream of cathode rays is produced by means of a H.T. supply of about 50,000 volts. Some of these cathode rays pass through a fine tube or pinhole in the anode and thence between the poles of a magnet and a pair of electrostatic deflecting plates. Variations of current in the magnet (or of voltage on the deflecting plates) produce corresponding deflections of the pencil of cathode rays, which ultimately falls on a screen of phosphorescent material and is thus rendered visible. The sensitivity of such an oscillograph is constant at *all* frequencies. Attempts to photograph wave-forms of A.C. by means of this oscillograph were unsuccessful until an automatic method was devised whereby the cathode rays were constrained to retraverse exactly the same track across the photographic plate any number of times in succession. In this way records of high frequency wave-forms were obtained by "multiple-exposure" of the plate to the cathode rays—such a photograph requiring about 100 traverses of the rays across the plate to be satisfactory.

Now when very high frequency or impulsive phenomena of short duration are to be studied, it is always a matter of great difficulty to obtain satisfactory results with the multiple-exposure method; in innumerable cases it is quite impossible to arrange for regular periodic repetition of the phenomena to be recorded. In such cases it is essential that a record should be obtained by a single traverse of the cathode rays across the plate.

An oscillograph which achieves this condition has recently been designed and constructed by M. Dufour. It is essentially an improved form of Braun tube, in which the cathode stream is much more intense than previously obtained. As in the case of the Braun tube, a cold cathode is employed and a trace of gas is left in the tube. The rays are produced by applying a p.d. of about 60,000 volts between cathode and anode. With this oscillograph Dufour has obtained beautiful records of frequencies of the order $10^5 \sim$/sec. In this respect it is a great advance on the original Braun tube, but it has one serious drawback, viz., its sensitivity is low. Cathode rays produced under a p.d. of 60,000 volts attain a very high velocity. High velocity cathode rays are more difficult to deflect than low-velocity rays, the deflection being inversely proportional to the voltage of the rays. Hence in Dufour's oscillograph large electrostatic voltages and magnetic fields are required to produce satisfactory deflections on the oscillograph records. On the other hand, it must be remembered that the photographic effect of high-velocity rays is greater than that of rays of low velocity. The experimental difficulties increase therefore as the velocity of the cathode rays diminishes.

In connection with certain experiments a sensitive cathode ray oscillograph has been employed by D. A. Keys, who was experimenting on a method proposed by Sir J. J. Thomson.* For these experiments it was essential that the oscillograph should have considerably greater sensitivity than that used by Dufour, for the voltage changes to be recorded were comparatively small. The increased sensitivity was obtained by using a p.d. of about 5,000 volts only across the tube, whilst the cathode was a tungsten filament heated by means of an insulated battery of accumulators. The use of a hot filament gives a powerful steady supply of electrons for the cathode stream, sufficient to produce the required photographic effects. With this apparatus Keys obtained a number of records of underwater and gaseous explosions, but the apparatus was rather fickle in its behaviour and left much to be desired as regards mechanical construction and adjustability.

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* Sir J. J. Thomson, Engineering, Vol. 107, 1919, pp. 543-544; and D. A. Keys, Phil. Mag., 42, pp. 473-488, Oct. (1921).

II. DEVELOPMENT OF LOW-VOLTAGE TYPE OF OSCILLOGRAPH.

Cathode ray oscillographs may be divided broadly into two classes, according as they employ :—

(*a*) A cold cathode—as in the Braun tube and Dufour oscillograph. These are of the high-voltage type (of the order 10,000 volts), are not electrically sensitive, but are very efficient photographically.

(*b*) A hot cathode—as in the oscillograph used by D. A. Keys. The voltages used in this case to produce cathode rays may vary according to requirements, and may range from 100 volts to 10,000 volts. Such oscillographs are of the low-voltage type, are very sensitive electrically, but are not sensitive photographically.

In what follows we are concerned with (*b*) the low-voltage type of oscillograph employing a hot cathode. The general arrangements of the electrical circuits of such an oscillograph are indicated diagrammatically in Fig. 1. The cathode C consists of a spiral of thin tungsten or lime-coated platinum wire, which is heated

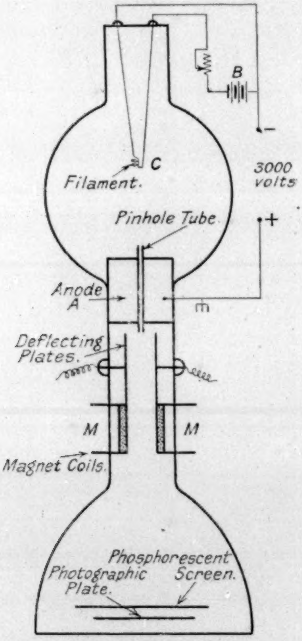
B
−
C
Filament.
3000 volts
+
Pinhole Tube
Anode A
Deflecting Plates.
M
M
Magnet Coils.
Phosphorescent Screen.
Photographic Plate.

FIG. 1.—OSCILLOGRAPH CONNECTIONS.

by means of an insulated battery B. The hot spiral is supported inside a cylindrical sheath (to which one end is connected) which exerts a focusing action on the cathode

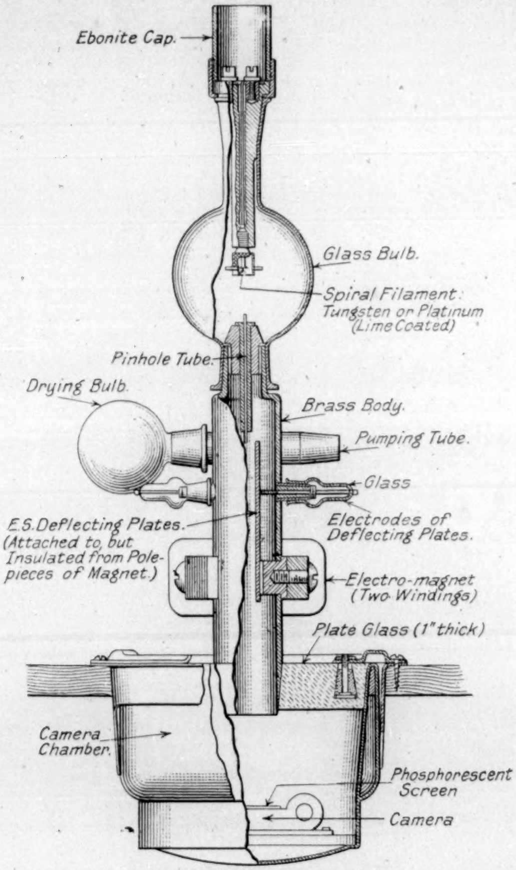
Ebonite Cap.
Glass Bulb.
Spiral Filament: Tungsten or Platinum (Lime Coated)
Pinhole Tube.
Drying Bulb.
Brass Body.
Pumping Tube.
Glass
E.S. Deflecting Plates. (Attached to, but Insulated from Pole-pieces of Magnet.)
Electrodes of Deflecting Plates.
Electro-magnet (Two Windings)
Plate Glass (1" thick)
Camera Chamber.
Phosphorescent Screen
Camera

FIG. 2.—CATHODE-RAY OSCILLOGRAPH.

rays as they travel towards the anode A. The rays are accelerated by means of an applied p.d. of, say, 3,000 volts, the positive end being connected to the anode A and earthed. After passing through the pinhole tube in the anode the cathode rays travel through parallel electrostatic and magnetic fields and ultimately fall on the phosphorescent screen. Removal of this screen exposes a photographic plate to the action of the rays. The lower the voltage at which it is required to generate the cathode stream the nearer together should be the cathode and anode. With rays of lower voltage than 1,000, however, considerable difficulty is experienced in obtaining a sufficiently intense beam to give a photographic effect.

Various methods may be employed in generating the time axis when an oscillograph record is to be made. In practice it is usually found inconvenient to employ actual mechanical movement of the photographic film, the same end being achieved much more conveniently by sweeping the cathode stream across the plate in a known manner either by electrostatic or electromagnetic means.

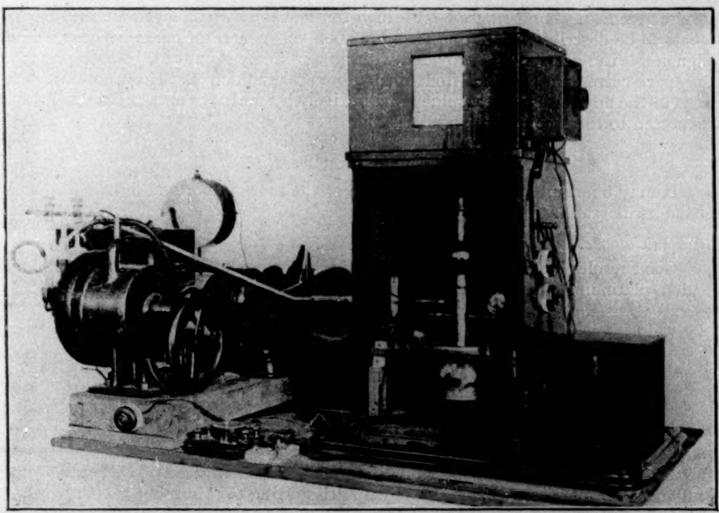

FIG. 3

On account of the extended applications of the above type of low-voltage oscillograph it was considered advisable to design a simple and robust form which might be of general service as a laboratory instrument. Fig. 2 indicates broadly the main features of a design proposed to fulfil such conditions. As far as possible fragile glass-work has been eliminated, the only glass parts liable to fracture being the main bulb carrying the heated cathode drying bulbs and a pair of small side connecting tubes. It will be observed that the tube containing the E.S. deflecting plates is made entirely of brass. The various parts of the apparatus are fitted together and rendered airtight by means of accurately fitting greased conical joints. The oscillograph is thus very easily assembled or taken apart, and spare fittings can be kept in reserve to replace quickly in case of accidental damage to any part of the apparatus. The oscillograph when assembled in this manner is found to be perfectly airtight, an X-ray vacuum being attained after about 10 minutes pumping with the standard Gaede and auxiliary vacuum pumps.

A photograph of the oscillograph mounted in its cabinet is shown in Fig. 3. The filament battery, control rheostat, ammeter, voltmeter and other fittings connected to the cathode and consequently at high potential (possibly up to 5,000 volts) are collected in the upper part of the cabinet, above the cathode ray tube itself, and insulated on an ebonite base. The high-tension supply employed in the experiments to be described is obtained from a 5,000 volt generator made by Evershed & Vignoles. The generator is provided with an automatic " cut-out," which prevents the space-current in the tube from rising above 20 milliamps. The vacuum conditions in the apparatus are examined qualitatively by means of an auxiliary test-bulb connected to the secondary of a spark coil.

III. Details of Design.

It may be of interest to consider some of the more important details of design of the oscillograph.

(*a*) *The Cathode.*

Good results have been attained by using filaments of either tungsten or lime-coated platinum. The latter have in general been found more satisfactory, for they emit a very powerful electron stream at a comparatively low temperature, and there are as a consequence fewer casualties owing to " burning out " filaments. A simple and efficient method of lime-coating the platinum filaments is to dip them in a solution of calcium nitrate,* then heat electrically to convert this into oxide. A filament once coated in this way will serve its purpose efficiently for several weeks constant use without recoating.

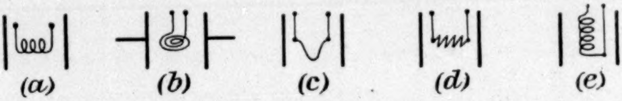

Fig. 4.—Typical Filaments.

The form of the filament is a matter of some considerable interest both from a physical and mathematical standpoint. The ultimate direction in which the cathode rays leave the filament depends primarily of course on the magnetic and electrostatic fields at the surface of the filament. These fields must have a very powerful initial effect, and as a consequence it becomes a matter of some importance to determine the best shape of the filament. The mathematical aspect of the question is under consideration, and it is hoped that we shall soon be able to design a filament which will bring a wide beam of cathode rays to a sharp focus. Such a concentrated pencil of rays would be ideal for oscillographic purposes. For the present, however, the design of the filament is based mainly on experimental evidence. Some of the many forms of filament used are shown diagrammatically in Fig. 4. Of these the flat spiral filament shown in Fig. 4 (*b*) has given the best results. It has been found a definite advantage to employ a cylindrical sheath shrouding the filament as shown.

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* The author has recently been informed that this method is improved by adding a small quantity of the salt of a noble metal to the solution—the oxide layer then sticks more tenaciously to the platinum filament.

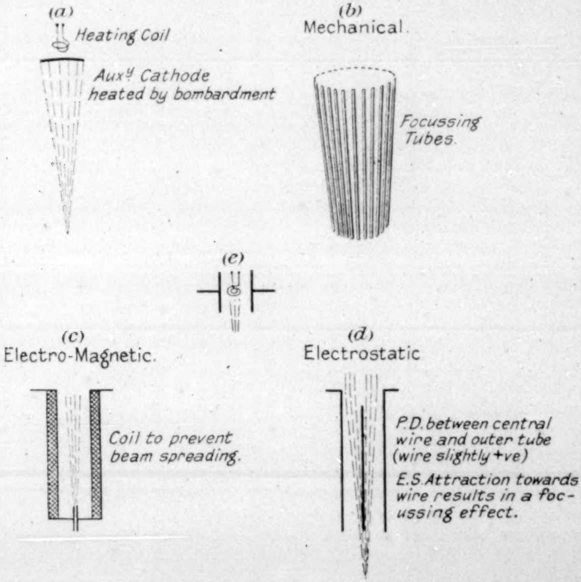
(a)
Heating Coil
Aux^y Cathode heated by bombardment
(b)
Mechanical.
Focussing Tubes.
(e)
(c)
Electro-Magnetic.
Coil to prevent beam spreading.
(d)
Electrostatic
P.D. between central wire and outer tube (wire slightly +ve)
E.S. Attraction towards wire results in a focussing effect.

FIG. 5.—TYPICAL METHODS OF FOCUSING A WIDE BEAM OF CATHODE RAYS.

The flanged sheath (as in the Coolidge tube) has been used, but it is not possible to state that it is definitely better than the unflanged form.

(*b*) *Focusing Devices.*

One of the chief difficulties encountered in the design of a low-voltage oscillograph is that of obtaining a sufficiently intense pencil of rays to affect the photographic plate when moving over it at high speeds. Two methods at once present themselves, (1) by increasing the intensity of the cathode ray stream, and (2) by increasing the sensitiveness of the photographic plate. We shall for the present discuss the first of these methods, leaving the question of photography till later.

The intensity of the cathode stream can be increased up to a certain point by increasing the current through the filament, but an upper limit in this direction is soon reached at a temperature just below the fusing point of the filament. Clearly, if very great advances are to be made, they must be in directions other than this. One alternative is to increase the intensity of the beam by focusing the rays from an extended area of filament. Typical methods of doing this are indicated in Fig. 5. These methods may be broadly divided into :—

(i) *Mechanical*—by means of concave cathodes, or by the use of cones of fine tubes, &c., as in Fig. 5(*a*) and (*b*).

(ii) *Electro-magnetic*—by means of a solenoid carrying a current and surrounding the anode and Fig. 5(*c*). Strictly speaking, this method is " anti-spreading " or " paralleling " rather than actual focusing.

(iii) *Electro-static*—by means of a positively charged central wire through the anode (*see* Fig. 5(*d*)). The wire attracts the negatively-charged rays, causing the beam to converge towards the further tip of the wire. Another method, shown in Fig. 5(*e*), is that employed in Coolidge X-ray tubes, where the filament sheath repels the electron stream towards the axis.

The mechanical method (i) shown in Fig. 5(*b*) whilst giving some evidence of focusing action did not result in any material improvement of the intensity of the cathode ray pencil. Methods (ii) and (iii) are considerably more promising, but as yet the experiments are not completed. The sheathed filament of the flat spiral form, shown in Fig. 4(*b*), has for the present been adopted, as it gives more satisfactory results than any others yet examined.

It is still a moot point how far any form of focusing device is advantageous, for the beam of rays streaming from a hot cathode always appears as a heterogeneous bundle of fine pencils of rays varying between wide limits of intensity. When it is possible to deflect one of the more intense of these pencils into and through the pinhole tube a brilliant small recording spot is obtained which is usually quite satisfactory for high speed oscillograph work. Of course, focusing devices will increase the probability of such an intense fine pencil entering the pinhole tube, but it is not at present certain that the intensity of the recording spot is greatly increased by the focusing action.

Another method of heating the cathode, in this case a concave disc of platinum or tungsten, is similar to that employed in the " pointolite " lamp in which a ball of tungsten is heated by cathode-ray bombardment (*see* Fig. 5(*a*)). As yet, however, no definite experimental data can be given.

(*c*) *Electrostatic Deflecting Plates and Electromagnet.*

In the oscillograph, shown in Fig. 2, the E.S. deflecting plates are mounted parallel to one another and to the axis of the tube, contact being made through side tubes of glass with spring contact fittings. The plates at present in use are 9 cm. long, 2 cm. wide and are situated a little over 1 cm. apart. For greater sensitivity, means are provided for bringing the plates closer together. Great care is taken to ensure that the electrostatic and magnetic fields are parallel, this being indicated when the oscillograph records show that the independent deflections (E.S.) and magnetic are mutually at right angles.

In the present oscillograph the pole-pieces of the electromagnet are silver-soldered to the main tube of the oscillograph, whilst the external case of the magnet consists of laminated iron attached to the pole-pieces as shown in Fig. 2. In a more recent design (*see* Fig. 7) the whole magnet, including pole-pieces, is external to and free from the tube of the oscillograph, and is situated above the E.S. deflecting plates. The magnet is provided with a double-winding the purpose of which will be indicated later.

(*d*) *The Camera.*

The original design of this important part of the oscillograph provided for external electrical operation of the camera by means of suitable electromagnetic fittings on the camera itself. Experience showed, however, that these operations are more easily performed by means of conical plugs fitted through the wall of the camera vessel. The camera itself was at first designed to take either a single glass plate or a roll-film (V.P.K. size). Rotation of one of the plugs opens the door of the camera and exposes the film or plate. Rotation of the other plug in its socket winds the film from one spool to another as in the ordinary Kodak. The advantage of a roll-film is obvious, for 8 to 10 exposures can be made without " opening up " and re-evacuating the apparatus between successive exposures. The chief disadvantage of the film is, however, the moisture which it contains and which it is reluctant to release. If a film is used, it should be previously dried by storing in vacuo with P_2O_5 for a period of about 24 hours before it is required for actual oscillograph work. For another reason also, which will be evident when the question of photographic plates is discussed, the film may prove unsuitable. This is unfortunate, of course, for the great advantage of the film is its adaptability for a large number of exposures whilst occupying a very small space in the vacuum vessel. In the latest design of oscillograph (*see* Fig. 7), the camera accommodates six plates fitted on the faces of a hexagonal block of aluminium. A conical plug fitted on the outer casting of the camera chamber is employed to rotate the hexagon and thus expose any required plate to the action of the cathode rays. When an exposure is to be made, another plug removes the door of the camera, this door serving also as the phosphorescent screen, its upper surface being coated with a layer of phosphorescent material (willemite or calcium tungstate). The camera containing the six plates is in itself a light-tight unit which can be inserted in or removed from the oscillograph in daylight. By means of a hinged cover plate, light is prevented from entering through the viewing window when an exposure is being made. With such an arrangement six oscillograph records can be made before " opening up " the apparatus and admitting air. This is a great advance on the old design shown in Figs. 2 and 3, where it was necessary to re-admit air after each exposure when glass plates were being used. In the new design provision is also made for a second pair of electrostatic deflecting plates, arranged at right angles to the first pair, as an alternative to the use of the electromagnet in certain circumstances. All operations can be carried out in subdued daylight without the use of a dark room except for the purpose of developing exposed plates.*

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* Experiments have been made, with some degree of success, to render plates insensitive to light whilst retaining maximum sensitivity to cathode rays. With such plates, *all* operations, including development, can be carried out in subdued white light.

(*e*) *Photographic Plates. Absorption of Cathode Rays in Gelatine Films.*

Reference has been made above to the possibility of increasing the photographic effect of the cathode rays by increasing the sensitiveness of the plates. Now a plate which is most sensitive to ordinary light is not necessarily most sensitive to the action of cathode rays, for the factors involved in the two cases are of a distinctly different nature. Early experiments indicated that the difference in photographic effect observed when plates varying in speed from 100 to 550 H. and D. when subjected to the same stream of cathode rays with exactly the same exposure, was too small to be of serious importance. There appeared to be a slight advantage in favour of the faster plates, but this advantage was lost in the greater tendency of such plates to fog. Consequently ordinary plates of speed about 150 H. and D. were usually employed.

The results were unsatisfactory, however, until a very powerful means of increasing the sensitivity to cathode rays was discovered, viz., the use of a "phosphorescent" photographic plate in December, 1920.* An ordinary plate was dusted over with a thin layer of calcium tungstate and exposed in the usual manner to the cathode rays in the oscillograph. The result obtained was much blacker than the usual faint record and indicated great possibilities in this method of increasing the photographic effect of the rays. The photographic action of the rays is of course two-fold. In the first place they produce their ordinary photographic action as they penetrate into the film, but more important still they produce a luminous effect in the phosphorescent material. This luminous effect, particularly when calcium tungstate is used, has a powerful photographic action on the film, with which it is in close contact.

A simple calculation, easily confirmed by experiment, shows that slow cathode rays (p.d. 3,000 volts for example) such as those with which we are now dealing, will only penetrate extremely thin films of matter. Thus it was found that the photographic effect of the rays after penetrating a layer of mica 0·001 in. thick was extremely small when compared with the direct effect on the bare photographic plate. It thus seems probable that the rays are not sufficiently penetrating to get deep down into the gelatine of an ordinary plate—the energy being expended on the gelatine as well as on silver granules. The phenomenon is exactly analogous to that of ultra-violet light incident on an ordinary gelatine plate. It therefore seemed advisable to use plates containing a minimum quantity of gelatine,† so that the cathode rays might expend their energy almost entirely on the silver granules. With such a plate, made in addition phosphorescent by means of calcium tungstate (rich in ultra-violet phosphorescence), we should anticipate a greatly increased photographic effect from the cathode rays. A quantity of calcium tungstate was therefore forwarded to Adam Hilger, Ltd., who very kindly supplied a number of ordinary Schumann plates and some specially prepared phosphorescent Schumann plates.

The results obtained with these plates have been very gratifying. Both the ordinary Schumann plate and the phosphorescent variety are greatly superior to the ordinary gelatine plate, yielding records of considerably greater intensity. The certainty of obtaining good records is now greatly increased, and there is less urgent need for obtaining increased intensity of the cathode ray pencil as indicated above. An examination of some of the records obtained with Schumann plates (untreated or phosphorescent) shows that a visible photographic impression is obtained when the velocity of the "spot" on the plate reaches 1,000 metres per second. There is no doubt that even this speed can be exceeded. With an ordinary gelatine plate, however, such a speed of recording would be quite impossible, for the records appear rather faint when the speed of the spot exceeds 50 metres per second. Examples of records with Schumann plates—ordinary and phosphorescent—are reproduced in Fig. 6.

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* This method has also been employed by Levy, West & Baker (*see* Röntgen Soc. Journ., 17, April, 1921), but the author was unaware of this work until a few months ago.

† Schumann plates for the ultra-violet possess this quality.

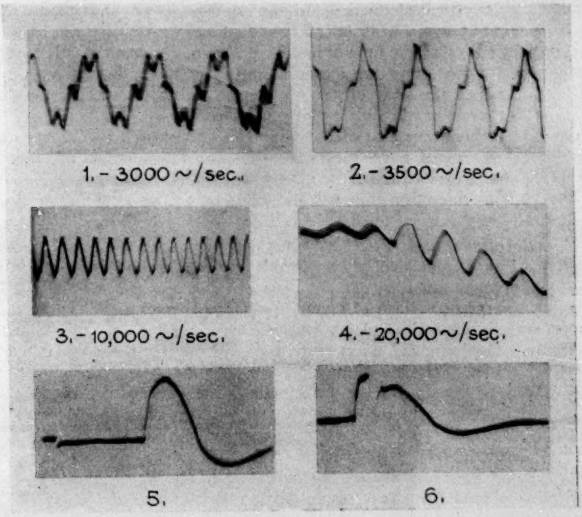
1. - 3000 ~/sec.
2. - 3500 ~/sec.
3. - 10,000 ~/sec.
4. - 20,000 ~/sec.
5.
6.

FIG. 6.

It appears from such observations that the most hopeful line of development of the cathode ray oscillograph for high-speed work lies in the direction of increasing the sensitiveness of the photographic plate, rather than in attempting to obtain further increases in intensity of the cathode ray beam. Investigations are in progress to improve the oscillograph in both these directions.

The importance of the use of Schumann plates in recording by means of low-gress to voltage cathode rays cannot be overestimated.

(*f*) *Generation of Time Axis of Oscillograph Record.*

In the majority of high-frequency electrical problems we are concerned mainly with the variation of current or voltage with time. The generation of a time axis in the cathode ray record is therefore of considerable importance. This can be accomplished by mechanical, electrostatic or electromagnetic means.

(i) *Mechanical Methods.*—This is the most obvious method. Here it is required that the plate or film is moved at a definite rate in a line at right angles to the direction of deflection of the cathode rays by the A.C. supply. For example, if the film were attached to the surface of a drum inside the vacuum, the drum could be driven either directly through a gland, magnetically or as an induction motor. If magnetic or electrical methods of driving are used, however, it is necessary that stray fields should be very weak (insufficient to affect the cathode ray beam) or that they should be cut off at the time of making a record, the drum in this case continuing to rotate by virtue of its own inertia. This latter method is the most promising but is not in actual use since other and simpler methods serve the same purpose equally well—especially when glass plates are used.

There is a possibility also of removing the film *outside* the vacuum and displacing it by the usual mechanical methods, but this method presents serious difficulties owing to the absorption and scattering of the cathode beam in passing from the vacuum through a thin window (mica or similar material) to the photographic film.

(ii) *Electrostatic and Electromagnetic Methods.*—(*a*) A simple method of generating a time-axis particularly applicable in recording impulsive or non-periodic phenomena is that used by D. A. Keys. The method employs sinusoidal alternating current or voltage of known frequency to excite the electro-magnet or the electrostatic deflecting plates—the cathode rays being thereby deflected sinusoidally. The impulsive effect to be recorded deflects the rays at right angles to the track traced in the above manner.

(*b*) The disadvantages of the A.C. method just outlined are obvious and the method is quite useless in recording A.C. wave-forms or other types of periodic oscillations. The following method, suggested by Dr. C. V. Drysdale and used also by M. Dufour, has proved much more satisfactory for this purpose. The method depends for its success on the finite rate of rise of current in an inductive circuit* when a p.d. is applied. In the early stages of rise of current the relation between current and time is practically linear. Hence if such a rising current is passed through the electro-magnet of the oscillograph, the deflection of the spot at any instant will be proportional to current and consequently to time.

In the practical application of this method it is found advantageous to employ an auxiliary winding to the deflecting electro-magnet, this winding carrying a steady current which deflects the spot off the plate in a direction opposite to that produced by the primary winding. The current through the latter is, however, sufficiently powerful to overcome the effects of that in the auxiliary winding and thus produce a much greater deflection of the spot in the opposite direction. Hence, on closing the main magnet circuit the spot is drawn from a position just off one edge of the plate to a position well off the other edge, *i.e.*, the spot trails across the plate from one edge to the other at a definite rate depending on L and R, and leaving a record of the A.C. wave-form which is being studied. Records obtained in this manner are quite satisfactory, as will be seen from the specimens reproduced in Fig. 6.

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* **Analogous considerations apply also in the case of the charging or discharging of a condenser.**

The same result as that obtained in the " inductance " method can, of course, be obtained more directly by varying the current through the electro-magnet or the p.d. across the deflecting plates in a manner proportionate to time. This may be done by means of a simple mechanical method of continuously varying the resistance of a rheostat—it being essential, of course, that no intermittent variation producing sudden " jumps " in current are permitted. A rotating potentiometer has served the purpose extremely well, making it possible to vary the rate of traverse of the spot across the plate over a very wide range of speeds. Using such a device records of impulsive and periodic phenomena have been obtained with a linear time scale.

IV. Experimental Results.

A number of typical records are reproduced in Fig. 6.

Nos. 1 *to* 4 are records of A.C. wave-forms, the source of A.C. being a valve-oscillator. Records of this character have been obtained up to a fundamental frequency of 50,000 ∿/per sec., this upper limit being fixed by the presence of the iron-core in the deflecting electro-magnet. In the new design of oscillograph at present in hand, this defect will be eliminated and it is anticipated that records at much higher frequencies will then be possible. It will be observed that some of the A.C. records are far from sinusoidal. Such records suggest interesting possibilities in the study of oscillations in valve circuits.

Nos. 5 *and* 6 are records of impulsive phenomena, representing the back e.m.f. at the break of current flowing in a coil of large inductance. The time-scale in this case was generated by the "inductance" method, and an automatic timing device, which it is unnecessary to describe here, was employed to ensure that the " break " occurred at the correct instant when the cathode rays were crossing the photographic plate. The time of the first semi-oscillation in this record is about 0·001 second. A potentiometer method of voltage-reduction was employed, the maximum deflection on the record corresponding to a back e.m.f. of about 6,000 volts on the coil. Record No. 6 is of a similar type, showing the result of a defect in the commutator which breaks the current in the main circuit.

V. Conclusion.

New Design of Oscillograph.

From what has been said above it will be realised that a commercial form of low-voltage oscillograph has tremendous possibilities in its applications to research and to electrical engineering problems. The satisfactory development of a sensitive high-frequency oscillograph opens up a wide field of investigation in wireless telegraphy. An attempt is also being made to develop a sealed form of cathode ray oscillograph with external photographic arrangements. For this purpose experiments are being made with various focusing devices, so as to obtain an intense beam of cathode rays which will give an appreciable photographic effect after passage through a " window."

With the knowledge gained in the use of the oscillograph described above a new oscillograph has been designed which it is hoped will remedy the defects of the earlier patterns.

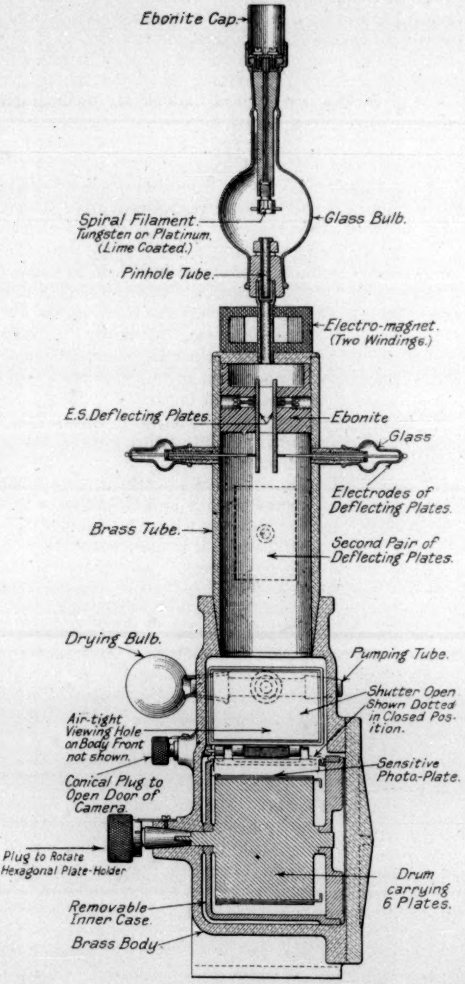
Ebonite Cap.
Spiral Filament.
Tungsten or Platinum.
(Lime Coated.)
Glass Bulb.
Pinhole Tube.
Electro-magnet.
(Two Windings.)
E.S. Deflecting Plates.
Ebonite
Glass
Electrodes of
Deflecting Plates.
Brass Tube.
Second Pair of
Deflecting Plates.
Drying Bulb.
Pumping Tube.
Shutter Open
Shown Dotted
in Closed Position.
Air-tight
Viewing Hole
on Body Front
not shown.
Sensitive
Photo-Plate.
Conical Plug to
Open Door of
Camera.
Plug to Rotate
Hexagonal Plate-Holder
Drum
carrying
6 Plates.
Removable
Inner Case.
Brass Body

FIG. 7.

In conclusion, I should like to take this opportunity of thanking the Admiralty for permission to publish this Paper, Dr. C. V. Drysdale for his constant interest and for many useful suggestions in the design of the oscillograph, and Mr. E. H. Lakey, B.Sc., for his invaluable assistance in obtaining the oscillograph records.

SUMMARY.

A description is given of a new form of Cathode-ray oscillograph of a pattern adapted for commercial production and laboratory use.

Details are given of the principal features of the instrument which is considerably more sensitive than the Dufour oscillograph. The increased sensitiveness is due to the production of the cathode rays at a comparatively low voltage, *viz.*, 3,000 volts, whereas the Dufour instrument employs 60,000 volts.

Various methods are described for focusing the cathode ray stream, and for increasing the sensitiveness of the photographic plates. The best results have been obtained with Schumann plates containing a proportion of calcium tungstate. This material phosphoresces with a light rich in ultra-violet, and consequently the secondary luminous effect on the Schumann plate is very great.

Mechanical, electrostatic and electromagnetic methods are described for generating a time axis on the records. For certain purposes this time-axis is sinusoidal, whilst for others it is linear.

Typical records of high-frequency A.C. wave-forms and of impulsive electrical phenomena are reproduced in illustration.

In conclusion, brief reference is made to the wide applications of the low-voltage type of oscillograph and to a new design of oscillograph.

DISCUSSION.

Mr. MINES inquired whether any measurement had been made of the potential gradient in the tube with a view to determining the distribution of potential along the path of the rays. Was anything known of the current density of the rays, and was there (as in the arc lamp) a limiting value to such current density, involving a maximum intensity for the spot of light beyond which it would be impossible to go? And had the Author tried the focussing effect of charging the cylindrical shield, as at (*d*), Fig. 5, to a positive potential of, say, 100 volts?

Mr. R. A. WATSON WATT (partly communicated): I would join in congratulating Dr. Wood on his very lucid account of a most important and valuable instrument of research. Perhaps the most outstanding feature of his work is the way in which the oscillograph has been converted into a robust engineering structure, which the "glass shy" engineer can approach with more confidence than he feels towards the older forms of blown glass tube, and into an instrument with a much higher expectation of life than these older forms possessed.

In considering cathode ray oscillographs in general, one should not lose sight of the fact that there is a wide field of application for sensitive tubes in which it is not necessary to resort to photography with its attendant difficulties and complications. Single axis working, in which the tube is valuable as a direct reading high-frequency voltmeter, is in itself sufficient justification for such tubes. But Dufour has shown that in wave form work one can obtain visual images of radio frequency wave forms from his high-voltage tubes, while recently, by the courtesy of the International Western Electric Co., I have been able to satisfy myself that a reliable visual image of a transient deflection of 10^{-4} sec. duration can be obtained with the 300-volt beam of their tube. The fluorescent spot had a translational speed of about 5×10^4 cm. per sec. in this experiment. This result, whether due to persistence of vision or to persistence of fluorescence, is much more satisfactory than one had the right, *a priori*, to expect.

On minor matters of detail it might be desirable in a Paper which will take an important place in the chronology of cathode ray oscillographs that the Author should make it clear that he does not assign priority for the introduction of the hot cathode, of the A.C. time base, and of the D.C. time base, to the workers referred to in this Paper. The Author's own familiarity with the history of the subject might lead to the appearance of neglecting the priority of Wehnelt (1906) as to the hot cathode, of Fleming (1913) as to the A.C. time base, and of Dufour (1914) as to the D.C. time base, which he superposed on the A.C. base, so that the final image is based on the plane projection of a cylindrical helix.

I should be grateful if Dr. Wood could say whether any data are available as to a tube working on the 100 volts, which he mentions as a lower limit. A tube of this extreme sensitivity would be most valuable, but I know of no references to a stably operating tube with an accelerating voltage below 300 volts.

Finally, I should like to ask the Author whether he has any experience of Van der Bijl's "positive ion" method of focussing, as used in the Western Electric tube. It appears to be a method of great value, particularly in view of the fact that the commonly used electro-magnetic "paralleling" method reduces the sensitivity of the tube. My experience of low-voltage tubes leads me to conclude that at the present moment the limitation of sensitivity for visual observation lies, not in the mechanism of beam production or concentration, but in the fluorescent materials used for the screen, and that progress in that direction can still be made.

Mr. R. S. WHIPPLE said that the beautiful instrument described would meet a real demand, as oscillographs using a cold cathode are cumbersome and costly, while the one under discussion should be within the means of most laboratories. The method of phosphorescent sensitisation opens up great possibilities. The Paper is written in such a way as to be unusually readable.

AUTHOR'S reply (communicated): In reply to Mr. MINES: No measurements have been made to determine the potential gradient along the path of the rays in the bulb. There is, of course, no further change of potential after the rays pass through the pinhole tube, and enter the deflecting chamber. The *total* current between the hot cathode and the anode is usually of the order of 1 milliampere. No direct measurements have been made of the current density of the cathode stream after passing through the pinhole tube. This could easily be done of course by a capacity-electrometer method. There is no evidence to indicate that a maximum intensity of the spot has yet been reached—the brilliancy apparently increases up to the point of breakdown of the bulb or of fusion of the filament.

With regard to focusing by means of a cylindrical shield and central wire as shown in Fig. *5d*, it is proposed to use a potentiometer device which will give a continuous voltage variation over any required range, thus providing a simple means of adjusting the position of focus of the rays.

In reply to Mr. WATSON WATT: The Wehnelt hot lime cathode is now so well known that I consider it unnecessary to assign priority to the inventor. Lime and thoria-coated filaments of platinum and tungsten are now in everyday use in thermionic valve manufacture.

I was not previously aware that the A.C. time base was first proposed by Fleming. In any case the D.C. time base is preferable whether it is generated by the inductance method or by the rotating potentiometer.

With suitable vacuum conditions (rather a soft bulb), a beam, sufficiently powerful for visual observations, can be obtained with a p.d. as low as 100 volts; but, as I stated in the Paper, considerable difficulty is experienced in obtaining a sufficiently intense beam for photographic purposes if the generating voltage falls below 1,000. If it is essential for certain purposes to use low voltage rays then the photographic action might be increased by further acceleration of the rays subsequent to deflection. For ordinary purposes, however, where a good photographic record is to be obtained, 3,000 volts generating potential has been found quite satisfactory. Other methods of increasing sensitivity are at present under consideration; these, however, will form the subject of a future Paper.

It should be remembered that cathode rays generated at low voltages, *e.g.*, 300 volts, are so much reduced in energy that they cannot penetrate sufficiently deep into the phosphorescent screen or the photographic film, hence the observed effects with such rays are always very much less than with rays at higher voltages. Since the square of the velocity of the rays is proportional to the applied voltage it might be assumed that the energy available to produce photographic or phosphorescent action is proportional to the latter. That is, the photographic effects of rays generated at 30,000, 3,000 and 300 volts would be in the ratio 100, 10 and 1 respectively.

It would seem on these grounds that improved visual or photographic effects can best be obtained by using screens or films of less dense material than such as is at present employed.

I am not familiar with Van der Bijl's "positive ion" method and should be grateful to Mr. Watson-Watt for information.*

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* *See* p. 127.

DEMONSTRATION of Some Applications of the Gyroscope. By M. PAUL SCHILOWSKY, *Chairman of the Gyroscopic Society, Petrograd.*

M. SCHILOWSKY said that the principles of the gyroscope are insufficiently taught in technical colleges, with the result that the subject has received very little attention from the engineering profession. Engineers are in consequence unduly sceptical as to the applicability of such principles to practical problems, although a staff of some 50 Russian experts under the speaker's chairmanship had evolved designs for the construction of a perfectly safe and practicable monorail system for a suburb of Petrograd. The models he was exhibiting were not to scale. In practice the gyrostatic apparatus would form from 3 per cent. to 5 per cent. of the load of a ship, and from 5 per cent. to 10 per cent. of the load of a monorail carriage. The underlying principle of his inventions is as follows: To stabilise a system in unstable equilibrium, a reaction must be set up between the system and the gyrostat of such a character as to *help* the precession of the gyrostat during the return of the system to normal. To check the oscillations of a stable system, the reaction must be such as to *oppose* such precession. The gyrostat must, of course, be power-driven to neutralise friction. The exhibits included the following:—

(*a*) A collection of apparatus for teaching purposes, comprising *inter alia* models illustrating the precession of the earth, a method of optically projecting an image of a spinning top, and small monorail models. The above-mentioned principle was illustrated by means of a top which, when spinning on a sharp point, exhibited precession. When, however, the point was furnished with a spherical knob, the top "slept" until its angular momentum was exhausted by friction, when it abruptly collapsed.

(*b*) Model of a rocking ship. The gyrostatic flywheel is mounted with its axle vertical in a frame, which can both rock about and slide along an axis transverse to the ship. When the ship rolls to starboard the axle of the wheel tilts in precession in a fore-and-aft plane, and at the same time the gyrostat slides bodily sideways under gravity, its frame engaging a rack which holds it with the axle in the tilted position. When the ship begins to roll back, the axle tries to reverse its fore-and-aft tilt, but is prevented by the engagement of the rack, which is spring-controlled. Thus precession is opposed during the return of the ship to normal and (the equilibrium being stable) rolling is checked by the resulting reaction.

(*c*) Model of an aeroplane carrying a gyrostat. The problem of combining automatic stability with mobility while avoiding dangerous stresses was discussed.

(*d*) Models of monorail gyrostatic apparatus. In the most recent design the flywheel is mounted with its axle vertical in a frame which can tilt in a fore-and-aft plane, and also slide sideways under gravity. The frame is surmounted by a pinion coaxial with and geared down from the flywheel; and the pinion lies between, but normally clear of, two parallel fixed racks mounted on the carriage, and having their lengths in a fore-and-aft direction. When the carriage tilts to one side, the axle of the flywheel tilts forward or backward in precession, and at the same time the gyrostat slides bodily sideways, so that the pinion engages one or other of the two fixed racks. The pinion then climbs back along the rack to its normal position—*i.e.*, in the same direction as the precession which it would exhibit during the righting of the carriage. Thus such precession is aided and (the equilibrium being unstable) the resulting reaction restores the carriage to the upright position.

(*e*) Angular-velocity indicator for aeroplanes. Angular velocity of the aeroplane about a vertical axis causes a tendency to precess in a gyroscope rotating about an horizontal axis. This tendency is balanced by a gravity control, and the angle moved through in attaining a balance is indicated by a pointer, affording a measure of the required angular velocity.

DISCUSSION.

Sir **GEORGE GREENHILL** expressed the indebtedness of the meeting to M. Schilowsky for his brilliant experiments and address.

Prof. **C. VERNON BOYS** expressed his pleasure at seeing a Russian scientist alive and well. He hoped M. Schilowsky would take back to his confrères in Petrograd a message of goodwill and sympathy. He admired the lecture-models, especially those illustrating the earth's precession. A monorail carriage for passengers seemed at first sight impracticable, but the designs which had been actually worked out must be accepted as evidence to the contrary. Such a system would effect great economies, and it would much improve the comfort of travelling by removing the constraint of a double rail, which is responsible for the shocks experienced in an ordinary railway carriage. In the long coach with a four-wheel bogey at each end, of which a model was shown, pitching as well as lateral movements should be eliminated. An example of the opposite extreme is afforded by the so-called mono-railway between Listowel and Ballybunion, which is monorail only in the Irish sense since it has five rails designed to give a maximum of constraint. The result is that the noise and shaking are excessive, a speed exceeding 15 miles an hour would probably destroy the track, and the design is such that the driver cannot cross from one side of the engine to the other without stopping the train. The designs of M. Schilowsky, on the other hand, aim at a minimum of constraint, with correspondingly superior results. The problem of replacing existing systems by the monorail, however, would involve questions of finance, and of vested interests which would defy any calculus known to science. One difficulty occurred to him as serious. It was clear that the carriage would run on a straight or uniformly curved track; but what would happen in the case of a track whose curvature was changing?

Sir **JAMES HENDERSON** expressed admiration for the experiments, and the skill and ingenuity of the practical designs. He fully agreed as to the necessity for educational propaganda, and had, in fact, introduced a course on the subject into the Royal Naval College some 17 years ago. The apparent fact that a mono-railway had been constructed in Petrograd was one of much interest.

M. **SCHILOWSKY**, in reply, said that no difficulty was experienced in negotiating short curves. (This fact was illustrated by means of a small model running on a curved track, and also on a turntable.) Long curves presented certain difficulties, which could not be discussed fully in the time available. In principle such difficulties are met by loading the instability of the system. Unfortunately the impression that the Russian Government had built a railway was imperfectly true. Some 10 kilometres of track had been constructed, but the scheme had got no further.

DEMONSTRATION of a Low Voltage Cathode Ray Oscillograph. By Mr. R. WEBB, *International Western Electric Company.*

THIS instrument, which is designed to work at 300 volts, is intended for manufacture on a commercial basis. The cathode consists of a hot platinum filament coated with certain oxides, and formed into a circle coaxial with the path of the rays. It is protected from bombardment by positive rays, which would disintegrate it, by a screen in which is cut a circular hole slightly less in diameter than the filament, and has a life of about 200 hours. The anode is a platinum tube through which the rays pass. The deflecting fields are electrostatic and are provided by two pairs of plates at right angles, but electro-magnetic deflections may be produced with the help of a pair of small coils fitted outside the tube. The bulb is in the form of a conical flask, the cathode being at the narrow end, so that the rays impinge on the flat bottom, which is coated inside with fluorescent matter, and the luminous trace of the rays can be seen from outside through the bottom of the flask. The spot can be focused by adjusting the filament current. This is brought about with the help of the gas in the tube which is ionised by the rays.* The apparatus was connected up† to show the characteristic of an oscillation-generating valve, so that the potential difference between one pair of plates was proportional to the grid potential, and that between the other pair to the plate current. The resulting trace consisted of a single curved line terminating in a loop of peculiar form at one end.

DISCUSSION.

From Mr. R. A. WATSON WATT (communicated): The method of focusing suggested by Van der Bijl, and applied by J. B. Johnson, in the very beautiful tube demonstrated at this meeting, employs positive ions derived from the residual gas, actually argon at about 0·005 mm. mercury pressure, to neutralise the spreading of the electron beam due to mutual repulsion and to initial divergence. The impact of the electrons in the beam on molecules in its path produces ionisation, and the relatively low mobility of the heavy positive ions results in a concentration of positive electricity along the path of the beam. The space charge of random electrons outside the beam aids the production of a radial field opposing divergence. The number of positive ions along the path of the beam depends on the electron current, which, under working conditions, is limited by the cathode temperature. The strength of the radial field, and consequently the focal distance of the beam, can thus be controlled by the rheostat governing the cathode heating current. Experimentally Johnson finds that a radial field of the order of 3 volts per centimetre is required in the type of tube which has been standardised for production. In practice one finds that a change of 5 per cent. in filament current reduces the diameter of the fluorescent spot (at a screen distance of 20 cm.) from about 5 mm. to less than 1 mm., so that the focal control is very sharp and sensitive. The presence of so many positive ions would, in the case of the older forms of tube, have led to very short tube life by destructive bombardment of the filament. Johnson minimises this trouble by making the free volume of the inter-electrode space less than 1 cubic cm., and by shaping the filament so that it is out of the "line of fire" of ions shooting through the canalising anode. A filament life of 200 hours is thus obtained. Reference may be made to J. B. Johnson's Paper on "A Low Voltage Cathode Ray Oscillograph" in the Journal of the Optical Society of America and Review of Scientific Instruments, Vol. 6, pp. 701-12, Sept., 1922.

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* J. B. Johnson, Journal of the Optical Society of America and Review of Sci. Instruments. September (1922).

† E. V. Appleton and B. van der Pol, Phil. Mag., August (1921).

XII. *A Supposed Relationship between Sunspot Frequency and the Potential Gradient of Atmospheric Electricity.* *By* C. Chree, *Sc.D., LL.D., F.R.S.*

Received November 9, 1922.

ABSTRACT.

In a recent Paper in *Terrestrial Magnetism*, Dr. L. A. Bauer draws the conclusion that both the range of the diurnal inequality of atmospheric electricity potential gradient and the mean value of the element for the year increase and diminish with sunspot frequency. This conclusion was based on observational data from the Ebro Observatory, Tortosa, Spain, between 1910 and 1920.

The present Paper investigates the subject more mathematically, employing Kew electrical data from two periods of years, in addition to the Ebro data utilised by Dr. Bauer. Magnetic data from Kew Observatory for the same periods are similarly treated, so as to have parallel results from an element for which the sunspot relationship is generally admitted.

The results obtained are on the whole not incompatible with Dr. Bauer's conclusion, but they indicate that if a relationship of the kind supposed exists, the sunspot influence must be very much less in the case of atmospheric electricity than in that of terrestrial magnetism.

In a recent number of *Terrestrial Magnetism*[*] Dr. L. A. Bauer announces the following conclusion :—

"*f.* The potential gradients of earth currents and of atmospheric electricity apparently vary during the sunspot cycle . . . the latter increasing with increased sunspot activity. The diurnal ranges of the potential gradients of earth currents, as well as of atmospheric electricity, just as is the case for the diurnal variation of terrestrial magnetism, increase with increased sunspot activity " (*l.c.*, p. 30).

The phenomena in terrestrial magnetism to which Dr. Bauer refers are generally accepted as holding all over the earth. We should thus naturally suppose his conclusions respecting atmospheric electric potential to be equally general. But the potential gradient results he dealt with were from only one observatory (Ebro, Tortosa, Spain), and he may have intended his conclusions to be limited to that one station. It is desirable in any case to see what happens at other stations, because it would be a very important step indeed if a general relationship of the kind indicated could be established, at all as decided as that between sunspot frequency and terrestrial magnetism. This has led me to see what further light can be thrown on the question by the Kew Observatory records of atmospheric electricity potential gradient.

A Kelvin water-dropping electrograph has been in action at Kew Observatory with little interruption since 1862. To translate the measurements of the photographic records into absolute measure, it is necessary to take absolute observations in an open spot, and to compare the voltages so measured with the corresponding curve readings. A system of absolute observations was not introduced at Kew until 1898. Since then it has been in regular operation. But there was an improvement in 1910, both in the apparatus and in the site of the absolute observations.

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[*] Vol. 27, pp. 1-30.

These improvements showed that the previous measurements gave too low a conversion factor, consequently the previously published voltages were too low. A conclusion was reached as to the factor by which the older figures should be multiplied to make them comparable with those based on the new apparatus. There had been no change in either apparatus or methods from 1898 to the end of 1909. So far as relative values are concerned, the data from these years are as satisfactory as the more recent data.

It appeared desirable to consider alongside the electrical data magnetic data for the same periods. The magnetic results will show what happens in a case where a relation with sunspot frequency is generally admitted. It also seemed desirable to make certain that the solar influence was not abnormally high or low during the periods under review.

The employment of magnetic data partly determined the periods selected. In 1910 Kew Observatory came under the Meteorological Office, and some changes were then introduced. Up to the end of 1910 the non-cyclic changes were not eliminated from the published diurnal inequalities of the magnetic elements, and the figures went only to 1γ in H (horizontal force) and to 0·1′ in D (declination). From 1911 onwards non-cyclic changes were eliminated, and the inequality figures went to 0·1γ in H and 0′·01 in D. This led to the adoption of 1911 to 1921 for one period. This is almost identical with the period 1910 to 1920, on which Dr. Bauer's conclusions rest. The difference between the mean sunspot frequencies for the two periods is only 0·5.

Dr. Bauer did not employ the actual range of the regular diurnal inequality, but a quantity c_r given by

$$c_r^2 = c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2,$$

where c_1, c_2, &c., are the amplitudes of the Fourier waves of periods 24, 12, &c., hours.

The calculation of Fourier coefficients for a number of years is troublesome. There were, however, available values of c_1 and c_2 derived from the mean diurnal inequality of potential gradient at Kew Observatory for each year from 1898 to 1912.

This suggested the period 1898 to 1912. An objection might, however, have been raised to the last three years of this period owing to the change of apparatus mentioned above; accordingly the period was confined to the twelve years 1898 to 1909.

To obtain values of c_r exactly equivalent to Dr. Bauer's would have entailed the calculation of the shorter period Fourier waves. The prospective labour was only one of the reasons for not doing this. If c_6 represents, as follows from a remark of Dr. Bauer's, a 4-hour wave, c_5 must answer to a period of 24/5 hours. As this is not an exact multiple of an hour, and the Kew curves had been measured at 1-hour intervals, the calculation would have presented difficulties. When Fourier waves are calculated from individual years' data at Kew Observatory, there are irregular variations in the amplitudes and phase angles from year to year even in the 8-hour and 6-hour waves, which suggests that accident plays rather a considerable part. This irregularity can be recognised in Dr. Bauer's Ebro results, especially those connected with c_3, c_5 and c_6. The contribution to the annual values of a c_r of Dr. Bauer's type from the shorter period waves at Kew would undoubtedly have possessed a considerable accidental element. Lastly, the contribution from these smaller terms, relatively considered, would have been trifling. Values of c_1, c_2, c_3, c_4 calculated at Kew for a group of years were as follows, in volts/metre :—

$$c_1=23{\cdot}2,\ c_2=48{\cdot}3,\ c_3=8{\cdot}0,\ c_4=6{\cdot}1.$$

Thence we get $(c_3^2+c_4^2)/(c_1^2+c_2^2)=1/28$ approximately.

The value of $(c_5^2+c_6^2)/(c_1^2+c_2^2)$ would naturally be considerably less, and at Kew $c_3^2+c_4^2+c_5^2+c_6^2$ would probably represent only about 5 per cent. of c_r^2 as defined by Dr. Bauer. Mathematical considerations alone suffice to show that if we take two c_r's, one as defined by Dr. Bauer, the other defined as the equivalent of $c_1^2+c_2^2$, neither could show a pronounced solar influence without the other also doing so. This is supported by physical considerations. While the phase angles given by Dr. Bauer for the less important waves, especially φ_3, φ_5 and φ_6 show considerable fluctuations, these are irregular and not suggestive of any decided solar influence. On the other hand, the fluctuations of the phase angles of the principal waves are small. The type of the diurnal inequality would seem to be nearly constant. If the type were quite constant, c_r, whether as defined by Dr. Bauer or as the equivalent of $(c_1^2+c_2^2)^{\frac{1}{2}}$, would bear a constant ratio to the range of the inequality, and the precise value of the constant would be immaterial for the present purpose. Accidental features would no doubt present themselves differently in the annual values of the two forms of c_r and in those of the inequality range, but it seems reasonably certain that any decisive solar influence would manifest itself to a similar extent in the three quantities. It was accordingly decided to take as one of the electrical quantities for the period 1898 to 1909, a c_r defined by

$$c_r^2=c_1^2+c_2^2.$$

The data employed in the investigation are given in Tables I. and II. All the data are given as differences + or − from the mean value of the quantity for the period considered. By comparing the signs and sizes of the corresponding sunspot and magnetic or electrical data in the same column, even the unmathematical reader can form a judgment as to the existence or non-existence of a close connection. The letter S is attached to sunspot data (Wolfer's frequencies), and the letters M and E to magnetic and electrical data respectively. The suffixes 1 and 2 serve to distinguish the data in Table I. from those in Table II. The last column gives in each case the mean value of the quantity for the period considered. S_1, S_2 and S'_2 refer respectively to the periods 1898-1909 (mean sunspot frequency 32·8), 1910-20 (mean frequency 39·0) and 1911-21 (mean frequency 39·5). M_1 and M_2 relate to the range R_h of the mean diurnal inequality for the year of horizontal force at Kew, while M'_1 and M'_2 relate to the range R_I of declination The noncyclic changes were duly allowed for in all cases where this had not already been done. The magnetic data were derived from the international quiet days, five a month. It is known that the sunspot influence on Terrestrial Magnetism is shown by all days, quiet or disturbed, though not necessarily to the same extent.

E_1 refers to the $c_r \equiv (c_1^2+c_2^2)^{\frac{1}{2}}$ at Kew calculated from the mean diurnal inequality of the year, the mean value of c_r from 1898 to 1909 being 54·3 volts per metre. E'_1 refers to the mean value of the potential gradient P for the year at Kew, on a scale supposed identical with that used in later years, but dependent on a different apparatus.

TABLE I.—*Sunspot, Magnetic and Electrical Data.*

	1898	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	Mean Value
S_1	−6·1	−20·7	−23·3	−30·1	−27·8	−8·4	+9·2	+30·7	+21·0	+29·2	+15·7	+11·1	32·8
M_1	−3	−4	−4	−5	−5	−3	−1	+7	+4	+6	+4	−1	25γ
M'_1	−0·1	−0·3	−0·5	−0·8	−1·0	+0·1	−0·1	+1·1	−0·1	+0·5	+0·9	0·0	7′·0
E_1	+4·1	+8·7	−2·1	−10·4	−9·2	+0·3	−5·6	+8·6	+5·3	−1·3	−2·3	+4·0	$54{\cdot}3v/m$
E'_1	+4	+38	−35	−6	−27	+5	+15	+15	−6	+7	−21	+9	$304v/m$

TABLE II.—*Sunspot, Magnetic and Electrical Data.*

	1910	1911	1912	1913	1914	1915	1916	1917	1918	1919	1920	1921	Mean Value
S_2	−20·4	−33·3	−35·4	−37·6	−29·4	+8·4	+16·4	+64·9	+41·6	+24·6	−0·3	...	39·0
E_2	−2·8	+0·1	−3·7	−1·0	−1·0	+1·6	−1·1	+4·9	+1·8	−1·2	+2·2	...	$25{\cdot}1v/m$
S'_2	...	−33·8	−35·9	−38·1	−29·9	+7·9	+15·9	+64·4	+41·1	+24·1	−0·8	−14·8	39·5
M_2	...	−6·0	−5·0	−7·1	−3·0	−0·4	+4·8	+8·8	+4·9	+2·8	+3·1	−2·6	$25{\cdot}2\gamma$
M'_2	...	−1·81	−1·29	−0·78	−1·52	−0·35	+1·08	+2·53	+1·58	+0·87	+0·26	−0·58	7′·65
E'_2	...	−29	−30	+5	+15	+24	+37	+24	+16	+1	−15	−49	$330v/m$
E''_2	...	+6	+1	+12	+21	+25	+3	+6	−9	−24	−26	−16	$148v/m$

E_2 refers to the value of c_r for the Ebro Observatory, as given by Dr. Bauer. The mean value accepted, however, 25·1 volts per metre, differs slightly from that assigned by Dr. Bauer, whose value answered apparently to the arithmetic mean values for c_1, c_2, &c., and was not the arithmetic mean of the values of c_r itself. Dr. Bauer had not made any exact numerical evaluation of the sunspot influence, and it seemed desirable to have numerical results for comparison with those for Kew.

E'_2 represents the mean annual value P of potential gradient at Kew, corresponding exactly to E'_1, except that it refers to a later epoch. E''_2 represents the range R_p in the mean diurnal inequality of potential gradient at Kew, and is thus a quantity of the same species as E_1 and E_2.

The data were all treated strictly mathematically. The values of b in the "regression equation" $y=bx$—where x and y are corresponding departures of the independent and dependent variables X and Y from their mean values—were calculated in the usual way from the equation

$$b=\Sigma xy\div\Sigma x^2,$$

and the "correlation coefficient" r was similarly calculated from

$$r=\Sigma xy\div(\Sigma x^2\,.\,\Sigma y^2)^{\frac{1}{2}}.$$

I have preferred to exhibit the results in formulæ of Wolf's type

$$\text{Quantity}=a+bS,$$

where S is the sunspot frequency of the year.

Take, for example, the case of M_2 (*i.e.*, the horizontal force inequality ranges in Table II.). The original form of the regression equation

$$R_h=25{\cdot}2+0{\cdot}144\ (S-39{\cdot}5)$$

reduces at once to

$$R_h=19{\cdot}5+0{\cdot}144S.$$

In this case $a\equiv19{\cdot}5\gamma$ is the range calculated for zero spot frequency. $100b\equiv14{\cdot}4\gamma$ represents the increase in the range associated with a sunspot frequency of 100, and

100 b/a is the ratio borne by this increase to the range found for the ideal year of no spots. The frequency usually falls short of 100 for the sunspot maximum year, and hardly ever falls to zero for the sunspot minimum year. Thus, if 100 b/a is small compared with unity, the sunspot influence is trifling.

Sunspot influence may, of course, be small and yet real. The criterion as to the reality of the correlation is r. If the correlation is close, r should approach unity. A small value of r implies that there is little if any real correlation between the two quantities compared.

The results of the calculations are a follows, magnetic data for Kew for the earlier period 1890-1900 being included for purposes of comparison :—

Quantity.	Period.	Formula.	100 b/a	r
M_1	1898—1909	$R_h = 19 + 0{\cdot}188\ S$	0·99	0·94
M'_1	1898—1909	$R_d = 6{\cdot}2 + 0{\cdot}0237\ S$	0·38	0·84
M_2	1911—1921	$R_h = 19{\cdot}5 + 0{\cdot}144\ S$	0·74	0·95
M'_2	1911—1921	$R_d = 6{\cdot}12 + 0{\cdot}0387\ S$	0·63	0·96
	1890—1900	$R_h = 18{\cdot}1 + 0{\cdot}194\ S$	1·07	...
	1890—1900	$R_d = 6{\cdot}10 + 0{\cdot}0433\ S$	0·71	...
E_1	1898—1909	$c_r = 50{\cdot}1 + 0{\cdot}129\ S$	0·26	0·46
E'_1	1898—1909	$P = 297 + 0{\cdot}225\ S$	0·08	0·24
E_2	1910—1920	$c_r = 23{\cdot}1 + 0{\cdot}051\ S$	0·22	0·72
E'_2	1911—1921	$P = 313 + 0{\cdot}440\ S$	0·14	0·55
E''_2	1911—1921	$R_h = 148 - 0{\cdot}118\ S$	−0·08	0·24

Taking the magnetic results first, we see that the different periods supply very nearly the same values for a, the range answering to zero sunspots, but differ rather notably in their values for b. When a formula of the type $R = a + bS$ is applied to the individual 12 months of the year, for a period of years, b/a is found to possess a large annual variation. In both declination and horizontal force it is higher for winter than summer months, but to an extent varying with the element. Two years having the same mean sunspot frequency may have the highest frequencies at different seasons. Thus a sensible variation in the values of b/a derived as above from different periods is not surprising. But the value of b/a deduced in the case of M'_1 is certainly on the low side, and the same is true of the corresponding value of r. This seems due in part to the inequalities having been taken out only to $0'{\cdot}1$. It so happened that the mean value of R_d for the period was almost equally distant from $6'{\cdot}9$ and $7'{\cdot}0$. The negative values appearing for M'_1 in Table I. for the years 1904 and 1906 are attributable to $7'{\cdot}0$ being taken instead of $6'{\cdot}9$. The sunspot maximum in the period 1898 to 1909 was also poorly developed, and was of a somewhat exceptional character, there being two nearly equal frequencies in 1905 and 1907, with a considerably lower value intercalated. The magnetic diurnal inequalities for some reason were poorly developed in 1906 and also in 1904.

While the values of b/a for the M quantities in Tables I. and II. may be on the whole a little below average, they are not outstandingly so, and the mean value of r for the four exceeds 0·92. There is thus no reason to anticipate a specially poor manifestation of the sunspot influence in the case of the electrical quantities, if a substantial real influence exists.

The resulting formulæ for E_1 and E_2 support one another, the values obtained for b/a being closely alike. Also the values of r, though markedly lower than those found for the M elements, are at least not very small. But E''_2 is, as explained above, a quantity of exactly the same type as E_1 and E_2, and applies to a period of years in which sunspot maximum was specially well developed, and in its case not only are b/a and r low, but the former is even negative.

In the case of E'_1 and E'_2, both representing the mean annual value of potential gradient, the results are again consistent with a positive sunspot influence; but the values of b/a are both very small, and the values of r are on the low side, especially that for E'_1.

The conclusion drawn as to the E elements will naturally depend on the temperament of the judge. The balance of the results derived from the Kew data is admittedly on the side of a solar influence such as Dr. Bauer describes. But the results are insufficient to prove that the influence exists, while they certainly do prove that it must, if existent, be of a very different order from the influence seen in the case of the magnetic elements. If, for example, we take the Kew case E_1, which supplied the largest value of b/a, and compare its yearly entries in Table I. with the corresponding entries for S_1, the lack of parallelism is at once apparent. The E_1 value in 1904 is not merely negative but numerically substantial, and the absolutely largest positive value occurs in 1899, when sunspot frequency was much below its average.

The E case most favourable to Dr. Bauer's conclusion is that of the Ebro, E_2; for, while the value of b/a is a trifle smaller than in the corresponding case, E_1, at Kew, the value of r is considerably bigger. But it is the divergence of r from unity that has to be considered, and that divergence in the case of E_2 is fully five times as large as it is in the case of the Kew magnetic elements for what is practically the same period. There are other not very re-assuring features in the case of E_2. The values of b/a and r are much influenced by the outstandingly high value of c_r in the year of sunspot maximum. The value of xy contributed by that one year represents fully half the net value of $\Sigma\,(xy)$ for the eleven years. The contribution from the sunspot maximum year, if a real correlation exists, ought of course to be considerably the largest, but its pre-eminence is relatively much greater in the case of E_2 than in the case of any Kew magnetic element. One would like to feel quite sure that there is no reasonable explanation of a more local nature than sunspots for the phenomenon. This is all the more so because the range of the diurnal inequality of potential gradient in 1917 was not outstanding at Kew, while at Eskdalemuir it was less for the quieter days than in any other year from 1915 to 1918.

The irregularities visible when we compare the sequence of values of E_2 and S_2 in Table II. are similar to those observed in the Kew E data. There are substantial deficiencies in E_2 in 1916 and 1919, years of considerable excess in S_2. On the other hand, the second largest excess in E_2 occurs in 1920 when S_2 was below the mean, and there is a slight excess of E_2 in 1911 when the deficiency in S_2 is very large.

A few general remarks seem called for.

There seems to be at least a general parallelism between simultaneous magnetic phenomena all over the globe. A year that is characterised by great disturbance or large diurnal variations at one station is, so far as is known, similarly characterised everywhere. There is thus nothing surprising in a *general* relation between sunspot frequency and the amplitude of the diurnal inequality of a magnetic element.

But, so far as I am aware, no such general similarity has been recognised between simultaneous electrical phenomena at different stations. Phenomena at Kew and Eskdalemuir, for instance, show no obvious parallelism. The electrical phenomena at a particular station are unquestionably much dependent on the weather and other purely local conditions. A year may be very dry at one station and very wet at another. Thus to establish satisfactorily any *general* relation between sunspot frequency and atmospheric electricity it would be necessary to have simultaneous data from a number of stations in different parts of the world.

Again, it has been found, at least at Kew Observatory, that the absolute value of potential gradient is considerably dependent on the visibility, and so purity, of the atmosphere. Various authorities have described widespread contamination of the atmosphere by dust arising from outstanding volcanic eruptions. There may, for all we know, be a correspondingly widespread effect on the potential gradient. A real solar influence corresponding to a value as small as 0·2 in 100 b/a might well be swamped by such an influence, or a fictitious effect of this size might be introduced. It thus seems very desirable that electrical data from several sunspot cycles should be contrasted.

There are special difficulties in the way of securing uniformity in electric potential data. The voltages actually recorded are dependent on instrumental conditions which it is hardly possible to keep absolutely constant. To secure really good results, it is necessary to have accurate and not infrequent scale value determinations, and also to determine with precision at not too long intervals the factor for reduction to the infinite plane. Instrumental troubles, especially inferior insulation, call for incessant care. The difficulties of securing really comparable data for a period of years are, in short, much more serious than in the case of terrestrial magnetism when a good magnetograph is available.

There is also a difficulty in the rapid large oscillations not infrequently encountered in potential gradient. If the sensitiveness of the electrograph is that appropriate for quiet days, the trace on disturbed days is apt to go off the sheet, or to be too faint to decipher. It is thus practically impossible at the ordinary station dependent on only a single electrograph to employ all days when calculating mean values or diurnal inequalities. Mean annual values and diurnal inequalities vary according to the choice of days. At Kew a choice is made of a uniform number of days, 10 a month, free from negative potential—the usual accompaniment of disturbed weather—in hopes of securing a fair representation of fine weather conditions. But how far the results can be regarded as fairly representative of the *year* must be a matter of opinion. Highly disturbed days seem also to be omitted at the Ebro Observatory, as the results are said to be derived from days of characters 0 and 1—highly disturbed days are usually described as of character 2—but the number of such days will naturally vary with the season. What the result would be if the Kew and Ebro procedures were interchanged no one can say.

In view of the interest which attaches to the theoretical question as to what is the origin of the negative charge on the earth's surface, evidenced by the prevailing positive sign of the potential gradient, it may be pointed out that if the mean annual value of potential gradient waxes and wanes all over the earth with sunspot frequency, so also must the earth's total negative charge.

With respect to Dr. Bauer's reference to earth currents, the well-known relation of magnetic storms and earth currents renders an 11-year period in the latter almost a foregone conclusion. This was pointed out in the article on Earth Currents in the *Encyclopædia Britannica*, 11th Edition (Vol. VIII., p. 815, section 8).

DISCUSSION.

Dr. A. Russell, in expressing the thanks of the meeting for an interesting and instructive Paper, said that few realise the very high value of the potential gradient which normally characterises their physical environment, and which may have important biological effects, not only in connection with electro-culture but also in human psychology. The "correlation coefficient" which Dr. Chree introduced was not familiar to all physicists, and appeared to be of very great interest as a test of the inter-relation of different phenomena.

Dr. D. Owen remarked that in one of the tests quoted, the water-dropper had been put out of action by freezing. In this respect at least the radio-active collector would have the advantage. The author's experience as to the relative advantage of the two methods would be of value. Though not arising strictly out of the Paper the question occurred as to whether the earth as a whole was charged, or whether the electric fields observed extended only between the surface and the upper atmosphere. Was it yet possible to decide this point ?

Author's reply : In a very severe climate freezing is a serious drawback to a water jet collector, and radio-active collectors are there probably much to be preferred. They have been used, for instance, successfully by Dr. G. C. Simpson in Lapland, and in the Antarctic. In temperate climates I think—though others will differ—that the balance of advantages lies with the water-dropper, at least when there is a suitable water supply. The efficiency of the water jet has no tendency to decay with time. It is also very markedly greater than that of any radio-active collector I have come across, an important point when insulation is imperfect. In very calm weather the radio-active collector may infect the immediate neighbourhood. Loss of record from freezing of the jet is a very rare occurrence at Kew. As a source of trouble it is not to be mentioned in comparison with spider webs or inferior insulation in damp weather. The object of exhibiting the trace which was interrupted by the freezing of the jet was to show the leakage due to the imperfect insulation.

Observations of potential gradient made from balloons in fine weather have shown a rapid decline in the gradient with increase of height above the ground. The inference is that the lower layers of the atmosphere contain a charge opposite in sign to that on the earth's surface. Balloon observations have not, so far as I know, ever shown an actual reduction to zero in the potential gradient. But the natural inference from the somewhat limited data available is that under normal conditions the lowest 10 km. of the atmosphere contain a charge opposite in sign to that in the earth's surface, and making at least a close approach to equality with it.

XIII. *A Further Improvement in the Sprengel Pump.*

By J. J. Manley, *M.A.*,
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Received July 10, 1922.

ABSTRACT.

The Paper relates to a further improvement in the pump recently described in the Proceedings of the Society,* which is designed to avoid irregularities due to air skins on the inner surfaces of the apparatus. The present improvement consists in means for providing a mercury seal during periods when the pump is out of use, whereby the formation of fresh air skins is prevented.

In a Paper* recently communicated to the Physical Society, I described an elaborated form of the Sprengel pump and detailed a plan whereby certain irregularities arising from the presence of air skins upon the interior surfaces of the apparatus may be entirely avoided.

Now although the pump when duly prepared for action functions exceedingly well, a critical examination of the apparatus and its condition leads us to infer that a very appreciable defect is still retained; and the reality and importance of the defect will probably be readily admitted by all who view the matter in the following way.

On readmitting air to the exhausted apparatus, the tube leading to the head of the pump is almost entirely freed from mercury; in consequence of this, the incoming air at once begins to form a skin upon the hitherto gas-free surface. When the pump is again used, the erosive action of the mercury flowing through the tube slowly wears away the newly formed skin and the gas, thus set free, accumulates in the form of small bubbles within the head of the pump from whence it is discharged at irregular intervals into the fall tube; a little of the gas is carried away by the dropping mercury, but the major portion penetrates in the direction of the vessel that is being evacuated; and the more perfect the existing vacuum the greater the ease with which the eroded gas finds its way into the exhausted apparatus. The defect has recently been remedied by interposing between the head of the pump and the apparatus to be exhausted two superposed barometric columns δ and ε, as shown in the accompanying figure. The pump in its newest and more perfect form is prepared and used in the following way:—

The entire apparatus is first made chemically clean and dry; and then for the air skin covering the whole interior surface is substituted an equivalent skin of carbon dioxide, the substitution being effected as already described in my former Paper (*vide supra*). Next, the several reservoirs of the pump, including β, are partially but suitably charged with mercury, and β slowly lifted until the gas within the body of the trap a has been expelled through the as yet open capillary tube σ; the capillary is then sealed by fusing its tip. Temporarily closing the tube L, the two chambers† serving as large air traps are successively evacuated with a Geryk pump; the one is then closed by mercury, but the other, D, is hermetically sealed by fusing the capillary through which the chamber was exhausted. Lastly,

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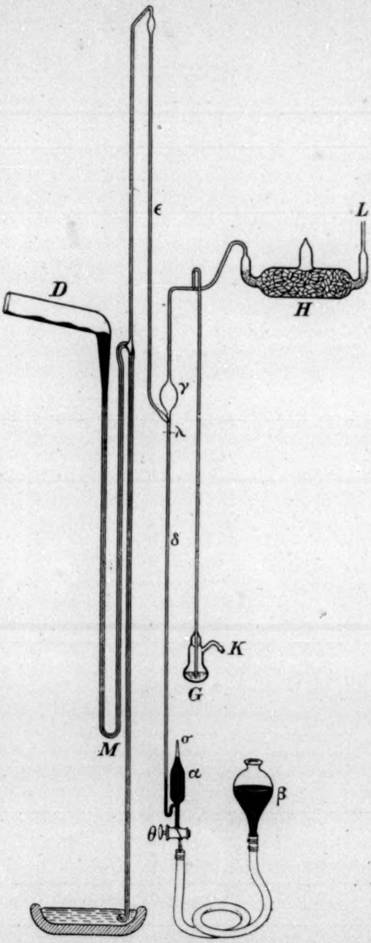
* Proc. Phys. Soc., Vol. 34, Part 3, p. 86.

† The two chambers are fully depicted in the former Paper.


ε
L
D
H
γ
λ
δ
K
G
M
σ
α
β
θ

the Geryk pump is connected with the tube *K* and the columns δ and ε, together with the chamber *H*, exhausted until the mercury has ascended in the column δ to a point λ but little below the barometric height. The reservoir β is now suitably raised and the bulb γ almost filled with mercury; on closing the tap θ, the Sprengel pump is fully prepared for use and any apparatus to be exhausted may be attached to the tube *L*.

On opening *L* prior to the attachment of a vessel, the barometric column which is at once set up within ε completely excludes air from the head of the pump; hence the formation of a new gas skin upon the interior surfaces of the tubes meeting in the head of the pump is rendered impossible. As the primary exhaustion of a vessel by a Geryk pump operating through *K* proceeds, the barometric column ε continues to shorten and ultimately retains a small value only; at this stage the tap θ is opened and the cistern β lowered until the mercury in the bulb γ has fallen to the level λ; free communication is thus established between the Sprengel pump and the partially evacuated vessel; the required vacuum can then be produced in the usual way.

XIV. *Null Methods of Measurement of Power Factor and Effective Resistance in Alternate Current Circuits by the Quadrant Electrometer.*

By D. Owen, *B.A., D.Sc., F.Inst.P., Sir John Cass Technical Institute, London.*

Received October 27, 1922.

ABSTRACT.

Zero methods are proposed, and expressions derived, for the measurement of power factor and effective resistance of alternating current loads. The methods are extended to high tension circuits.

The effect of "electrical control" of the needle of the quadrant electrometer is discussed, and it is shown that the usual formula for the instrument is applicable only when the needle is maintained at its mechanical and electrical zero. The further advantages of null methods are emphasised.

Illustrative tests are recorded.

Introductory.

The measurement of electrical power by the quadrant electrometer has until recently been made exclusively by deflectional methods. When first applied to this purpose the procedure consisted in taking two readings, the second being in the nature of a correction on account of the power consumed in the auxiliary resistance in series with the load. Miles Walker* showed how the latter reading could be avoided by connecting the needle to the mid-point of a non-inductive resistance (or transformer) placed as a shunt across the main leads; the reading is then proportional to the power required. The law of the instrument may under these conditions be written

$$W = K\theta \quad \ldots\ldots\ldots\ldots \quad (1)$$

where W denotes the power in watts, θ the angle of deflection of the needle (a unifilar torsional control being assumed), and K is a multiplying factor to be ascertained by a calibration test on a purely non-inductive load. In practice, for accurate work, it is necessary to calibrate the scale of the instrument at every point. Furthermore if, as is usually the case, the needle experiences a torque depending on its voltage even when the quadrants are at one potential—in other words, if *electric control* is present—the scale value in watts varies according to the root mean square potential of the needle.

If, however, the voltages on needle and quadrants are adjusted so that the deflection of the needle is zero, no calibration is necessary, and the only condition to be satisfied is that

$$\overline{(V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right)} = 0 \quad \ldots\ldots\ldots\ldots \quad (2)$$

where V_1 and V_2 denote the potentials of the quadrants and V that of the needle at any instant; the bar over the expression on the left denotes that the mean value is to be taken. The proof of this is given in an appendix to the present Paper.

Two modes of connection of the electrometer to the circuit will be considered, with either of which the power supplied may be deduced when the adjustment of the needle to zero has been made.

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* Journ. Am. Inst.E.E., p. 1035 (1902).

Now in a balance method it is clear that this balance should be independent of the voltage across the circuit, except in so far as the power factor is itself dependent on that voltage. It therefore appears that balance methods should lead to the determination of the power factor, and similarly of the effective resistance, without the necessity of knowing either the current or the applied voltage, as would be required when the actual power is measured. It thus seems desirable to regard these methods from this point of view. Indeed, for many purposes the knowledge of power factor, and its variation with conditions of temperature, applied voltage, etc., is of more interest than that of the power itself. It will be shown that null methods do in fact lead to the calculation of both power factor and effective resistance.

The increased accuracy of which they permit, in virtue of non-interference by electrical control, the fact that extreme steadiness of voltage is not to the same extent necessary, and the freedom from laborious calibration, give them a clear advantage over deflectional methods. In addition, they possess the merits of null methods in general—namely, freedom from errors arising from imperfect elasticity of the suspension, and increased sensibility whenever the deflection in the corresponding deflectional method would be off the scale.

The methods referred to may be applied to high-tension circuits, with connections suitable for use with the usual low voltage electrometer.

METHODS OF CONNECTION AND FORMULÆ.

Method I. (Double-shunt Connection).

H. Parry* proposed a mode of connection involving a shunt resistance, which admits of the adjustment of the deflection of the needle to zero, the data at balance being sufficient to enable the power to the load plus that to the series resistance to be calculated. In the course of the discussion on the Paper it was pointed out that the method could be improved by connecting the needle to the mid-point of the shunt, in which case the term for the power in the series resistance is eliminated. The circuit as modified is shown in Fig. 1. By adjusting the ratio $N = OA/OP$ of the full voltage V across the circuit to that between O (which may

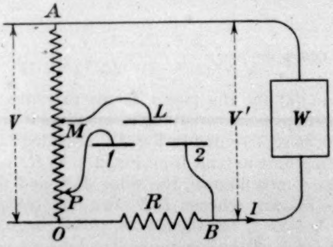

FIG. 1.—MODIFIED PARRY CONNECTION.

OA = shunt resistance; W = load; L = needle; quadrants denoted by 1 and 2.

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* Proc. Phys. Soc. Lond., p. 217 (1921).

conveniently be regarded as at a constant zero of potential) and the travelling point P, the condition of zero deflection may be secured for any value of R, and the power W is given by

$$W=\frac{V^2}{R}\cdot\left(\frac{1}{N}-\frac{1}{N^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where R denotes the value of the series resistance. The measurement of the power thus requires a reading of the voltage across the mains, as well as knowledge of the shunt-ratio and the series resistance.

To obtain the power factor an auxiliary balance is necessary, the connection of the needle being removed from M to O, and a new shunt ratio N' observed. This balance is simply one of voltages; the voltage V/N' pulling the needle towards quadrants 1, whilst the voltage RI across the series resistance pulls the needle towards quadrants 2. The balance implies equality of these voltages, quite irrespective of their relative phases. Hence we can write

$$V/N'=RI, \text{ or } V/I=\text{impedance of load including } R=N'R \quad . \quad . \quad . \quad (4)$$

where I denotes the load current.

The two balances supply all the data for calculating the power factor cos φ, and the effective resistance R_e (defined by the relation $W=I^2R_e$) as well as the power W. Denoting the voltage across the load (the series resistance being excluded) by V', we may write

$$W=V'I\cos\varphi=I^2R_e=\frac{V^2}{R}\left(\frac{1}{N}-\frac{1}{N^2}\right).$$

Combining with (4) we obtain

$$R_e=R\,.\,(N-1)\,.\,N'^2/N'^2\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$\cos\varphi=\frac{N'^2\,.\,(N-1)/N}{\sqrt{N^2\,(N'^2-1)-2N'^2\,(N-1)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The expression in (6) for the power factor in terms of N and N' is rather cumbrous, and it is perhaps more satisfying to calculate it in steps, with the guidance of the vector diagram, as represented in Fig. 2. Dividing each side by the current I we have Fig. 3, differing only in scale from Fig. 2. As R, R_e, N and N' are the data supplied by the balance, now known, the sides $A'B'$ and $A'D'$ of the right-angled triangle $A'B'D'$ are known, whence $B'D'$ can be calculated; $B'C'$, the load impedance, may now be found, and thence cos φ($=R_e$/impedance of load).

When the supply is a high-tension one it is no longer possible, using the ordinary low-voltage type of electrometer, to place this tension (or half of this) on the needle. If, however, the needle connection be made to a point F on the shunt, such that OF is a fraction $1/m$ of the circuit voltage; and if at the same time quadrants 2 are connected across the fraction $R/\frac{1}{2}m$ of the series resistance, as shown in Fig. 4, then balance may be obtained by adjustment of P, the connection to the other pair of quadrants, and the formula (3) applies without alteration. The value of m is to be chosen so that the voltage applied to the needle is near to its normal working voltage. (*See also* Paper by Miles Walker, *loc. cit.*)

As before, a second balance is obtained on transferring the connection of the needle to *O*. The formula $V/N'=2\,IR/m$ then applies, differing from that used on low-voltage circuits for the reason that the fraction $2/m$ of the total drop across *R* is being utilised between the needle and the second pair of quadrants. With this difference the calculation of power factor or effective resistance proceeds as before.

The use of the very high resistance shunts needful on high-tension circuits is, however, expensive, as well as undesirable on other grounds. This difficulty has often been met in practice by the use of step-down transformers. In view of the accuracy with which the transformation-ratio and the phase error between primary and secondary voltages of a suitably designed transformer can be determined,* their employment appears feasible provided they are used within the prescribed limits of their calibration. Assuming, then, the use of a transformer with step-down ratio *m*, we may apply the voltage V/m direct to the needle, and obtain balance by connecting the quadrants 1 to the travelling point *P* on a non-inductive shunt

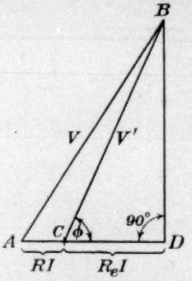

FIG. 2.—VECTOR DIAGRAM OF VOLTAGES.

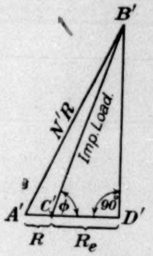

FIG. 3.—VECTOR DIAGRAM OF VOLTAGES WHEN $I=1$ AMP.

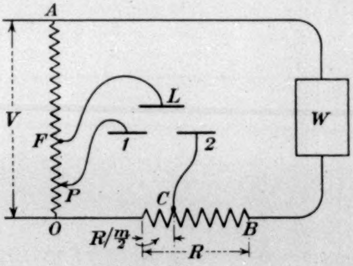

FIG. 4.

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* Rosa and Lloyd, and Agnew and Fitch, Bull. Bureau of Standards, Washington, Vol. 6 (1909).

placed across the secondary, quadrants 2 being connected to the intermediate point on R as already specified. The diagram of connections is shown in Fig. 5.

To illustrate: Suppose $V=20{,}000$ volts, load current $I=0{\cdot}2$ ampere, and $\cos\varphi=0{\cdot}01$. Choosing m as 100, the voltage applied to the needle becomes 200. The resistance R might be taken as 1,000 ohms, making the voltage drop applied to the quadrants 2 equal to $200/50=4$ volts. This would admit of an accuracy of 1/10 per cent. in the power factor.

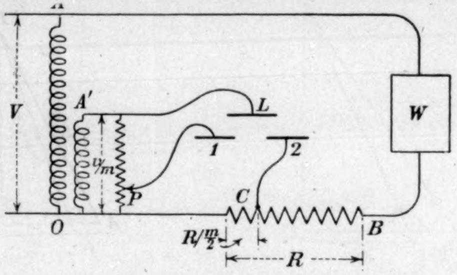

FIG. 5.—USE OF STEP-DOWN TRANSFORMER.

Method II. (*Single-shunt Connection*).

This mode of connection, which has been known for some time (*see* Russell, Alt. Curr., Vol. 1, Ch. 9, p. 196), has the appearance of greater simplicity, and, moreover, enables the effective resistance of the load to be determined from a single balance only. The accuracy obtainable is very satisfactory when the power factor is low, as on condenser loads; owing, however, to a certain lack of symmetry it loses sensitivity as the power factor approaches unity, and the accuracy is not then comparable with that obtainable by the previous method.

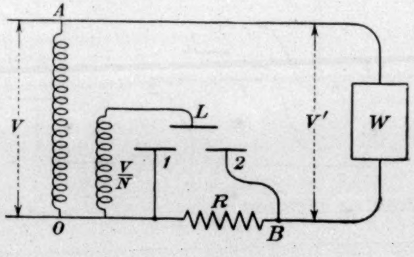

FIG. 6.—SECOND NULL METHOD—THAT OF SINGLE-SHUNT CONNECTION: BALANCE OBTAINED BY ADJUSTMENT OF R.

On high-tension circuits it has the advantage that the use of a non-inductive shunt may be avoided altogether if the effective resistance only is required. The diagram of connections is then as shown in Fig. 6. Using a step-down transformer of ratio N, the point O is common to one side of the primary, one side of the secondary, and one side of the series resistance. Keeping the voltage on the needle fixed, a balance may be effected by varying the value of R. The condition for balance is expressed in the formulæ

$$W = I^2R_e = I^2R \,.\, (N-2)/2 \quad \ldots\ldots\ldots \quad (7)$$

whence

$$R_e = R \,.\, (N-2)/2 \quad \ldots\ldots\ldots \quad (8)$$

Thus let $V=20{,}000$ volts, $I=0{\cdot}2$ ampere, $\cos\varphi=0{\cdot}01$; then choosing N equal to 100, voltage on the needle is 200 volts, and the value of R required for balance is 0·816 ohms, obtainable by a low resistance shunted by an adjustable high resistance. The accuracy of the determination of effective resistance, which works out at 1,000 ohms, would be about one quarter of 1 per cent.

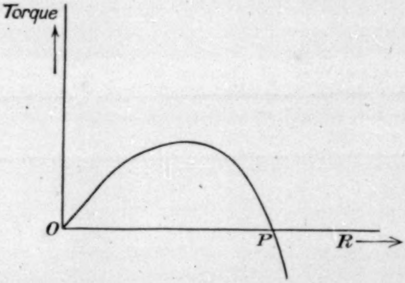

FIG. 7.

To obtain the power factor, a second balance exactly similar to that required in the first method is made, this needing the use of a shunt across the secondary, a tapping from which is connected to quadrants 1, whilst the needle is connected to O. If at balance the fraction $1/N'$ of the full voltage V is applied to quadrants 1, we have

$$RI = V/N' \quad \ldots\ldots\ldots \quad (9)$$

N', which is greater than N, is obtained as the product of N into the tapping ratio of the shunt on secondary. The power factor $\cos\varphi = C'D'/C'B'$ may be calculated, as in the previous method, by reference to the vector diagram of Fig. 3, using the data of balance according to equations (8) and (9).

It should be noted that there are two values of R for balance. The curve connecting the torque on the needle with R is very nearly a parabolic one, as shown in Fig. 7. The balance required is, of course, that corresponding to the point P of the diagram.

EXAMPLES OF TESTS.

The following examples of actual tests are adduced, the method being that of double-shunt connection (*see* Fig. 1) :—

(1) Load consisting of a standard mica condenser, capacity 1 μF, in series with a known non-inductive resistance *r*. Frequency of supply 96 ~. R=80 ohms.

V	r	N	N'	R_e	cos φ	W	$R_e - r$
(volts)	(ohms)			(ohms)		(watts)	(ohms)
75·5	20·06	754·4	13·89	20·48	0·0184	0·0919	0·42
74·5	10·06	1454	13·98	10·75	0·00964	0·0477	0·69
74·5	2·06	5480	13·90	2·80	0·0025	0·0092	0·74
74·5	0·06	26700	13·90	0·56	0·0005	0·0025	0·50
							Mean = 0·60 ± 0·1 ohm

The choice of the above artificial loads of low power factor allows of a severe check on the accuracy of the method. The last column gives the values of the effective resistance of the condenser alone, inferred as the difference between R_e and *r*. The agreement between them is very satisfactory, and leads to a power factor of 0·0006 for the condenser alone, corresponding to a phase lead of current over voltage amounting to 1′ 56″ short of 90°. This is estimated to be correct within 20″ of arc ; but the sensibility of the electrometer used might have been improved considerably by working with a smaller distance between the upper and lower plates of the quadrants, and this error correspondingly reduced.

(2) Commercial paraffin condenser, capacity 1·09 μF. At 96 ~. R=80 ohms.

V	N	N'	R_e	cos φ	W
74·5	2857	12·93	4·59	0·00445	0·0238

(3) Small transformer, closed iron circuit, on open secondary. At 96 ~. R=110 ohms

V	V'	N	N'	R_e	p.f.	φ
76·0	74·0	38·64	20·30	1143	0·526	58° 15′
38·0	36·9	37·02	14·39	598	0·390	

Though the values of voltage V are given in the above tables, it should be remembered that this is an unnecessary datum so far as the calculation of effective resistance or power factor is concerned.

The author desires to express his obligations to Mr. F. I. G. Rawlins, who assisted in the tests recorded in the Paper, and to Mr. G. L. Addenbrooke, who kindly placed one of his electrometers at his disposal.

APPENDIX.

The Formula for the Quadrant Electrometer.

Though it has been known for some time, it is not generally realised that in addition to the mechanical controlling couple on the needle there is an electrical control, of magnitude depending on the voltage on the needle. (Reference may be made to a clear exposition of this, due to Dr. R. Beattie, in *The Electrician*, p. 729, 1910.) The following derivation of the formula may be of service as showing the origin of the term for electrical control, as well as the fact that the existence of finite electrical control, though affecting the constant to be used in deflectional measurements, does not enter into the conditions which hold when a balance, *i.e.*, zero deflection of the needle, is attained.

Let C denote the capacity between two conductors differing in potential by V. The mutual torque is $\frac{1}{2}V^2 \cdot dC/d\theta$, where θ denotes the appropriate angle. Applying this to the quadrant electrometer, if C_1 and C_2 denote the respective capacities between a pair of quadrants and the needle, and V_1, V_2, and V the potentials of quadrants and needle, we have

$$\text{torque on needle} = \tfrac{1}{2}(V-V_1)^2 \cdot dC_1/d\theta - \tfrac{1}{2}(V-V_2)^2 \cdot dC_2/d\theta \quad . \; . \quad (1')$$

Writing

$$C_1 = a_1 + b_1\theta + c_1\theta^2 + \quad . \; . \; .$$
$$C_2 = a_2 + b_2\theta + c_2\theta^2 + \quad . \; . \; .$$

the expression for the torque becomes

$$\text{torque on needle} = \tfrac{1}{2}(V-V_1)^2(b_1 + 2c_1\theta + \; . \; . \; .) - \tfrac{1}{2}(V-V_2)^2(b_2 + 2c_2\theta + \ldots) \quad . \quad (2')$$

If $V_1 = V_2 = V$, the needle comes to rest at the "mechanical zero." If

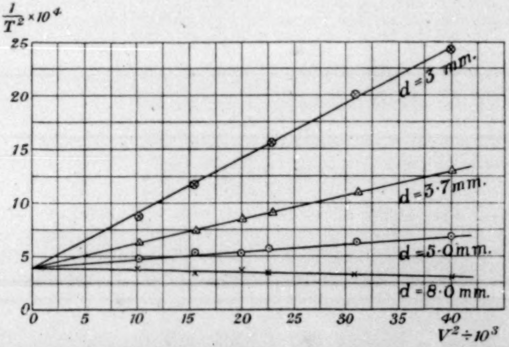

FIG. 1′.—EFFECT OF ELECTRICAL CONTROL ON PERIOD OF NEEDLE.
V = voltage on needle ; T = period in secs.

$V_1 = V_2 = 0$, but $V \neq 0$, the rest position of the needle is the "electrical zero." When the instrument is adjusted for coincidence of these zeros (as is necessary in all modes of use of the instrument), the effect, as may be seen by inspection of (2′), is to make $b_1 = b_2 = b$, say.

The derivation of the usual Maxwell formula involves the assumption that the capacities are linear functions of θ, in which case the electrical control vanishes. In general this simple assumption is unjustified; agreement with observation may, however, be secured to a close degree by taking the capacities as quadratic functions of θ. Supposing a unifilar suspension to be employed, (2′) thus becomes

$$-k\theta = \tfrac{1}{2}b[(V-V_1)^2 - (V-V_2)^2] + c_1(V-V_1)^2\theta - c_2(V-V_2)^2\theta \ldots$$

Since V is generally large compared with V_1 or V_2, this may, to a near approximation, be written

$$-k\theta=b[(V-V_1)^2-(V-V_2)^2]+(c_1-c_2)V^2\theta,$$

or

$$[k+(c_1-c_2)V^2]\theta=b(V_1-V_2)\left(V-\frac{V_1+V_2}{2}\right) \quad . \quad . \quad . \quad . \quad (3')$$

The controlling torque, proportional to θ, is thus made up of two terms, one with constant k representing the mechanical control; the other with constant (c_1-c_2) representing the electrical control, which may either assist or oppose the mechanical control. The expression for the deflection may be written

$$\theta=\frac{K(V_1-V_2)\left(V-\frac{V_1+V_2}{2}\right)}{1+AV^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4')$$

where K and A are constants; the sign of A may be positive or negative.

On alternating current circuits this becomes

$$\theta=\frac{K\,.\,\overline{(V_1-V_2)[V-\frac{1}{2}(V_1+V_2)]}}{1+A\overline{V^2}} \quad . \quad . \quad . \quad . \quad . \quad (5')$$

In place of the constant K of the simple formula it is necessary to use a coefficient $\dfrac{K}{1+A\overline{V^2}}$ which involves the root-mean-square value of V.

But in the null use of the instrument $\theta=0$, and the condition for this is

$$\overline{(V_1-V_2)[V-\tfrac{1}{2}(V_1+V_2)]}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6')$$

which involves a knowledge neither of K nor of A. No calibration of the instrument is required, nor is the presence of electrical control any source of error.

It may be of interest to add a graph (Fig. 1′) showing the magnitude of the electrical control as found in an actual instrument (of the Addenbrooke pattern), for various distances between the upper and lower plates of the quadrants, and for various voltages on the needle. The simplest means of measuring the control is to observe the period of vibration of the needle, with the quadrants at the same potential, for any chosen potential of the needle.

The graphs, which connect the reciprocal of square of the periodic time and the square of the needle voltage, prove to be straight lines, in accordance with the formula (3′). For a distance d of 6·8 mm. between the plates the electrical control vanishes, the period being independent of the voltage on the needle; for smaller distances the electrical control is positive, and for greater distances negative. The electrical control may prove to be actually greater than the mechanical control: it only vanishes for a particular value of the sensibility of the instrument.

DISCUSSION.

Dr. E. H. **Rayner** congratulated the author on his valuable additions to the many uses of the Quadrant Electrometer. This wonderful instrument was invented over half a century ago by Lord Kelvin, but is still unsurpassed in its utility, being applicable to the accurate measurement of power, insulation, phase-angles, and many other quantities. The speaker took the opportunity to point out some details as to which care is necessary in the practical use of the Electrometer. (1) With high voltages the mechanical force on the needle is considerable and may bend it, leading to inconsistent results at low power factors. (2) Referring to Fig. 1 of the Paper, the high resistance AO generally has an appreciable distributed capacity, with the result that the voltage across MO is not in phase with the current. If conditions permit, the easiest remedy is to take as much current along OA as possible; for instance, if the current in this branch be 1/20 ampere, the power factor in a common case would be 0·1 or 0·2 per cent., but on increasing the current to 1 ampere the phase lag might become negligible. A similar error has to be contended with where a step-down transformer is used, as shown in Fig. 5, and it must be remembered that for small phase-angles an error of a few minutes of arc may represent a large percentage error. (3) An extremely important point when high voltages are applied to the needle is that the faces of the quadrants should be perfectly flat. To this end they should be ground on cast iron after they have been fixed in place.

Mr. G. L. ADDENBROOKE referred to his Papers published in the ***Electrician*** in 1901 as relevant to some of the points raised by the author. He added that it is convenient to arrange a switch whereby the point P, Fig. 1 of the Paper, may be connected at will to the point O. In this way the instrument may be converted into an ammeter. He had used deflectional methods because they permitted " seeing what was going on."

Dr. A. RUSSELL congratulated the author on discovering so many theorems and applying them so usefully, and expressed appreciation of Dr. Rayner's helpful suggestions.

Capt. R. DUNSHEATH (communicated): This Paper is very opportune at the present time when so many investigators are seeking the best method of measuring dielectric losses, and is full of useful suggestions. I do not agree with the author, however, that it is desirable to eliminate both voltmeter and ammeter. His methods give power factor only, but a figure for actual watts lost is generally required. Also, due to the importance of the dependence of power factor and losses on voltage, it is usual to decide at the commencement of a test what voltage shall be adopted, and a voltmeter is essential. The ammeter is not so necessary as, having V, N', and R, in formula (4), the value of the current follows at once. Proceeding in this manner $\cos\varphi$ is obtainable without the use of equation (6). It is, of course, necessary to switch one side of the voltmeter from O to B, but this is a simple matter.

I notice that Dr. Owen estimates the error of the figure obtained for phase angle on a $1\mu F$ mica condenser at about 16 per cent. Much smaller condensers than this are usual in certain branches of industrial work, and it would be interesting to know the sensitivity of the instrument used, and what sort of accuracy might be expected if the method were applied to capacities of the order of $0{\cdot}01\mu F$.

Mr. HUBERT PARRY (communicated): The most interesting part of Dr. Owen's Paper is, I think, the extension of the null method to other ratios of line voltage to needle voltage than 2. I think it is a very valuable step. I would point out that when taking the needle voltage as some fraction of the line voltage, if the resistance of the potential divider has a large value per volt the capacity of the electrometer and leads may cause the voltage on the needle not to be in phase with the line voltage, and this phase displacement enters directly into the result obtained. When the needle is at line voltage the potential divider accuracy for phase is not so important.

I do not think Method II. as accurate as Method I., as the phase angle of variable series resistances is somewhat uncertain. This is especially so if the resistance value is low. This also applies to shunting one resistance by another. I think this will be realised when working at about 90° phase difference, where one or two minutes' error means a large percentage difference in the result.

I doubt if the phase error of a potential transformer would be sufficiently constant for precision measurement of, say, condenser losses; a potential divider on the lines of Örlich and Schultz would be better, and there probably would not be such a lot of difference in the cost.

In practice $\cos\varphi$ in equation (6) could be expressed by watts/VI without serious error, provided the voltage drop on the series resistance is not large compared to the needle voltage.

I do not think it is quite correct to say that " the presence of electrical control is no source of error " in the zero method; if the voltage on the needle varies the zero varies, and this will lead to incorrect balancing.

Mr. A. ROSEN (communicated): A great advantage of balance methods which measure power factor directly is that $\cos\varphi$ varies approximately as V^{n-2}, when the power W varies as V^n, so that fluctuations in voltage are of less importance, and the voltmeter need not be so accurately calibrated. The figure given for the accuracy when testing at 20,000 volts—viz., 1/10 per cent. of cos $\varphi=0{\cdot}01$—is no doubt deduced from tests taken at low voltages, and appears somewhat optimistic. When working to such a high degree of accuracy, factors enter which might otherwise be ignored—*e.g.*, has the author considered the effect of the time-constants of the resistances in the various parts of the circuit? Another difficulty is the effect of speed. Although frequency is not mentioned explicitly in any of the equations, it enters as follows: On a condenser load, the impedance is approximately $1/\omega c$ ($\omega=2\pi\times$frequency, c=capacity)$=N'R$ from (4). $\therefore N'$ varies approximately as $1/\omega$ since c is constant. Assuming for the condenser that cos φ is roughly constant with frequency, we have from (6) N varies approximately as $1/\omega$. Thus, to measure cos φ to 1/10 per cent., the speed must be controlled and measured to within this figure, an ideal not obtained in practice with the comparatively large machines needed for testing cable at high voltages. Possibly speed variation accounts for the difference of 3 per cent. of 0·01 in the figures quoted for test (1). If so, it is obvious that increasing the voltage or the sensitivity of the electrometer will not, in this case, produce greater accuracy. However, 1 per cent. is sufficient for practical work, and is better than can be obtained with certainty using the wattmeter in the usual way.

The AUTHOR (in reply to the discussion): The remarks of Dr. Rayner will be valued by practical workers. The question of phase error in the shunt resistance, and methods of compensation have also been treated by Örlich and Schultze. Mr. Addenbrooke's desire to follow what was going on by watching the deflection was, of course, quite natural. In the present methods this could always be done in the preliminary tests by slipping P into coincidence with O (*see* Fig. 1); for the final reading the zero balance would confer a distinct gain in accuracy. In reply to Capt. Dunsheath's inquiry the sensitivity of the electrometer used in the tests quoted was such that with 100 volts on the needle and one-tenth of a volt across the quadrants the deflection was about 60 mm. at a metre scale-distance. This could have been multiplied three or four times possibly. Measurements on a 0·01 μF condenser could be conducted with much the same accuracy as those with $1\mu F$, since the resistance R could be increased in inverse proportion to the capacity.

In reference to Mr. Parry's last remark, the zero of needle should not vary as the voltage on needle varies; nor have I found that it does in my own experiments. Mr. Rosen's observations in regard to effect of frequency are of value, and show the necessity of securing constancy of speed of machines in proportion to the accuracy aimed at.

XV. *The Eötvös Torsion Balance. By Captain* H. SHAW, *M.Sc., F.Inst.P., and* E. LANCASTER-JONES, *B.A.* (*Cantab*).

RECEIVED NOVEMBER 10, 1922.

ABSTRACT.

The Eötvös Gravity Balance in the Science Museum, South Kensington, is described, and the theory of its operation considered. A full account is given of certain preliminary experiments made on the torsion wires, with a view to ascertaining the daily variation of the equilibrium position, due to strains set up in the wires during manufacture. Until this daily shift has been eliminated the instrument cannot be employed successfully. The experiments show that a preliminary treatment of the wires is desirable, and a test was also made of the "baking" method recommended by Eötvös for ageing the wires. This method was found to give the desired result, and after this treatment the reading of the instrument remained practically constant from day to day.

Tables are given showing the variation in the reading of the instrument when its beam is rotated into different azimuths, and the constancy of these readings when repeated. The differences in the readings as the azimuth varies are sufficiently marked to indicate the changes in the local gravitational field of the laboratory, and further experiments are being carried out to determine these changes in detail, as a preliminary to field tests; it is hoped to communicate the results of these experiments at a later date.

PREVIOUS to the introduction of this instrument, researches on gravity were confined almost exclusively to investigations with the pendulum and the bubble level, but in 1888 a new line was followed by Eötvös, who endeavoured to measure the variation of the force of gravity in the vicinity of a point, or more exactly, to determine the derivatives of its components, while analogous methods were also employed to determine the derivatives of the magnetic force.

As the variations of gravity are extremely small in comparison with the total force, Eötvös concluded that the method to be employed should measure the differences of gravity directly, rather than the force of gravity itself.

This method, it is claimed, is extremely sensitive, and is capable of giving results which are unobtainable by pendulum methods. According to Eötvös, it is possible by this method to estimate the desired magnitudes at any place in the course of one night, with an accuracy of 1×10^{-9} C.G.S. units, while under favourable weather conditions the accuracy may be increased still further, but it is unfavourably affected by rapid variations of temperature.

In the construction of an instrument sufficiently sensitive for this purpose, the main characteristic required is a long period of swing, and it is due to this consideration that an instrument has been produced, capable of observing and measuring these small variations of force.

The balance was designed in 1888 by Baron Roland Eötvös, Professor of Physics at the University of Budapest, and was constructed by the firm of Ferdinand Süss. It is intended to be used as a field instrument, and is therefore designed with a view to portability, while at the same time it is capable of determining variations of the above-mentioned order, with a considerable degree of accuracy.

It consists of a fine torsion wire, carrying a lever which supports at its extremities two weights, at different vertical heights, the whole being enclosed in a double-walled metal case which can be rotated about a vertical axis. An azimuth circle enables the positions of the case to be determined and the orientation of the balance arm relative to it is observed by the aid of a telescope. The system has a period of swing exceeding 1,200 seconds, and after having been disturbed returns to rest in its equilibrium position in approximately two hours. The position of equilibrium and the motion of the beam are found to be remarkably stable and relatively constant, so that the instrument can be used not only in a well-protected laboratory, but also at night in the open air, with the protection only of a canvas tent.

(A.) THEORY.

The beam is freely suspended under the action of two force-systems, viz. :—

(*a*) The force of gravity, which is compounded of forces due to earth-attraction and forces due to earth-rotation.

(*b*) Torsional forces due to twisting of torsion-wire.

As regards the system (*a*) we make the following assumptions :—

1. The complete system has a potential function U, which is *uniform* in the neighbourhood of any point external to the earth's masses, and of which the first derivatives $\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}$, and the second derivatives $\frac{\partial^2 U}{\partial x^2}, \frac{\partial^2 U}{\partial x \partial y}, \frac{\partial^2 U}{\partial y^2}, \frac{\partial^2 U}{\partial z^2}$, &c., are also uniform at such points for any system of rectangular axes.

2. The masses of the torsion balance beam are so arranged that it has only a tendency to twist about the torsion wire.

As regards (*b*) we assume :—

3. The force due to twist is proportional to the angle through which the beam is twisted from its position of zero torque. The zero position is usually unknown until the series of observations is complete.

Choose rectangular axes of co-ordinates (x, y, z), with the origin O at the centre of gravity of the balance beam, the axis Oz directed vertically downwards, the axis Ox towards the geographical north and Oy towards the east.

Let (x, y, z) be any point in the suspended system.

Let (X, Y, Z) be the forces at (x, y, z) per unit mass due to system (*a*) resolved along the axes.

Then, since a potential function U is assumed to exist for these forces, we have

$$X=\frac{\partial U}{\partial x}, \quad Y=\frac{\partial U}{\partial y}, \quad Z=\frac{\partial U}{\partial z}.$$

At the origin O, we have $X_0=Y_0=O$, and $Z_0=g_0$, since the resultant force is assumed to be along the Oz axis at O.

At any other point (x, y, z) we have, expanding by McLaurin's Theorem,

$$\left.\begin{aligned} X&=X_0+x\left(\frac{\partial X}{\partial x}\right)_0+y\left(\frac{\partial X}{\partial y}\right)_0+z\left(\frac{\partial X}{\partial z}\right)_0+\text{ higher orders} \\ Y&=Y_0+x\left(\frac{\partial Y}{\partial x}\right)_0+y\left(\frac{\partial Y}{\partial y}\right)_0+z\left(\frac{\partial Y}{\partial y}\right)_0+\text{ higher orders} \\ Z&=Z_0+x\left(\frac{\partial Z}{\partial x}\right)_0+y\left(\frac{\partial Z}{\partial y}\right)_0+z\left(\frac{\partial Z}{\partial z}\right)_0+\text{ higher orders} \end{aligned}\right\} \quad . \; . \; . \; . \; . \quad (1)$$

If we assume that, in the region covered by the balance, the forces X, Y, Z are so nearly uniform that we can neglect terms involving x^2, y^2, &c., and the second derivatives of X, Y, Z, we have

$$\left.\begin{aligned} X &= x\left(\frac{\partial X}{\partial x}\right)_0 + y\left(\frac{\partial X}{\partial y}\right)_0 + z\left(\frac{\partial X}{\partial z}\right)_0 \\ &= x\left(\frac{\partial^2 U}{\partial x^2}\right)_0 + y\left(\frac{\partial^2 U}{\partial x \partial y}\right)_0 + z\left(\frac{\partial^2 U}{\partial x \partial z}\right)_0 \\ Y &= x\left(\frac{\partial^2 U}{\partial x \partial y}\right)_0 + y\left(\frac{\partial^2 U}{\partial y^2}\right)_0 + z\left(\frac{\partial^2 U}{\partial y \partial z}\right)_0 \end{aligned}\right\} \text{since } X_0 = Y_0 = O. \quad . \quad . \quad . \quad (2)$$

The torque F_a due to system (a) about axis Oz will be

$$F_a = \int (Yx - Xy)\, dm,$$

where the integral is taken over the whole suspended mass system.

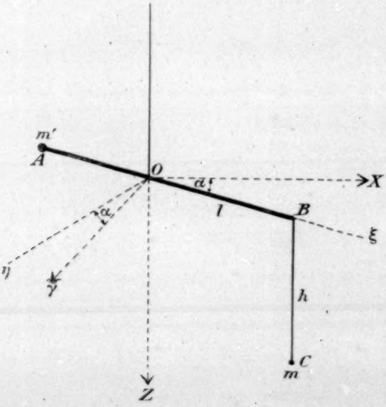

FIG. 1.

Using equations (2), we get

$$F_a = \left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)_0 \int xy \, . \, dm + \left(\frac{\partial^2 U}{\partial x \partial y}\right)_0 \int (x^2 - y^2) \, . \, dm + \left(\frac{\partial^2 U}{\partial y \partial z}\right)_0 \int zx \, dm$$

$$- \left(\frac{\partial^2 U}{\partial x \partial z}\right)_0 \int yzdm \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let the beam when in equilibrium be inclined at an azimuth angle α to axis Ox, and take new axes O (ξ, η, z), such that $O\xi$ lies along the axis of figure of the beam, $O\eta$ perpendicular thereto (both in the horizontal plane xOy).

Then, if (ξ, η, z) denote the co-ords. corresponding to (x, y, z) for every point in the suspended mass, we have

$$x=\xi\cos a-\eta\sin a$$

$$y=\xi\sin a+\eta\cos a$$

$$\therefore \qquad \int xydm=\tfrac{1}{2}\sin 2a\int(\xi^2-\eta^2)\,.\,dm+\cos 2a\int\xi\eta\,.\,dm$$

$$\int(x^2-y^2)dm=\cos 2a\int(\xi^2-\eta^2)\,.\,dm-2\sin 2a\int\xi\eta\,.\,dm$$

$$\int xz\,.\,dm=\cos a\int\xi z\,.\,dm-\sin a\int\eta z\,.\,dm$$

$$\int yzdm=\sin a\int\xi z\,.\,dm+\cos a\int\eta z\,.\,dm$$

By symmetry about axis $O\xi$ and plane ξOz, we have

$$\int\xi\eta\,.\,dm=O,$$

$$\int\eta z\,.\,dm=O,$$

$$\int\xi z\,.\,dm\ (\text{for } m' \text{ and beam})=O,$$

$$\int\xi z\,.\,dm\ (\text{for } m \text{ alone})=mhl,$$

where l = distance OB from origin to point of suspension of lower weight.
h = distance BC from pt. of suspension of lower weight to its centre of gravity

Also we may put $\int(\xi^2-\eta^2)\,dm=K'$

Although K' does not equal the moment of inertia K of the whole system about Oz, it is usually assumed to do so, since η^2 is negligible in comparison with ξ^2.

$\therefore$ on this understanding we have for equation (3)

$$F_a=\frac{K}{2}\,.\,\sin 2a\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)+\mathrm{K}\cos 2a\left(\frac{\partial^2U}{\partial x\partial y}\right)+mhl\left[\frac{\partial^2U}{\partial y\partial z}\cos a-\frac{\partial^2U}{\partial x\partial z}\sin a\right]\ . \qquad (4)$$

where the suffix $[i.e. (\ )_0]$ has been dropped, as there is no further ambiguity and the quantities $\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)$, $\frac{\partial^2U}{\partial x\partial y}$, &c., are understood to have their values at the origin O.

Now, since the beam is in equilibrium under the torque F and the resistance to twist, which latter is equal to $\tau\theta_a$, where θ_a is the twist in the wire for the position a, we have

$$F_a=\tau\theta_a.$$

Let n_a = the scale reading (n scale units of 0·5 cm.) corresponding to the a position of equilibrium.

,, n = scale reading when there is zero torque (*i.e.*, $F=O$).

,, D' = distance (in scale units) of mirror from scale.

Then $n_\alpha - n = 2D'\theta_\alpha$.

Equation (4) becomes

$$n_\alpha - n = \frac{2D'K}{\tau}\left[\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)\frac{\sin 2\alpha}{2} + \frac{\partial^2 U}{\partial x \partial y}\cos 2\alpha\right] + \frac{2D'mhl}{\tau}\left[\frac{\partial^2 U}{\partial y \partial z}\cos \alpha - \frac{\partial^2 U}{\partial z \partial x}\sin \alpha\right] \qquad (5)$$

putting

$$\frac{D'K}{\tau}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) = A$$

$$\frac{2D'K}{\tau} \cdot \frac{\partial^2 U}{\partial x \partial y} = B$$

$$-\frac{2D'mhl}{\tau} \cdot \frac{\partial^2 U}{\partial z \partial x} = C$$

$$\frac{2D'mhl}{\tau} \cdot \frac{\partial^2 U}{\partial y \partial z} = D$$

we have

$$n_\alpha - n = A \sin 2\alpha + B \cos 2\alpha + C \sin \alpha + D \cos \alpha \quad . \; . \; . \; . \; . \; . \; . \quad (6)$$

(B.) DETERMINATION OF CONSTANTS.

In the equation (6) the quantities A, B, C, D and n are unknown factors, independent of the azimuth angle α of the beam. By taking readings of n_α in 5 different positions of α, these factors can be evaluated from the resultant 5 equations.

The factors $\frac{2D'K}{\tau}$ and $\frac{2D'mhl}{\tau}$ are purely instrumental constants, independent of the position of the balance, and are evaluated previous to the observations in the following way.

To find τ.—An accurately turned lead sphere to attract the suspended weight m is placed alternately on the left and right side of m, in a line perpendicular to the direction of the beam and at a measured distance from m as small as possible (about 10 cm.). Then if

$n'-n$ = displacement of zero at a setting of lead sphere
D' = distance of scale from mirror
G = gravity constant
M = mass of lead sphere
m = ,, suspended cylinder
λ = length of suspended cylinder
ρ = mean distance of centre of sphere from axis of cylinder
l = distance OB

we have

$$\tau \cdot \frac{n-n'}{2D'} = G\frac{Mm}{\rho^2} \cdot \frac{l}{\sqrt{1+\frac{\lambda^2}{4\rho^2}}} \quad . \; . \; . \; . \; . \; . \; . \quad (7)$$

neglecting effect of sphere on mass m'.

To find K.—The quantity $\frac{K}{\tau}$ can be found by ascertaining the period of swing of the system in two positions of the balance arm, $\alpha=0$, and $\alpha=\frac{\pi}{2}$. Owing to the long period, however, it is preferable to substitute a stiffer and shorter wire, and to find the periods of swing T_0 and T_1, firstly for the original suspended mass system (m', m), and secondly when extra weights m_1, m_2 are suspended on the beam. The moment of inertia of the added weights can readily be calculated, and the unknown coefficient of torsion of the wire can thus be eliminated.

Thus, if the new wire has a coefficient of torsion τ', we have for the first oscillation, with only m and m' suspended on the beam,

$$\frac{K}{\tau'}=\frac{T_0^2}{4\pi^2}$$

where T_0 secs. represents the complete undamped period of torsional oscillation, neglecting all effects due to varying gravity field owing to the unimportance of these when the wire is sufficiently rigid. This could be tested by oscillating in different azimuths.

For the second oscillation, with added weights m, on each side, and a resultant added moment of inertia I,

$$\frac{K+I}{\tau'}=\frac{T_1^2}{4\pi_2^2}$$

$$\therefore \quad \frac{K+I}{K}=\frac{T_1^2}{T_0^2}$$

$$\frac{I}{K}=\frac{T_1^2-T_0^2}{T_0^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which gives K in terms of I.

By means of the above preliminary experiments, and ordinary measurements and weighings for D', m, h, l, we find the instrumental constants $\frac{2D'K}{\tau}$ and $\frac{2D'mhl}{\tau}$. Hence, by observing n_α in five positions of the beam and thus obtaining A, B, C and D we can get the four magnitudes

$$\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right), \quad \left(\frac{\partial^2 U}{\partial x \partial y}\right), \quad \left(\frac{\partial^2 U}{\partial y \partial z}\right), \quad \left(\frac{\partial^2 U}{\partial z \partial x}\right) \text{ at the origin } O.$$

(C.) NUMERICAL VALUES.

To show the magnitude of the quantities concerned, one may give the values in C.G.S. units, of the above quantities for the normal Bessel Ellipsoid, where the semi-axes of the ellipsoid a, b, are

$$a=637{,}739{,}700 \text{ cm.}$$

$$b=635{,}607{,}800 \text{ cm}$$

and by Helmert's formula for g,

$$g=978{\cdot}00(1+0{\cdot}00531 \sin {}^2\varphi).$$

For latitude $\varphi=51°30'$ (London) we get

$$g=981\cdot1806$$

$$\frac{\partial g}{\partial x}=\frac{\partial^2 U}{\partial z\partial x}=7\cdot9376\times10^{-9}$$

$$\frac{\partial g}{\partial y}=\frac{\partial^2 U}{\partial y\partial z}=0$$

$$\frac{\partial^2 U}{\partial x\partial y}=0$$

$$\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)=4\cdot005\times10^{-9}$$

In one of Eötvös's instruments, the instrumental constants $\frac{2D'K}{\tau}$ and $\frac{2D'mhl}{\tau}$ amounted to $0\cdot10323\times10^9$ and $0\cdot14087\times10^9$ respectively. If this instrument were set at $\alpha=\frac{\pi}{2}$ at a place where gravity was "normal" in latitude 51°30′, the equation would be

$$n_\alpha-n=-(4\cdot005\times0\cdot05162)-(0\cdot14087\times7\cdot9376)=-1\cdot3 \text{ approx.},$$

whilst at $\alpha=0$, we should have

$$n_\alpha-n=0$$

$\therefore$ the reading at $\alpha=\frac{\pi}{2}$ would differ from that at $\alpha=0$ by approximately 1·3 scale divisions, each division being 0·5 mm. This would be easily appreciable. Where, however, there are great local attracting masses, the differences in readings as α varies often amount to ten times the above quantity. For example, in a room of the Physical Institute of the University of Budapest, Eötvös got the following readings for different azimuths, of the beam.

α	n
0°	204·5 [scale unit=0·5 mm.]
72°	200·7
144°	193·2
216°	183·2
288°	199·1

The corresponding values of $\frac{\partial^2 U}{\partial x\partial z}$, &c., being

$$\frac{\partial^2 U}{\partial x\partial z}=+21\cdot12\times10^{-9}$$

$$\frac{\partial^2 U}{\partial y\partial z}=-66\cdot89\times10^{-9}$$

$$\frac{\partial^2 U}{\partial x\partial y}=+10\cdot25\times10^{-9}$$

$$\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)=+66\cdot08\times10^{-9}$$

(D.) DESCRIPTION OF THE INSTRUMENT.

Torsion Wire.

The balance arm G is supported by a 60 c.m. length of fine platinum-iridium wire B of 0·04 mm. diameter, and containing 20 per cent. of iridium. Special precautions are taken to prevent deformation of the wire, which is supplied by the makers on a reel of not less than 25 cm. diameter. The ends of the wire are soldered to brass eye-plates which serve to attach them to the beam, and the torsion head A. The wire, which is capable of supporting a weight of 120-130 gr. is stretched under a load of 80 gr. which is approximately the weight which it has to carry in the instrument, after which it is subjected to a special baking treatment, which consists in gradually raising its temperature to 100°C. in an oven, and allowing it to cool slowly. A wire treated in this manner appears to lose the greater portion of its remanent torsion, and according to Eötvös may be used the following day, although it still exhibits a slow displacement of the equilibrium position which amounts to a few minutes of arc per day, but is sufficiently regular to enable its value to be calculated and a correction applied. This variation gradually decreases, but apparently is only eliminated after several years.

A far greater disadvantage arises from the varying susceptibility to temperature effects possessed by the torsion wires according to the permanent twist set up during manufacture. The position of equilibrium thus varies with the temperature, but after baking, this variation is found not to exceed a few tenth minutes per degree centigrade, and can be corrected by the use of an estimated temperature, coefficient.

Quartz fibres were tried by Eötvös subsequently, but were found by him to be not so sensitive as platinum, as he found great difficulty in getting a quartz fibre capable of supporting 100 grm. and having a torsion coefficient per metre length, of less than unity, whereas that for his platinum wire was 0·3 C.G.S. units. According to Eötvös the constancy of the equilibrium position of similar well-drawn platinum wires was found to be so satisfactory that he preferred them, especially in portable field apparatus, to the quartz fibres, which are too delicate and fragile.

Torsion Head.

The upper end of the torsion wire is attached to a torsion head A which enables the wire and beam to be adjusted, while the central member to which the torsion wire is attached is threaded through an annular collar, which is itself threaded into the body of the torsion head, thereby enabling vertical adjustment of the wire and beam to be secured. The torsion head is graduated on its circumference in intervals of 5 deg., a vernier enabling readings to be taken to degrees. A three-screw clamping device is also provided, by which the torsion head may be adjusted and clamped in position

Beam.

A light aluminium rod forms the balance arm G, which is rectangular in section and 40 cm. in length. From a V-notch near one end of this beam a cylindrical platinum weight K is suspended by a fine platinum wire J. This cylinder, which weighs about 25·5 grm., has a length of 27 mm. and a diameter of 8 mm. and is suspended with its centre of gravity about 65 cm. below the beam. To the other end of the beam a 30 grm. counterweight D is riveted, consisting of a platinum

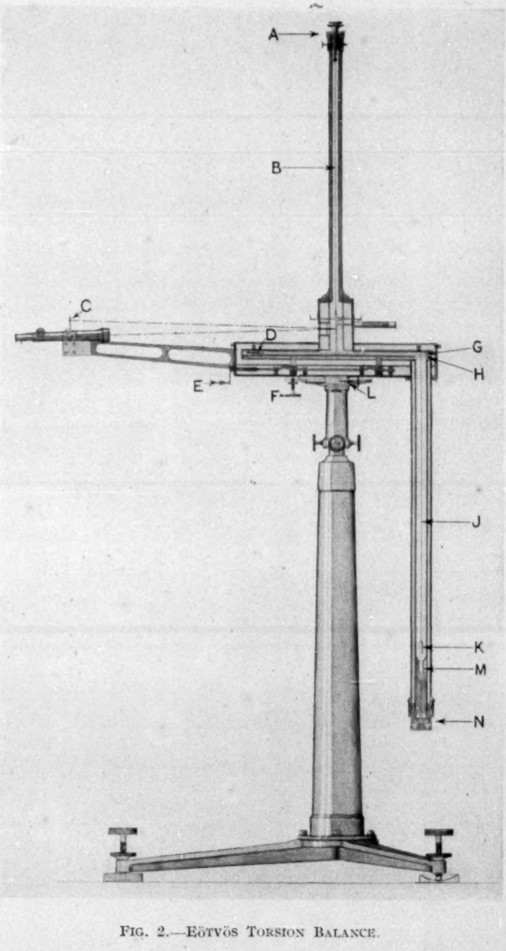
A
B
C
D
E
F
L
G
H
J
K
M
N

FIG. 2.—EÖTVÖS TORSION BALANCE.

To face page 158.

strip 85 mm. by 12 mm. bent in the form of a U, and fixed with its maximum dimension in the direction of the axis of the beam.

A light vertical rod 10 cm. long rises from the centre of gravity of the beam, and serves to carry a small circular mirror, while a small screw near its extremity facilitates the attachment of the torsion wire.

Case.

The sensitiveness of the instrument is such that special protection is required against all external influences which are able to affect in any way its operation and equilibrium position. It is possible to realise those conditions by using double-walled metallic boxes which, without interfering with the motion of the bar, enclose and protect it as much as possible. The boxes are of brass, with walls 3 mm. and 4 mm. thick respectively and separated by an air gap of 1 cm. It is claimed that with this construction temperature changes penetrate uniformly into the interior of the instrument and consequently do not give rise to air currents while the metallic envelope, homogeneous on all sides, also furnishes protection against electrical action and radiations of all kinds. The degree of stability obtained in this manner is such that the instrument is capable of furnishing satisfactory results in the field, protected only by a canvas tent. The effect of solar radiation, however, is a factor of importance, and the best results are obtained at night time or on cloudy days.

In this instrument the horizontal balance beam *G* is enclosed in a double-walled rectangular box, the inner and outer covers of which are easily removable, thereby exposing the beam. A double-walled box enclosing the mirror and its support is split centrally into two portions which are easily removable, giving ready access to the mirror and the lower end of the torsion wire. In one of these portions of the box a glass window is provided, enabling the reflection of the scale *C* in the mirror to be read by the telescope.

Screwed to the upper surface of the mirror box a double-walled vertical tube terminating in a torsion head provides protection for the torsion wire, while a similar tube projecting downwards from one end of the beam case protects the lower weight and its suspension.

The inner box enclosing the beam has a movable base *H* which is capable of adjustment vertically by means of a handle *E* at one end of the case. In this way the base of the box can be lowered or raised for damping the motion of the beam, and also forms a platform upon which the weight of the beam can be taken, thus releasing the tension of the torsion wire for the purpose of transport. The beam is also clamped in position by means of two forked arms projecting horizontally into the interior of the mirror case, into which they may be pushed so as to clamp the aluminium support of the mirror.

A clamping device is also provided for the suspended weight *K* in the form of a clamp and rest *M* carried on a vertical screw working in a fixed collar at the lower end of the enclosing tube and operated by a screw cap *N*. Rotation of this cap in one direction causes the rest to be raised sufficiently to enclose the platinum weight and to raise it slightly, thus removing the tension from the suspension. The platinum weight is automatically clamped in this position by a vertical wire carried on the rest, and operating in a vertical groove in the inner tube. A projection on the end of this wire moves inwards above the platinum weight when the latter has been taken up by the rest, and so holds it in position, should the instrument be overturned during transport.

The instrument is read by a telescope carried on an arm hinged to the case, and also carrying above the telescope a scale graduated in half-millimetres, at a distance of 60 cm. from the mirror. The telescope arm is adjusted by means of a screw F bearing on the under surface of the beam case, while the telescope itself may be adjusted by means of two set screws so that the graduated scale is visible in the mirror. By this means the scales can be read to 0·05 mm., which is the equivalent of 8·6 secs. of arc.

Mounting.

The case and telescope can be rotated about a vertical axis mounted on a fixed horizontal stage carrying a circle graduated in thirds of a degree and provided with a vernier L, by means of which the azimuth of the case can be read to an angle of one minute. The instrument stands on a heavy tripod base, to which is bolted a vertical centre column, the upper end of which terminates in a cylindrical rod of smaller diameter, and upon which the upper portion of the instrument is carried. Four flats on the exterior of this rod enable four adjusting screws on the upper part of the instrument to be employed for levelling purposes, after the centre column has been adjusted vertically.

A later type of instrument is provided with two suspended beams arranged side by side, so that a complete set of readings can be made by taking three observations, inclined at angles of 120 deg. In this way the period required for the observations at any place is reduced very considerably, enabling the determination to be made in one night.

An automatic device has also been introduced for recording the reading photographically, and turning the instrument into the required azimuth positions at suitable intervals. The balance can then be set up during the day, and left to make its own observations during the night, so that it may be transported and erected at the next station the following day.

The practical use of the instrument lies in its ability to locate large subterranean masses, which differ considerably in density from their surroundings, and it is understood to be in use by a number of oil companies in various parts of the world for the location of salt domes.

(E.) PRELIMINARY TESTS OF THE INSTRUMENT.

A fundamental assumption made in the theory of the balance is that for a constant gravitational torque, the beam takes up a fixed equilibrium position, in which the torque is balanced by the torsion of the suspending wire. It is, however, well known that for some time after their manufacture, thin torsion wires are subject to elastic "after effects," owing to the gradual recovery of the wires from the twist they sustain in being drawn. The consequence of this is that the equilibrium position of the beam gradually changes even when the remainder of the instrument is fixed.

The first test made therefore was to determine this "shifting of the equilibrium position" from day to day, firstly in the case of a torsion wire used just as it was received from the makers, and, secondly, in the case of a wire which had been heated repeatedly to 100°C., and allowed to cool before being used, this being the treatment employed by Eötvös to render his wires stable. The results of this preliminary test are represented graphically in Fig. 3, which gives the day to day shift of the equilibrium position for two wires *A* and *D*, during the periods indicated. The wire *A* was not subjected to any treatment before use, and was observed for a period of two months, the instrument remaining undisturbed throughout. The wire *D* was specially treated by being slowly heated under a load of 80 gm. to 100°C. in an oven, and gradually cooled, the operation being repeated 50 times during a period of 10 days, after which it was transferred to the balance and its behaviour observed for about 10 days.

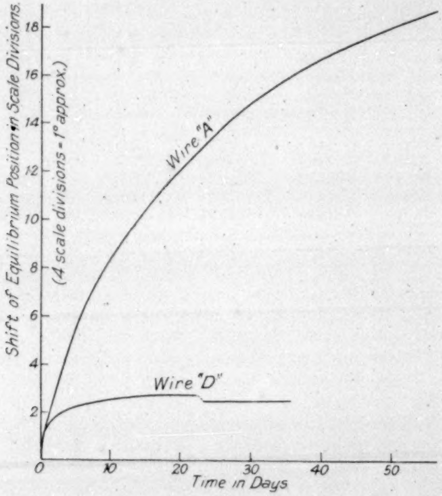

FIG. 3.

From the above test the advantage of treating the wire is obvious and it is apparent that in all cases the wire must be subjected to some previous treatment in order to stabilise it, if results are to be obtainable within a reasonable period of time. The method of treatment here adopted may be regarded as fairly satisfactory, although several days must elapse before the wire reaches a serviceable stage.

SECOND TEST.

The next test was intended to establish the fact that, for a fixed position of the instrument, the equilibrium position of the beam varies in different azimuths, but is constant for any given azimuth. The following tables give the scale readings corresponding to the six azimuth positions, 0°, 60°, 120°, 180°, 240°, 300°, the upper portion of the instrument being rotated through 60° in a clockwise direction after each observation, and allowed to come to rest in its new position.

TABLE I.—*For "D" Wire.*

0°.	60°.	120°.	180°.	240°.	300°.	Date.
21·14	20·17	21·85	22·38	19·91	20·30	May 28 to June 14.
21·13	20·15	21·83	22·35	18·89	20·29	
21·13	20·17	21·88	22·37	19·96	20·34	
21·13	20·15	21·83	22·30	19·89	20·30	
21·13	20·16	21·85	22·35	19·91	20·31	Mean.

TABLE II.—*For "D" Wire.*

0°.	60°.	120°.	180°.	240°.	300°.	Date.
21·50	20·52	22·22	22·74	20·27	20·69	July 12-17.
21·50	20·51	22·24	22·75	20·28	20·69	
21·51	20·51	22·23	22·75	20·28	20·70	
21·50	20·51	22·23	22·75	20·28	20·69	Mean.

The differences evident from a comparison of corresponding columns of these tables, are probably due in the main to the liability of the wire, even after having been stable for several weeks, to sudden "shifts," after which it again takes up a stable position; another contributory factor was the removal of a number of heavy objects from one adjoining room to another during the intervening period.

The results indicated in Table II. agree more closely than those in Table I., and this may be ascribed to the increased age of the wire, the observations being taken a month later. These tables also give an indication of the sensitivity of the balance and the magnitude of the differences between the various deflections, which with a properly stabilised torsion wire enable results to be obtained to within approximately 1 per cent., as claimed by Eötvös.

The instrument on which the tests were made is the property of the Science Museum, South Kensington, the experiments being conducted in the basement of that institution, and our thanks are due to the Director, Col. H. G. Lyons, F.R.S., who has afforded us every facility in this work.

The conditions of the test as regards constancy of temperature were very favourable, but the reverse was true as regards liability to shocks and variation in neighbouring masses. Notwithstanding this, the reading of the instrument remained very stable.

Further experiments are being undertaken to determine the constants of the instrument, and to investigate the variations of the gravitational force within the laboratory in order to gain experience with the instrument under laboratory conditions, before employing it in the field.

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DISCUSSION.

Col. H. G. LYONS said that it was some ten years since, as Director of the Science Museum, he first endeavoured to obtain an example of this instrument, but the execution of his plan had been delayed by the war. The authors had been working in unfavourable circumstances owing to the disturbing effects of alterations which were being carried on at the Museum, so that it was very creditable to have achieved consistent results. It was important to know whether in their experience the instrument is sufficiently shielded from the effect of temperature variations, as it is very desirable that it should be used in field observations. Prof. Soler claims to have obtained results in the field which showed a marked difference in the readings at stations quite close together, the readings being repeated consistently on a subsequent survey. The balance is proving of use commercially, for instance, in connexion with the survey of oil fields in Egypt. The oil companies, unfortunately, appear to have been under the impression that to obtain the requisite calculations it was necessary to send the observations to Germany!

Sir GERALD LENNOX-CUNNINGHAM said he was astonished to see such a sensitive instrument working so satisfactorily in the demonstration given by the authors. It might well have been expected that without a special floor and special means of protection irregular results would be obtained. He inquired as to the damping arrangements adopted.

Dr. C. CHREE said that he first became acquainted with the instrument twenty years ago, when he read a German account of it which struck him as very obscure and difficult to follow. The present treatment in English will be of great value. The fact that it has remained practically unknown in this country for so long seems to show the need of a Geophysical Institute, for at present it is no one's business to keep the scientific world informed of progress in this branch of physics. Col. Lyons has done a public service in securing a trial of the balance, which may have the greatest value industrially.

Mr. J. GUILD referred to the assertion that the balance can measure the difference between the values of g at places a decimetre apart. As the dimensions of the instrument itself are of this order, it is not quite clear how this claim is substantiated.

Mr. F. E. SMITH also commented on the extreme accuracy claimed for the balance which amounts to an error of only one part in a million million. The accurate determination of gravitational acceleration was undoubtedly a difficult problem. He recalled how the value of g was measured by an Antarctic expedition at Melbourne, then at Antarctic stations, and subsequently at Melbourne again. The values at Melbourne were found to be different on the two occasions, and this difference was attributed to a real change in g; but it might have been due to instrumental errors. Is there any record in the Continental literature of a comparable set of observations? He agreed as to the need for a Geophysical Institute. The measurement of gravity had made little progress in this country since the time of Kater.

Dr. E. H. RAYNER said that it was important to consider elastic fatigue of the suspension in an instrument of this kind. In his own experience of electrometers elastic fatigue appeared to be due mainly to the attachment at the two ends of the suspension wire, for a wire giving perfectly consistent results will show fatigue if owing to any accident it has to be resoldered. It would be valuable if authors in this and any accurate work involving fine torsion wires could record the effect of altering the fixing of the wire; such differences as those shown in Fig. 3 of the Paper might be explicable on these lines. Tungsten wires can now be obtained, and it might be an improvement to substitute them for the platinum-iridium wires used.

Mr. L. F. RICHARDSON recalled that tungsten wire, at least that available in 1914, showed a tendency to break up into irregular strands.

Capt. H. SHAW gave an account of some improved forms of the balance. The improvements have been mainly directed to the elimination of errors due to asymmetry of the instrument and to enabling observations to be taken more rapidly. A double instrument is used so that the six readings can be taken in the time previously required for three, and a photographic recorder makes it possible to determine the static deflection with greater certainty. With these improvements it has been found possible in Germany to deal with three stations a day.

Mr. E. LANCASTER-JONES, replying to the discussion, said that the damping could be regulated by raising or lowering the bottom plate of the balance casing and so varying the air space between it and the beam; but it was found impracticable to bring the beam to rest in less than 1½ hours. Temperature effects are not troublesome, a change of a few degrees producing results small compared with those due to the g-gradient. Whereas five observations completely determine the gradient, it is usual to take six so that a check may be obtained, and the results are consistent to 1 per cent. Field observations are taken at night to avoid the sun's heat, and no more protection is required than that afforded by a canvas tent. As regards the claim to accuracy, what is deduced is the gradient of g, assumed to be uniform in a given locality, and this gradient multiplied by a decimetre gives a quantity greater than the probable error. What is actually measured, however, is the integral of the effect of the gradient over a length of about 60 cm.

XVI. *The Crystalline Structure of Anthracene.* *By* SIR W. H. BRAGG, *F.R.S.*

RECEIVED DECEMBER 21, 1922.

ABSTRACT.

The author in an address recently published* put forward evidence for regarding the benzene ring as an actual structure of ascertainable size and form, and deduced that the unit cells of naphthalene and anthracene should have two of their axes equal, the third axis being longer for anthracene than for naphthalene by the width of one ring. The experimental data then available supported this hypothesis only roughly; in the present Paper it is shown that more reliable data, subsequently obtained from small but perfect anthracene crystals, agree with it very closely. It is pointed out that the X-ray data furnish a new and accurate method of determining the density of a crystal.

IN an Address which as President I gave to this Society on November 11th, 1921, I described certain experiments on the structure of organic crystals. Among these were measurements on the dimensions of the crystal units of naphthalene and anthracene. The comparison of the two sets of results was used to show that the molecule in each case lay with its longest dimension in the direction of the *c* axis. The anthracene data were not entirely satisfactory, since only one X-ray measurement could be made, and that not very accurately: this was the spacing of the cleavage plane 001. The dimensions of the unit were calculated also from the crystallographic data given by Groth and from a determination of the specific gravity quoted in Kaye and Laby's tables. The results were, however, sufficiently accurate to show that the major length of the molecule lay along the *c* axis, because the difference between the lengths of the *c* axes of naphthalene and anthracene was 2·9, while the width of the benzene ring as found in the diamond was 2·5. It was assumed that the molecule of anthracene might be expected to be longer than the molecule of naphthalene by the width of the extra ring, and that the small difference between the two values did not affect the argument.

Recently, through the kindness of Dr. Brady, I have had the opportunity of measuring the anthracene constants, using some minute but perfect crystals which he gave me. The results are set out below in a form which allows comparison with the naphthalene constants. It will be seen that there is now much better agreement between results as calculated and as expected. The *c* axis of anthracene is really 11·18, not 11·6 as calculated from the data previously available. Using, therefore, the X-ray data alone, we find that the difference between the *c* axes of the two crystals is 2·5 exactly, which was the figure anticipated.

The quite considerable difference between the new and the old results is due for the most part to the value previously adopted for the specific gravity. Now that the constants of the crystal unit are more accurately known, it is possible to calculate the specific gravity of a perfect crystal. It is to be expected that this will always be higher than the specific gravity found by ordinary means. It is, in fact, usual for crystallographers to make several measurements of specific gravity and to accept the highest as the most correct (Tutton, "Crystallography," new edition, page 625). The value of the specific gravity is now shown to be 1·255.

There is remarkable similarity between the naphthalene and anthracene observations. Not only are the crystals isomorphous, but the same planes are in each case the best reflectors, and the spacings of the 100 and the 010 are almost exactly the same; it is only in the length of the *c* axis that there is any marked difference. The cell contains two molecules in each case, while the symmetry number is 4, that is to say, if the molecule were unsymmetrical it would be necessary to employ four of them to make up a cell having the symmetry of the monoclinic prismatic class to which the crystals belong. As two molecules are able to give fourfold symmetry, each must have a twofold symmetry. The nature of that symmetry is indicated by the results. In both cases the 010 plane is exactly halved. Also in the 010 zone, that is in the case of the planes passing through the *b* axis, 100 and $10\bar{1}$ are halved, 001 and $20\bar{1}$ are not halved. These results show that the crystal molecule possesses a centre of symmetry, and that if one set of molecules all alike to each other be placed at the corners of the cell, the centres of the other molecules lie at the centres of the *ab* face.

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* Proc. Phys. Soc., Vol. 34, Dec. (1921).

ANTHRACENE.

Monoclinic prismatic. Two molecules in the crystal unit. Assume:—

$$a=8{\cdot}58\,;\ b=6{\cdot}02\,;\ c=11{\cdot}18\,;\ \beta=125°0'$$

	Assumed.	*Observed.*	
100	7·02	3·51	Strong
010	6·02	3·01	Moderate
001	9·16	9·16	Strong. Also 2nd & 4th orders, 3rd order weak.

	Calculated.	*Observed.*	
$10\bar{1}$	8·34	4·16	
$20\bar{1}$	4·17	4·16	Moderate
110	4·58	4·57	Very strong
$11\bar{1}$	4·88	4·90	Weak
$21\bar{1}$	3·43	3·43	Strong
210	3·04	3·04	Strong

Axial ratios in accordance with the above are

$$a:b:c=1{\cdot}423:1:1{\cdot}857,\ \beta=125°0'$$

Groth gives (Chemische Krystallographie V., p. 437):

$$a:b:c=1{\cdot}4220:1:1{\cdot}8781,\ \beta=124°24'$$

Specific gravity (calculated from the X-ray data) = 1·255
Specific gravity (redetermined by using Dr. Brady's crystal) = 1·250.

NAPHTHALENE.

(*See* Structure of Organic Crystals, Proc. Phys. Soc., 34, December 15, 1921.)
Monoclinic prismatic. Two molecules in the unit cell.

$$a=8{\cdot}34\,;\ b=5{\cdot}98\,;\ c=8{\cdot}68\,;\ \beta=122°44'$$

These values are in better agreement with the experimental results described in the Paper referred to than the values calculated from the crystallographic data; the differences are small, however.

DISCUSSION.

Dr. D. **Owen** inquired whether the measurements show identical dimensions for all the unit cells of a given substance, whatever the specimen taken ; or whether the results vary with the state of strain of a crystal, which might be expected to affect its density.

Dr. E. A. **Owen** said that metallurgists would agree with the remark made by the author in presenting his Paper, that X-ray measurements were more dependable than direct measurements of the density of a crystal. In alloys particularly the densities of the component crystals would be difficult to find.

Mr. J. **Guild** said he was interested in the statement that by X-ray measurements it was possible to determine the density of a crystalline substance despite the presence of impurities which would vitiate ordinary density measurements. Would it be possible in a matrix of two or more crystalline substances to determine the density of each ? For example, could the density of the cementite in a pearlitic steel be determined ?

Dr. H. **Borns** said that in neutralising H_2SO_4 with lime in organic work chemists often obtain fine crystals of various shapes which after all turn out to be impure $CaSO_4$. Have such and similar crystals been examined as to the identity of their unit cells ?

Dr. F. L. **Hopwood** said that the structure of mixed crystals has been investigated by Vegard,* who showed that the lattice of Thorite (Th SiO_4), which belongs to the Zircon group, was completely broken down, only the outer form being preserved.

Author's reply : Such evidence as is available supports the idea that the size and form of the crystal unit cell is always the same to a high degree of accuracy, except, of course, that strain or expansion of the crystal is shared by the cell. Each kind of crystal in a matrix gives its own spectrum quite independently. For instance, if cementite crystals occurred in sufficient quantity in a powdered mass the Debye-Hull photograph would show the spectrum of cementite with others due to other crystals. If the cementite lines could be disentangled and interpreted, and if the chemical composition is known, the density is found.

The author is not aware of any systematic measurement of $CaSO_4$ crystals, but does not doubt that they all have the same unit cell no matter what the outside form may be.

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* Phil. Mag. (1916).

XVII. *On the Frequency of Vibration of Circular Diaphragms.* *By* J. H. POWELL, *M.Sc., F.Inst.P., and* J. H. T. ROBERTS, *D.Sc.*

RECEIVED JANUARY 24, 1923.
(COMMUNICATED BY SIR ERNEST RUTHERFORD F.R.S.)

ABSTRACT.

The Paper describes measurements of the natural frequency of diaphragms of various sizes having a rigid rim and a central boss for attachment of a microphone or receiver. In air the frequency was found by means of a monochord, but under water the resonance-frequency was noted in the neighbourhood of a subaqueous transmitter of variable pitch. The results are in good agreement with the theoretical conclusions of Prof. H. Lamb.* In many cases harmonics were observed round about a fifth or an octave above the fundamental, but their occurrence was capricious, and their pitch inconsistent with theory. The resonance peaks of the frequency curves are more or less of the same area, being high and narrow or low and broad.

The effect of increasing the pressure on one side of the diaphragm was studied, and the pressure-displacement curve was found to be linear up to the elastic limit, while the pressure-frequency curve is of the saturation type. A large diaphragm is less affected by pressure (and therefore by immersion in deep water) than a small one of the same natural frequency, in consequence of its greater thickness.

PART I.

THE object of the research described in the following Paper has been to make an experimental investigation of the theory of vibration of circular diaphragms and also to study the vibrations of such diaphragms under various conditions.

In a recent publication,* Prof. H. Lamb has given an account of a mathematical investigation of the frequency of vibration of thin circular diaphragms rigidly clamped at their circumference; he has considered the vibration of these diaphragms under various conditions in air and water and has deduced certain expressions by means of which the frequency of a diaphragm of any given material can be readily calculated.

The diaphragms used in our experiments were turned out of solid ingots of the metal (steel or bronze) with a rim ½ in. broad round the circumference, and with a small boss at the centre for the attachment of a detector.

It will be seen therefore that the conditions are only approximately those assumed in Prof. Lamb's calculations, but it was shown that variations in the rigidity of the clamping of diaphragms with a rim of the dimensions used did not produce a variation of more than ¼ per cent. in the frequency. The boss at the centre of the diaphragm also produced a variation in the frequency for which a correction had to be applied before the results could be compared. For ease in calculation of this correction the boss was kept of the same size in all the diaphragms considered, with the exception of the 6½ in. diameter diaphragms, which were of a special type. The essential parts of this type of diaphragm are shown to scale in Fig. 1, and it will be observed that it was not necessary to attach this diaphragm to any holder while making the observations, as it was itself sufficiently massive. The smaller diaphragms were, for testing purposes, clamped down rigidly to a massive steel holder, illustrated in Fig. 2.

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* Proc. Roy. Soc., A., Vol. 98, p. 205 (1920).

Experimental Methods.

A very accurate measurement of the frequency of vibration of a diaphragm in air was possible by simply striking the diaphragm with a soft rubber hammer and tuning a monochord to the same pitch. In order to test the accuracy of this method, the diaphragms were excited electrically by a small electro-magnet, through which an alternating current of controllable frequency was passed.

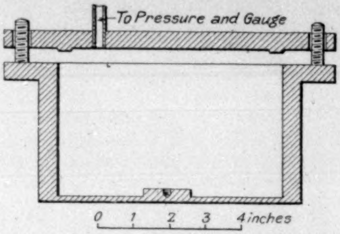
To Pressure and Gauge
0 1 2 3 4 inches

Fig. 1.—Standard 6½in. Diaphragm with Cover Plate. (Cover raised slightly.)

It was found that in every case these two methods gave values for the frequency which did not differ by more than 0·5 per cent. in the most extreme case.

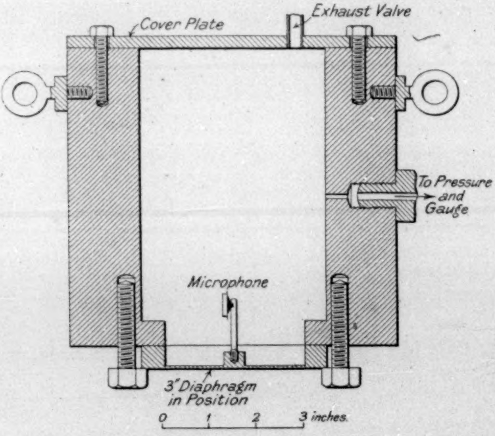
Cover Plate
Exhaust Valve
To Pressure and Gauge
Microphone
3" Diaphragm in Position
0 1 2 3 inches.

Fig. 2.—Diaphragm Holder Fitted for Pressure Experiments.

In water the method was not so simple, for the damping produced by the water made it much more difficult to estimate the pitch of the note emitted by the diaphragm. The method adopted was therefore to lower the diaphragm in its holder into water, in a tank constructed for the purpose, so that the diaphragm was immersed with one side in contact with the water. It was then excited by the vibrations transmitted through the water from a telephone sounder also immersed in the tank. It was found that the resonance frequency of a diaphragm in water was independent of the depth of immersion provided the depth was not less than a few millimetres, and provided the depth was not great enough for the pressure of the water to produce an appreciable distortion of the diaphragm. As the total depth of the water in the tank was about two feet, this difficulty did not arise, and for the purposes of these experiments measurements were always taken at a depth of from 6 in. to 10 in.

The vibrations of the diaphragm were examined by the use of a Brown microphone or telephone attached to the boss at the centre of the diaphragm, as shown in Fig. 2. The microphone acts as a small load on the diaphragm, but the telephone, the spring reed only being attached to the diaphragm, was found not to act as a load on the latter.

A simple Dolezelak[*] alternator was used to excite the telephone sounder in the tank, and by measuring the speed of rotation an accuracy of 0·1 per cent. was obtainable in the frequency measurements.

In all cases where microphones or telephone receivers or sounders were used, resonance points always occurred due to their natural frequencies, which were superposed on those of the diaphragm. These, however, were eliminated by comparing the resonance points using different sounders and detectors with the same diaphragm, when the diaphragm resonance points were easily identified.

In Air.—Prof. Lamb has shown that for a rigidly clamped uniform circular diaphragm of any given material vibrating in air the frequency (n) is given by the expression

$$n=\frac{p}{2\pi}=0{\cdot}4745\ \frac{hc_1}{a^2}$$

where h=thickness of the diaphragm, a=radius of the diaphragm, and where c is the velocity of propagation of extensional waves in an infinite thin plate of the same material and thickness and is equal to

$$\sqrt{\frac{E}{(1-\sigma^2)\rho_1}}$$

For an iron plate $E=2\times10^{12}$, $\rho_1=7{\cdot}8$, $\sigma=0{\cdot}28$ c.g.s. units, and we have $c_1=5{\cdot}27\times10^5$ cm. per sec.

Thence
$$\frac{p}{2\pi}=2{\cdot}50\ \frac{h}{a^2}\times10^5 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

In Water.—The effect of water is virtually to increase the inertia in the ratio $(1+\beta)$, where $\beta=0{\cdot}6689\ \frac{\rho}{\rho_1}\cdot\frac{a}{h}$, ρ being the density of water.

The frequency of a diaphragm with water on one side only, becomes therefore

$$\frac{p}{2\pi}=\frac{0{\cdot}4745}{\sqrt{1+\beta}}\ \frac{hc_1}{a^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

With water on both sides, the value of β is doubled.

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[*] F. Dolezelak Zeits. Instrumentenk., XXIII., 240 (1903).

CORRECTION FOR LOAD.

If a small load on is added to the centre of a diaphragm its effect is to increase the kinetic energy of the plate. In the Paper previously referred to, Lamb gives the kinetic energy of an unloaded plate as being equal to that of a mass $M/5$, concentrated at the centre, where M is the total mass of the diaphragm. Consequently a small additional mass m at the centre is equivalent to a uniformly distributed load of $5m$. The kinetic energy is therefore increased in the ratio $\left(1+\frac{5m}{M}\right)$.

The frequency of a loaded diaphragm in air is therefore

$$\frac{p}{2\pi}=\frac{0{\cdot}4745}{\sqrt{1+\frac{5m}{M}}}\cdot\frac{hc_1}{a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and with water on one side

$$\frac{p}{2\pi}=\frac{0{\cdot}4745}{\sqrt{1+\beta+\frac{5m}{M}}}\cdot\frac{hc_1}{a^2}, \text{ where } m \text{ is small} \quad . \quad . \quad (4)$$

Expressing $(1+\beta)$ as a load on the diaphragm it follows that the quantity of water moving with the diaphragm is equal in volume to a hemisphere whose diametrical plane is the surface of the diaphragm.

The experimental conditions therefore may be assumed not to differ materially from the theoretical in which an infinite medium is postulated.

EXPERIMENTAL RESULTS.

1. *Diaphragms in Air.*—The results obtained by the method already described may be summarised in the following table :—

TABLE I.—*Steel Diaphragms.*

Diameter of plate in inches.	Thickness of plate in mm.	Observed values.	Calculated values.		
		Frequency loaded in air. (Measured.)	Frequency unloaded in air. (Calculated.)	Frequency loaded.	Load in grams
3	1·0	1,416 ~	1,722 ~	1,420 ~	4·3
3	1·5*	2,520	2,580	2,580	Zero.
4	1·5	1,350	1,450	1,330	4·3
4	2·0	1,810	1,940	1,790	4·3
6½	3·0	953	1,100	942	36·3
6½	4·0	1,330	1,475	1,310	36·3
6½	4·5	1,500	1,650	1,500	40·0
6½	6·0	1,922	2,195	1,895	67·0
7	5·0	1,464	1,585	1,450	36·3
7	6·0	1,730	1,900	1,766	36·3
7	6·6	1,902	2,095	1,958	36·3

* This diaphragm was soldered to its rim and was not turned out of a solid ingot.

A large number of other diaphragms were also examined, including many of intermediate size—but the results were in general within the limits of accuracy indicated in the typical results quoted in the above table.

Irregularities were sometimes found in individual diaphragms, but these were undoubtedly due to alteration in the elastic constants of the material due to stresses set up during the turning process.

(2) *Diaphragms with One Side in Water and the Other Side in Air.*

A similar series of measurements was made with the same diaphragms supported in their holders below the surface of the water in the tank. The following table gives the results obtained for typical diaphragms :—

TABLE II.

Diameter of Plate in inches.	"*a*" Radius in centimetres.	Thickness of Plate in millimetres.	Frequency with water on one side.			Load in grams.
			Observed.	Calculated.		
			Loaded.	Unloaded.	Loaded.	
3	3·815	1·0	788	825	770	4·3
3		1·5	1,348	1,448	1,358	4·3
4	5·08	1·5	...	675	658	4·3
4		2·0	956	1,018	993	4·3
6½	8·25	3·0	535	602	543	67·4
6½		4·0	910	884	840	36·3
6½		4·5	1,000	1,028	983	36·3
6½		6·0	1,545	1,488	1,425	36·3
7	8·9	5·0	1,066	995	961	36·3
7		6·0	1,250	1,263	1,222	36·3
7		6·6	1,450	1,430	1,383	36·3

It will be observed from the preceding tables that a close agreement exists between the values of the frequencies of diaphragms calculated by Prof. Lamb and those actually found by direct measurement, and for diaphragms vibrating in air the expressions given can be taken as giving the true frequency within 2 per cent. In the case of diaphragms vibrating with one side immersed, a similar agreement between theory and experiment was observed particularly with the smaller type of diaphragm.

With the thicker 6½ in. and 7 in. types, however, it was found that the observed frequencies were all considerably higher than was anticipated from the theory. A closer examination of the frequency of a large number of 6½ in. diaphragms was therefore carried out with the results plotted in Fig. 3. Curves *A* and *B* represent the theoretical variation with thickness of the frequencies in air unloaded and with 56 grs. load. *C* shows the variation in water with the same load.

The values of the frequencies actually found in water are plotted and lie on a line corresponding more nearly to a value of

$$\beta = 0{\cdot}546\ \frac{\rho}{\rho_1}\ .\ \frac{a}{h}\ \text{rather than}\ \beta = 0{\cdot}669,\ \frac{\rho}{\rho_1}\ .\ \frac{a}{h}$$

The thickness of each diaphragm was measured with a large micrometer to the nearest $\frac{1}{10}$ millimetre, but by calculating indirectly from the frequency in air it was possible to estimate the thickness to 0·01 mm. This method, however, was only used when it was required to produce a diaphragm accurately tuned to a particular frequedcy.

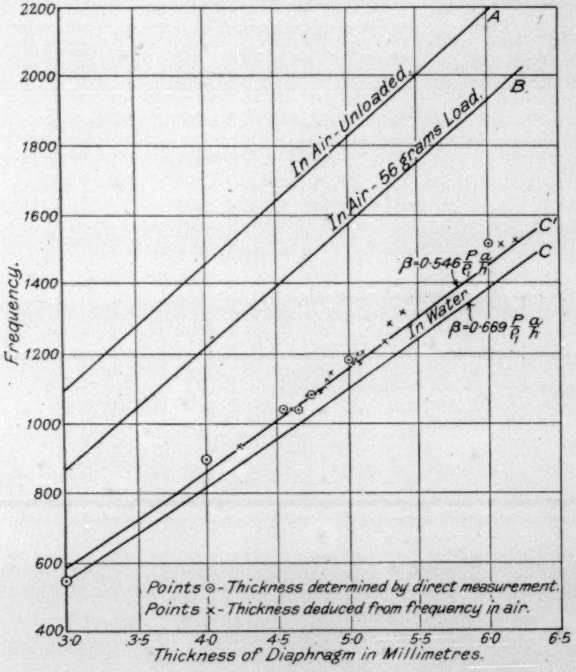

FIG. 3.—FREQUENCY OF 6½IN. DIAPHRAGMS IN AIR AND WATER.

EFFECT OF LOAD.

The effect of adding a load at the centre of a diaphragm was fully examined and in this case the value for the increase in the inertia was found to agree remarkably closely with that found experimentally. Fig. 4 illustrates the effect of applying additional loads up to 113 grams at the centre of a 6½ in. diaphragm whose normal load is 67 grams. The loads consisted of flat discs of lead of diameter equal to that of the boss and screwed tightly down upon it. The values of the frequency anticipated from theory are plotted as a continuous curve while the experimental values are given as points—and the close agreement between theory and experiment in this case is at once apparent.

Higher Harmonics.

Attempts were made to observe the more complicated modes of vibration of the diaphragms, but owing to very considerable variations in the behaviour of individual diaphragms it was impossible to formulate a definite rule. However, as a result of a great number of observations of many diaphragms, a first harmonic frequently occurs in water at a frequency 1½ times that of the fundamental, and a second harmonic at about double the frequency. These frequencies do not fit in with Prof. Lamb's calculations either for vibrations with a nodal circle or nodal diameter, which give much higher values, but as the occurrence of upper harmonics was very capricious, an agreement was hardly to be expected, particularly when a relatively heavy boss was attached to the centre of each diaphragm and must have exerted considerable control on its mode of vibration.

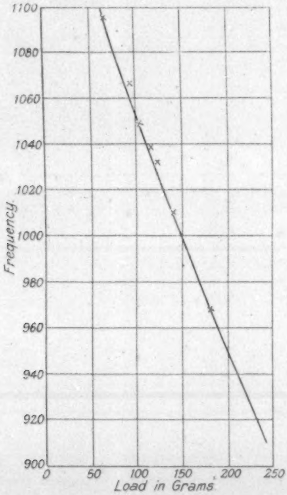

Fig. 4.—Effect of Load at Centre of 6½in. Diaphragm.

PART II.

ON THE EFFECT OF PRESSURE ON THE VIBRATION OF DIAPHRAGMS.

The object of the present research has been to determine to what extent the frequency and general behaviour of a diaphragm as a receiver of sounds in water is governed by its depth of immersion, in other words, by the external pressure applied.

The diaphragms examined were those described in detail in the first part of this Paper, and for testing purposes were clamped into their massive steel holder—which was covered by a thick steel plate and compressed air admitted, the pressure being read by means of a gauge as indicated in the diagram Fig. 2. The large 6½ in. diaphragms of the special massive type already described were also tested and were simply arranged for the application of pressure by screwing on the stout cover plate forming part of the diaphragm unit.

The method of investigating the frequency of the diaphragms was precisely that described in Part I, the apparatus being lowered below the surface of the water in the tank and excited by the sounder of controllable frequency exactly as before.

DISTORTION OF DIAPHRAGMS UNDER STATIC PRESSURE.

The bulging of the diaphragms under static pressure was studied by means of an index attached to the central point, the motion of which was observed through a reading microscope.

The displacement of the centre of different diaphragms at various pressures is shown in the following table:—

Diaph.	Zero Pressure.	5 lb.	20 lb.	35 lb.	50 lb.
Steel—					
3 in. 1·0 mm. ...	0	0·083 mm.	0·257 mm.	0·431 mm.	0·647 mm.
3 ,, 1·2 ,, ...	0	0·058 ,,	0·117 ,,	0·175 ,,	Distorted
4 ,, 1·0 ,, ...	0	0·20 ,,	0·83 ,,	1·35 ,,	1·80 mm.
4 ,, 1·2 ,, ...	0	0·13 ,,	0·46 ,,	0·74 ,,	Distorted
4 ,, 1·5 ,, ...	0	0·07 ,,	0·25 ,,	0·45 ,,	Distorted
4 ,, 2·0 ,, ...	0	0·041 ,,	0·125 ,,	0·225 ,,	0·325 mm.
Bronze—					
3 in. 0·8 mm. ...	0	0·125 mm.	0·515 mm.	0·734 mm.	Distorted
3 ,, 1·4 ,, ...	0	0·66 ,,	0·24 ,,	0·393 ,,	0·556 mm.
4 ,, 3·0 ,, ...	0	0·033 ,,	0·108 ,,	0·183 ,,	0·257 ,,

Some of these results are shown plotted in Fig. 5, from which it will be seen that the displacement is proportional to the pressure, up to a certain point, beyond which the displacement becomes less rapid. It was found that at this pressure the diaphragm had begun to receive a "permanent set" and its resonance frequencies were permanently raised; its response to its resonance frequencies beyond this point was always found to be very much impaired. Provided a diaphragm is not strained beyond this critical point, it can be repeatedly subjected to pressure, and the displacement is always proportional to the pressure.

This is in accordance with the result given by Love,[*] who deduces the expression :—

$$\text{Displacement of centre of diaphragm } w=\frac{1}{64}\frac{Pa^4}{D}$$

where

$$D=\frac{2}{3}\frac{Eh^3}{(1-\sigma^2)}$$

h being in this case equal to half the thickness, and P the applied pressure.

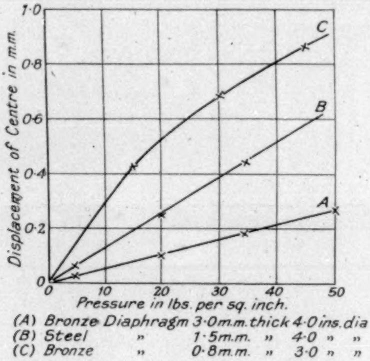

FIG. 5.

In the case of a steel diaphragm

$$w=6{\cdot}08\times10^{-9}\frac{a^4}{h^3}P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where P is expressed in lbs. per square inch.

For 4 in. 1·5 mm. steel diaphragm, w has the value 0·0238 for a pressure of 20 lb. per square inch. The value obtained experimentally in this case was 0·025 cm.

THE EFFECT OF PRESSURE ON FREQUENCY.

The curvature of the diaphragm, due to the pressure, causes the natural frequencies of the diaphragm to be raised in all cases, both in air and in water. The percentage rise in frequency due to pressure is approximately the same in air as in water, and the aproximate equality of the figures is shown in the following table :—

---

[*] Love, Mathematical Theory of Elasticity, 3rd Edition, equation (83), p. 490.

Diaphragm.	Fundamental frequency.	Percentage rise at		
		10 lb. pressure.	30 lb. pressure.	40 lb. pressure.
Steel 4 in..........	Air 1,048 ~	1·7%	10·0%	14·3%
1·0 mm.	Water 481	2·1%	10·4%	13·6%
Steel 4 in..........	Air 1,212	1·5%	5·05%	8·8%
1·2 mm.	Water 602	0·5%	5·70%	9·9%
Steel 4 in..........	Air 1,304	To small to measure.	1·20%	1·7%
1·5 mm.	Water 892*		2·70%	3·9%
Bronze	Air 1,120	4·6%	15·4%	Distorted.
3 in. 1·0 mm. ...	Water 478	4·9%	15·7%	Distorted.

* In this case the 1st harmonic was observed.

The figures for the fundamental frequency given in the second column of the above table are the actual experimental figures. The effect of load is allowed for in calculating the percentage rise.

The percentage rise of frequency for diaphragms of different dimensions is shown in the following table ; in this table the diaphragms are all made of steel :—

Diaphragm.	Rise in Frequency.				$\frac{a^4}{h^4}$	Anticipated rise in frequency in terms of 4 in. 1 mm. diaph.
	30 lb. pressure.		40 lb. pressure.			
	Percentage.		Percentage.			
	Actual.	In terms of that of 4 in. 1 mm. diaph.	Actual.	In terms of that of 4 in. 1 mm. diaph.		
3 in. 1·0 mm. ...	3·47%	33·3	7·3%	31·8	$2{\cdot}10\times10^6$	31·8
3 in. 1·2 mm. ...	1·34%	12·9	2·8%	12·8	$1{\cdot}00\times10^6$	15·2
4 in. 1·0 mm. ...	10·4%	100·0	Distorted		$6{\cdot}63\times10^6$	100·0
4 in. 1·2 mm. ...	5·05	48·0	8·8%	38·4*	$3{\cdot}20\times10^6$	48·3*
4 in. 1·5 mm. ...	2·0%	19·2	3·0%	13·1	$1{\cdot}31\times10^6$	19·8
6·5 3·2 mm. ...	0·64%	6·2	1·59%	6·9	$0{\cdot}443\times10^6$	6·7

* The discrepancy here was due to the fact that the diaphragm was just beginning to be permanently distorted.

The theory of the vibration of a circular diaphragm when, under flexure is very involved and has not been worked out, but it is important to find some simple relation to co-relate the effects of pressure in different diaphragms, as found by experiment.

The simplest relation is :—

$$\text{Percentage rise in frequency} = A\frac{a^4}{h^4}P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

where A and h are expressed in centimetres and P in lbs. per square inch, A being an empirical constant depending on the material. For steel, with these units, A has a mean value deduced from a large number of trials of $6{\cdot}0\times10^{-8}$.

A few further experiments were carried out using bronze diaphragms and, as would be anticipated, a rise in frequency occurred which was much greater than with steel diaphragms, being 10 to 12 times as large for corresponding diaphragms of the same dimensions.

These relations are shown by the preceding table, two columns of proportional values being included for comparison. It will be observed that they do not hold when the diaphragm is strained beyond its elastic limit which is reached when the rise in frequency reaches about 10-12 per cent.; this figure seems fairly uniform for steel diaphragms of all dimensions.

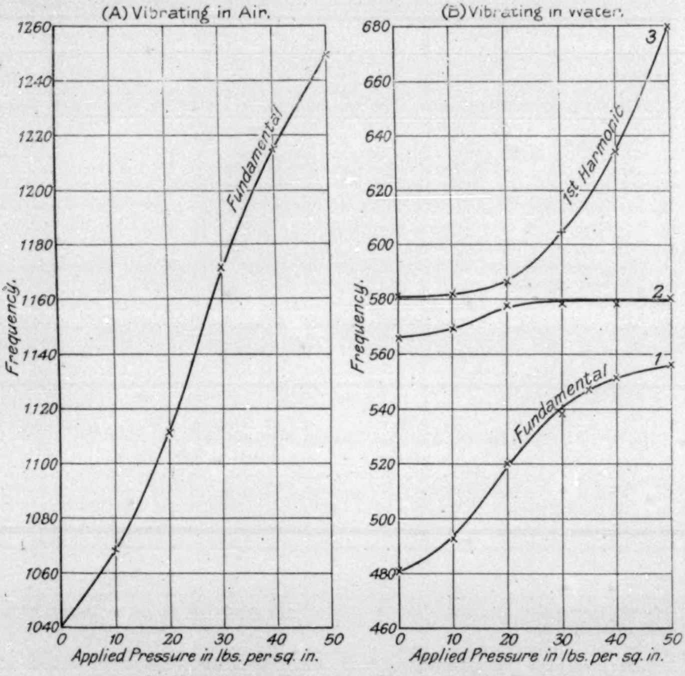

FIG. 6.—VARIATION OF RESONANCE FREQUENCY WITH PRESSURE.
Steel Diaphragm 1·0 mm. thick, 4·0 inches diameter.

General Observations.

In all cases the frequencies of the resonance points of a diaphragm are raised by increasing the pressure on one side of the diaphragm. (*See* typical curves, Fig. 6.)

The percentage variation in frequency with increase of pressure is the same in air as in water for the same diaphragm, and for different diaphragms, other things being equal, it is less the thicker the diaphragm.

The displacement of the centre of the diaphragm under uniform pressure is proportional to the pressure up to the point where the diaphragm begins to acquire a permanent " set" when the rate of displacement with increase of pressure begins to fall off.

When a resonance becomes diminished in loudness, it increases in area or extent, and when it increases in loudness it diminishes in extent ; *i.e.*, becomes sharper, as though the sum of energy in the resonance tended to remain constant. In some cases the energy appears to distribute itself between different resonance points, one sound-maximum increasing in loudness at the same time that another is diminishing.

In addition to the fundamental and harmonics, smaller maxima were frequently present initially. These were not easily followed, as they behaved irregularly. They were no doubt due to local strains, &c., set up when the diaphragms were being turned on the lathe. They usually diminished or disappeared on the application of pressure. Curve 2, Fig. 6 (B), is an example of such a maximum. As the pressure was still further increased, this maximum became very faint and, at the same time the fundamental and first harmonic increased in loudness.

With the fundamental of all the diaphragms which have been examined, the curve showing the rise of frequency with pressure resembles a saturation curve, the position of the resonance point rising much more slowly with pressure beyond a certain critical pressure.

With first harmonics and others, the initial portions of the curve are similar to the initial portions of the curves for fundamentals, but the corresponding features occur at higher pressures. Curve 3, Fig. 6 (B), represents a typical first harmonic.

At the pressure at which the curve begins to turn to the horizontal, the diaphragm begins to acquire a permanent " set," and the amplitude of vibration diminishes.

Since the percentage rise in frequency for a given pressure and material varies as $\frac{a^4}{h^4}$ and the fundamental frequency varies as $\frac{h}{a^2}$, it follows that for diaphragms of the same material and different dimensions, of a given frequency (*i.e.*, $\frac{h}{a^2}$=constant), the percentage rise in frequency for a given pressure varies as $\frac{a^2}{h^3}$.

From this it is seen that a large (and thick) diaphragm would be less affected by pressure than a small (and thin) one of the same material and frequency. For example, a steel diaphragm of fundamental frequency 481 and dimensions 4 in. by 1 mm., rises in frequency at a pressure of 30 lb. per square inch, by 10·4 per cent., whereas a steel diaphragm of fundamental frequency 535 and dimensions 6·5 in. by 3·2 mm., rises in frequency at the same pressure by only 0·64 per cent.

These results are now published by permission of the Admiralty. The experiments were carried out for Naval purposes during the war under the close direction of Sir Ernest Rutherford, F.R.S., to whom the authors have great pleasure in acknowledging their indebtedness.

DISCUSSION.

Dr. W. S. Tucker inquired whether double resonance had been observed. He had found that it was hard to avoid this phenomenon in the neighbourhood of air cavities, which form a coupled system with the diaphragm.

Mr. J. H. Powell replied that double resonance had been observed, and that it accounted, for instance, for the form of curve 2 in Fig. 6 of the Paper. In general, however, it was avoided by suppressing air cavities.

Dr. F. Ll. Hopwood said that much work has been done on multiple resonance, the results being analogous to those found for coupled electrical circuits. The resonance points can be varied at will by altering the coupling. No satisfactory theory has been given for overtones produced under water as it has not been found possible to deal analytically with the flow of water from one segment to another vibrating in opposite phase. It is doubtful whether the breadth of the resonance peaks has much significance, since the measurements of amplitude are scarcely reliable although the frequencies are accurately known.

Dr. A. O. Rankine questioned the propriety of calling the overtones "harmonics." In modern music such relations may be of small importance, but for scientific purposes a distinction should be drawn between a perfect octave, for instance, and a mere overtone such as an approximate fifth or octave.

Prof. C. L. Fortescue suggested that the multiple resonance was due neither to harmonics nor to non-harmonic overtones, but to the coupling of two elastic systems, not necessarily including the air cavities mentioned by Dr. Tucker. The structure supporting the diaphragm would form one such system, and its being mechanically coupled to the diaphragm would perhaps account for what the authors had described as the fundamental and first harmonic in Fig. 6.

Dr. Chree, in a written communication, pointed out that the formula $p/2\pi = 0{\cdot}4745hc_1/a^2$, ascribed to Prof. Lamb, is only an approximation—though a good one—the exact value of the numerical factor being 0·4693, as stated by Prof. Lamb himself. He would have expected the more exact formula to be used. He also pointed out that "iron" being so variable a substance, it would be desirable for high accuracy to determine c_1^2 directly for the actual material employed.

Reply to the Discussion by Dr. J. H. T. Roberts: Double resonance was in some cases observed, but special efforts were always made to avoid air-bubbles, as it was found that the presence of even a minute air-bubble seriously interfered with the sensitivity of the diaphragm for the reception of sound from water. The behaviour of the resonances was also found to be capricious when there were minute bubbles adhering to the system.

The theory of the mode of vibration of a diaphragm under the conditions necessary for the production of overtones is very complicated, and it is possible that the apparent overtones observed (or at any rate, some of them) might be due to a mechanically coupled system, as mentioned by Prof. Fortescue. But it is unlikely that this is the whole explanation, as not only was this possibility foreseen, but cases were observed in which this coupled action was definitely taking place, and in consequence steps were taken to prevent it.

XVIII. *A Radio-Acoustic Method of Locating Positions at Sea**: *Application to Navigation and to Hydrographical Survey.*

By A. B. WOOD, *D.Sc., F.Inst.P.,* *and Captain* H. E. BROWNE, *O.B.E., R.N*

RECEIVED FEBRUARY 9, 1923.

ABSTRACT.

A description is given of a series of experiments carried out to test the possibilities of the Radio-Acoustic method as a means of locating positions at sea. In this method, as applied to navigation, for example, the ship requiring her position makes a W/T "dash" at the same instant as a small charge is fired in the sea. A station on shore records the arrival of the W/T signal and also of the explosion wave at a number of hydrophones suitably disposed in known positions on the sea-bed. The times of travel of the explosion wave, and hence the distances from the charge to each hydrophone, are indicated by an Einthoven galvanometer photographic recorder.

The method permits of great accuracy and has important applications in navigation and hydrographical survey. For navigational purposes great accuracy is sacrificed to speed—it being possible to give a ship her position within a radius of half a mile within 10 minutes of receiving her request for a location. A 9 oz. charge of explosive can be located at 40 miles. In hydrographical survey work the method has already been used successfully in fixing accurately the positions of certain buoys and light vessels.

The possibility of screening and distortion effects produced by sandbanks has been investigated.

The radio-acoustic method has been thoroughly tested under working conditions. It has proved accurate and reliable by day or night in rough or in foggy weather and at all seasons of the year. Many locations have been given by the method and no failures have been experienced.

The R/A system should be regarded as a serious competitor to the method of location by directional wireless.

I. INTRODUCTION.

THE principle of measurement of distance by means of two waves propagated with different velocities has long been known; but it is not until recently that the method has received practical application in the determination of positons at sea. Joly[†] has described the method in some detail, and has proposed the use of various forms of it for navigational purposes. The underlying principle of the method is briefly as follows: The ship to be located emits simultaneous signals having different velocities of propagation. These signals are reecived in a suitable manner at a fixed station, the time interval "t" between the arrival of the two signals being measured.

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[*] Work carried out at Admiralty Station, St. Margaret's Bay, Dover, under the joint direction of the Director of Torpedoes and Mining and the Director of Scientific Research, Admiralty. The authors desire to acknowledge their indebtedness to the Department of Scientific and Industrial Research for financial aid in respect of the investigation. Thanks are also due to the Admiralty for permission to publish the results of the experiments, to those who were concerned in the laying and surveying of the hydrophones, and to many others who rendered assistance and advice.

[†] See Phil. Mag., 36, June (1918), and Proc. Roy. Soc., A,94, August (1918).

Knowing V_1 and V_2, the velocities of propagation of the two types of signal, the distance D of the ship from the fixed station is at once obtained from

$$t=\frac{D}{V_1}-\frac{D}{V_2}.$$

Thus, if the ship originated simultaneous acoustic waves in air and in water, the respective velocities being 338 and 1,510 metres/sec., the distance D in metres would be approximately $435t$ from the receivers. If light waves and acoustic waves in air were simultaneously emitted (as in the case of a gun firing), V_1 is 338 metres and V_2 is 3×10^8 metres per second; then D is equal to V_1t practically—*i.e.*, $338t$ metres.

If two or more fixed receiving points in *known* positions are employed, it is a simple matter to locate the source of the double signal, for the observations give the ranges of the source from each of the known fixed points.

In the experiments described in this Paper a method has been developed which employs the simultaneous emission of a wireless signal in air and an explosion wave in water. The distances measured in such a case are solely dependent on a knowledge of the time interval and velocity of propagation of the explosion wave in water, for the time of propagation of the wireless wave is negligible by comparison. Recent experiments by the authors at St. Margaret's Bay, Dover, have given very accurate data for the velocity of propagation of an explosion wave in sea water.

Between the temperature-limits 6° to 17°C. and with the salinity in the neighbourhood of 35%, the velocity is expressed by the relations

$$V=4756+13{\cdot}8t-0{\cdot}12t^2 \text{ (ft./second)}$$

or

$$V=4626+13{\cdot}8t-0{\cdot}12t^2+3{\cdot}73S \text{ (ft./second)},$$

where t is the temperature in degrees centigrade and S the salinity in parts per thousand.

With this information as a basis, ranges have been measured and positions located accurately at distances greater than 50 miles.

II. METHOD.

The location of a source of simultaneous wireless waves (W/T) and acoustic waves is a simple application of the above method of range measurement. There is required—

(*a*) A means of producing the double signal—*e.g.*, a means of producing a W/T dash at the instant of explosion of a charge.

(*b*) Two or more acoustic receivers under water—the positions being accurately known—and a W/T receiving set at the recording station; and

(*c*) A means of recording and timing the radio-acoustic signals.

Transmission of the Double Signal.

(*a*) The radio-acoustic signal can be produced in several ways, according to the degree of accuracy requred in the location or "fix."

In the simplest case, where only an approximate fix is desired, the W/T operator on the ship presses his transmitting key when he hears or feels the shock of the explosion. In such a case the explosive may be a small charge of guncotton fitted with a fuse, the charge being thrown overboard at the position to be located. Such a method, of course, involves the personal error of the *W/T* operator, there being necessarily a certain, somewhat variable, lag between the explosion of the charge and the pressing of the *W/T* transmitting key.

For more accurate work a double key can be used, one part of the key firing an electrical detonator in the charge, the other part transmitting the *W/T* dash. If still greater accuracy is required, a simple automatic arrangement can be employed, whereby the shock of the explosion automatically closes a circuit and transmits the *W/T* signal. Up to the present the double-key method has been found quite sufficient to meet all practical requirements. In that method it is essential, of course, that a sufficiently high voltage be employed to fire the detonator in order to reduce time-lag to a minimum. Usually 80 volts are applied to the detonator, whereas 2 or 3 volts would be sufficient to fire it; the lag in the former case is of the order of 0·001 second.

Reception of the Double-signal.

(*b*) The acoustic receivers employed to detect the arrival of the explosion impulse through the sea are hydrophones of the microphone type, laid in carefully surveyed positions. Four hydrophones are used, laid on a line approximately N. & S., just to the eastward of the Goodwin Sands. The positions are given in the following table :—

TABLE I.

Hydrophone No.	Latitude North.	Longitude East.
1	51° 24′1·5″	1° 36′38·5″
2	51° 20′26″	1° 35′27″
3	51° 16′38·5″	1° 36′33″
4	51° 12′29″	1° 35′55″

These positions are shown on Fig. 1.

The distances apart and relative orientations of the various pairs of hydrophones are given below :—

TABLE II.

Hydrophone Pair.	Distance apart, Feet.	Angle from true North (measured clockwise).
1—2	22,279	11° 43·5′
1—3	44,850	0° 26·5′
1—4	70,245	2° 15·5′
2—3	23,413	349° 42′
2—4	48,409	357° 54′
3—4	25,456	5° 27·5′

The four hydrophones are connected by twin-core armoured cable to the recording station.

Recording and Timing the Signals.

(*c*) The electrical circuits of the hydrophones consist essentially of a battery in series with the microphone and primary of a transformer, the secondary of which is connected directly to one of the strings of a six-stringed Einthoven galvanometer set. Four of the strings of the galvanometer are connected in this way to their corresponding hydrophones, a fifth string records (through a microphone circuit) the half-second ticks of a Greenwich chronometer, whilst the sixth string records the W/T signals. The latter are received by a suitable aerial and standard receiver, amplified by six valves in cascade, rectified by a seventh valve and then passed through the line circuit of a Brown microphone relay. The circuit of the microphone of this relay consists of a dry cell, an adjustable resistance and the moving coil of a Weston relay. The resistance is so adjusted that the local circuit of the Weston relay is *just* closed, and a small current flows through the contacts and the galvanometer string. When a W/T impulse is received, the microphone of the Brown relay is disturbed and its resistance increases, causing the local contacts of the Weston relay to open and cut off the current through the galvanometer string. With this arrangement W/T signals can be recorded quite satisfactorily provided the rate of transmission of the signals is not too rapid for the Weston relay to recover in the intervals between the signals.

The photographic record (on bromide paper) is crossed by a series of fine lines marking seconds, tenths, and hundredths—for very accurate work thousandths of seconds can be estimated. The time marks are made by a phonic wheel controlled by a 50 ∾ tuning fork electrically maintained.

With the apparatus outlined above, a radio-acoustic signal will give the following information :—

(1) The total time interval from the charge to each hydrophone ; and (2) the time of passage of the explosion wave between each pair of hydrophones.

From (1) we can at once deduce the distance of the charge from each hydrophone, whilst from (2) we obtain a bearing of the charge from the mid-point of each pair of hydrophones.

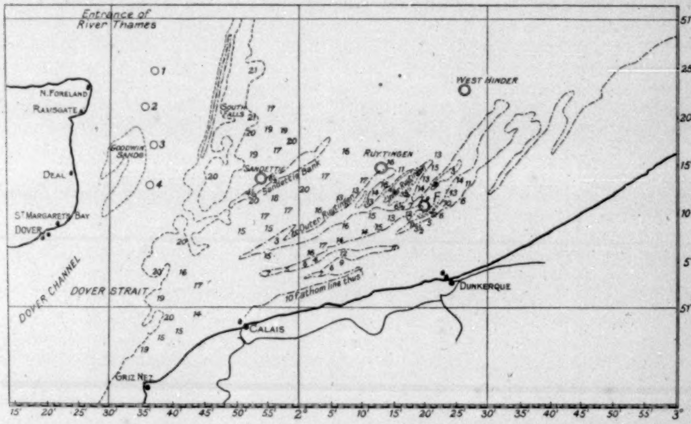
When the method is used in navigation where rapid "fixes" are required, the total distances and bearings need only be approximate, in which case the bearing may be taken as the asymptote of the hyperbola for the corresponding time interval. In accurate work both total distances and bearings must be accurate ; consequently, all bearings must have the asymptote correction applied. The location by plotting in the radio-acoustic method permits of great accuracy in that it is in all cases determined by lines crossing almost at right angles. The bearing lines from the mid-points of hydrophone pairs, always cut the range arcs from the individual hydrophones in a large angle (comparable with 90°), whereas the intersection angles of the various bearing lines or of the range arcs amongst themselves respectively are usually small. At long ranges this point is of great importance, and indicates the fundamental reason why radio-acoustic sound ranging is so much more reliable than the three-point system, where only time-differences and therefore bearing lines only are obtained.

In the R/A method the intersecting lines which locate the explosion are mutually at right angles, whereas in the three-point method they tend at greater distances to become parallel and the location correspondingly indefinite.

III. PRELIMINARY EXPERIMENTS.

The first experiments were made with the object of testing the accuracy of the method in giving rapid and approximate locations of the charge fired by a destroyer in known positions. Many such observations were made, charges being located at distances ranging from 10 to 40 miles from the hydrophones. In these preliminary tests the time intervals on the record were read to ±0·05 second only, in order to reduce the total time taken to obtain a location. An average time from calling up by the destroyer to giving the position was about 10 minutes. In all cases, with certain exceptions to which we shall refer later, the position given by the R/A method agrees, within the limits of error of the destroyer's estimate, with the position of firing the charge.

During these preliminary experiments a study was made of the screening effect of sandbanks, and it may be of interest to refer briefly here to the main results obtained. Charges were fired at various points around the Goodwin Sands. In no case when a charge was fired to the westward of the sands (the hydrophones lying to the eastward—*see* Fig. 1) was any effect recorded, the explosion wave failing to penetrate the sands or to pass over the shallow water (3 or 4 feet) covering them. The distance of the charge in this case from the nearest hydrophone was about 5 miles, the same hydrophone having recorded the effect of a charge of the same size at 50 miles when no shoals intervened. Firing charges in other positions only partially screened by the sandbank indicated that the latter was casting a very efficient acoustic shadow. In one experiment some evidence of distortion at the edge of the sandbank was obtained, the record suggesting that an acoustic wave (of long wave length) resulting from the explosion was either diffracted round the edge of the sandbank or that the main pressure wave was retarded in velocity and changed in wave form as it passed over the shallows at the edge of the bank.


Entrance of River Thames
N. Foreland
Ramsgate
Deal
St Margarets Bay
Dover
Goodwin Sands
O1
O2
O3
O4
South Falls
Sandettie
Sandettie Bank
Ruytingen
Outer Ruytingen
West Hinder
F
Dover Channel
Dover Strait
10 fathom line thus
Calais
Griz Nez
Dunkerque
51°30'
25'
20'
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30'
35'
40'
45'
50'
55'
3°

FIG. 1.

The observations in general, however, indicated the complete screening effect produced by a sandbank such as the Goodwins. A distortion effect such as that quoted is a rare exception which would be unlikely to occur in ordinary practice, and even then would easily be detected. Other factors which influence the range of detection of the explosion wave have been examined experimentally, but the results of the investigation will be published later.

IV. Navigational Experiments.

The preliminary experiments just outlined indicated at once the possibilities of the radio-acoustic method when applied to the navigation of ships. These experiments showed that it was possible to give a vessel an accurate position within 10 minutes of receiving her request for a "fix." Consequently several trial cruises were organised in which the destroyer should proceed to any position selected by her navigating officer, and, of course, within range of the station, and ask for a "fix." The results of these experiments were entirely satisfactory, in every case the rapid R/A "fix" agreed with the position estimated by the navigating officer of the destroyer. A long list of observations of this character could be given in evidence of this agreement, but they are omitted to save space. A specimen record and location is, however, shown in Fig. 2. On one occasion the destroyer found herself between dangerous sandbanks off the Belgian coast (*see F* in Fig. 1), a mist preventing her from distinguishing shore objects. An R/A fix, however, cleared up all doubt as to her position and she returned to Dover Harbour in safety.

Such experiments as these were carried out throughout the winter of 1921-22, in very rough and in misty weather, and on every occasion the R/A locations have been found more reliable than those determined by the ship's navigating officer by ordinary methods (*e.g.*, by time-distance-bearing, or by sighting light-vessels).

There should be little further difficulty in applying the R/A method to ships in general. Most ships requiring to make a good landfall after a long voyage carry wireless apparatus. It has been found that a small 9 oz. charge can be located up to about 40 miles, and this would be a convenient size to carry.

The R/A method of locating ships at sea in foggy weather should now be considered as a competitor to the Directional Wireless Method. The R/A method possesses the very great advantage of being equally accurate and reliable day or night, at all seasons of the year and under all weather conditions—whereas directional W/T locations are often open to doubt, peculiar errors of several degrees in bearing being of frequent occurrence.

V. Hydrographical Survey.

In the foregoing remarks particular importance has been attached to the "rapid" and approximate R/A location for navigational purposes, rather than to the more accurate location which the Einthoven galvanometer record is capable of revealing.

On certain occasions when the destroyer "Thruster" asked for a "fix" whilst in the neighbourhood of the West Hinder Lightship (*see* Fig. 1), the rapid R/A location appeared to be considerably in error. The experiments were consequently repeated, the destroyer estimating her position as accurately as possible, and the Recording Station calculating the position of the charge by the accurate method. In this method the recorded times are estimated with an accuracy of $\pm 0{\cdot}001$ second, and the asymptote correction is applied to the calculated bearings. The calculated position of the charge was still found to disagree with the destroyer's estimated

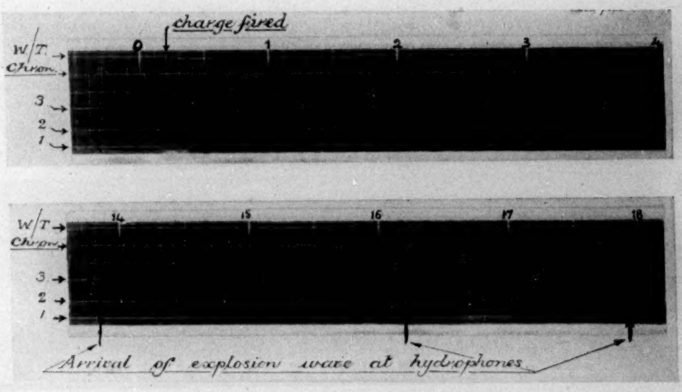


FIG. 2.—RECORD OF CHARGE No. 54.

Charge 2¼ lbs. G.C., 40 feet deep.
Rapid R/A.—Charge 1: 15 Nautical miles.
Charge 3: 11½ ,, ,,
Bearing 1-2: 124¾°.
Bearing 2-3: 109½°.

∴ Whence position 51° 14′ 15″ N., 1° 54′ 30″ E., as compared with 51°14′ 12″ N., 1°54′ 18″ E., estimated by "Thruster."

To face page 188.

position, the divergence being much greater than the possible error in the destroyer's estimate. These observations led the authors to suspect that the light vessel was not in her charted position. Consequently further R/A observations were made in the neighbourhood of other light vessels, with the results shown in the following table :—

TABLE III.

Light Vessel.	Charted Position.	R/A Position.	Error in charted position.
" West Hinder " ...	51° 22′ 30″ N. 2° 26′ 20″ E.	51° 23′ 16″ N. 2° 27′ 50″ E.	1¼ naut. miles N.E.
" Sandettie " ...	51° 13′ 24″ N. 1° 53′ 42″ E.	51° 13′ 25″ N. 1° 53′ 43″ E.	Nil.
" Ruytingen " ...	51° 14′ 28″ N. 2° 12′ 58″ E.	51° 14′ 38″ N. 2° 13′ 59″ E.	0·7 naut. mile 78° true.

Later observations of " Sandettie " light vessel, after a period of rough weather showed her to have moved about half-a-mile from her charted position. Further observation showed that she remained fixed in this position throughout the remainder of the winter. The R/A position of " Ruytingen " light vessel was checked by Lt.-Comdr. Archer by accurate bearing observations of the light vessel and Dunkerque. The R/A position required the bearing to be 156·5° true, whilst the charted position indicates 154° true. Direct observation gave 157° true—in good agreement with the R/A prediction. This was subsequently confirmed by the French hydrographer, in response to inquiries, who gave the position of the " Ruytingen " light vessel as 51° 14′ 50″ N., 2° 13′ 52″ E., which closely agrees with the R/A location quoted in the above table. " West Hinder " light vessel was driven from her moorings in a storm soon after she had been located by R/A ; no comparison of positions has therefore been possible.

The above illustrations serve to show how the R/A method might be applied to the location of buoys, light-vessels, &c., in hydrography. It is simple, direct and time saving. An accurate " survey " location of a buoy can be worked out in about two hours by the R/A method, whereas long and laborious observations and calculations are required in other methods of survey.

Suggestions have been laid before the Admiralty for a simple form of portable R/A set to be carried by survey ships as part of their equipment, and set up wherever required for the purpose of a particular survey.

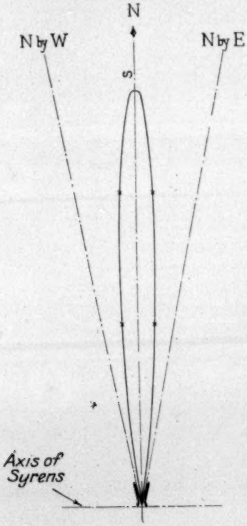
It was considered possible that temperature differences in a horizontal plane in the sea under winter conditions might conceivably affect the R/A locations, since temperature differences of as much as 2°C. (corresponding to about 20 ft./sec. change of velocity) were known to occur. Under such conditions the wavefront of an explosion wave would to some extent be refracted, with consequent errors in range and bearings deduced from the records. Experience has shown, however, that such errors are too small to be of any importance in navigational locations, but may possibly be not entirely negligible in hydrographical survey work where greater accuracy is required. For this reason it is recommended that surveys should preferably be carried out in settled weather when temperature fluctuations are a minimum.

DISCUSSION.

Col. H. F. **Towler** said there was no doubt as to the great need for aids to navigation in foggy weather. The method might find a further application in the prevention of collisions by a ship's emitting simultaneous fog-horn and wireless signals for observation by other ships. The direction could be got by wireless D/F, as the sound might be subject to refraction in the conditions mentioned. This arrangement would have two great advantages over that in which the locations are given by a shore station, viz., that the navigator would feel more confidence in the observations if they were taken by himself, and there would be no risk of jamming when a number of ships simultaneously required locations in a sudden fog. The speaker considered that for navigation near the shore the wireless D/F shore stations can hold their own against the R/A stations, as the errors ascribed to the former are not usually manifested at the ranges considered, viz. 30 to 40 miles. D/F stations can give a location in as good time as that claimed for R/A (10 minutes), they already exist in considerable numbers, and they are useful for other than marine purposes, *e.g.*, for aircraft.

Mr. F. **Twyman** said it might not be irrelevant to mention an acoustic directional transmitting system* which he had devised, but had not yet constructed. It comprises a row of synchronised syrens the total length of which is great compared with the wave length, and emits sound which is practically confined to the plane bisecting the row at right angles. The distribution of intensity has been worked out by Dr. Silberstein on the basis of the ideas put forward by Lord Rayleigh.†

Localisation of Sound



Number of Syrens 11. Vibrations per second 224. Distance apart 2 ft. 6 in.

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* British Patent Specification 4797/14 (Twyman and Another).

† Theory of Sound, 1896, Vol. 2, p. 103.

It might be expected that a beam of sound thus produced would be of more stable intensity in windy weather than a spherical wave from a point source, and that its intensity would fall off less with distance. A directional receiving apparatus might perhaps be designed on analogous lines.

Dr. C. V. **Drysdale** said that the R/A method would be an invaluable aid to navigation, but he agreed that seamen would be disinclined to rely on observations taken by other persons. The solution seemed to be that the lightships should be fitted as transmitting stations and the ships as receiving stations. For direction finding it would be preferable to use subaqueous sounds, detected by hydrophone, rather than those emitted by a foghorn, which are liable to changes of direction by refraction or reflection.

Mr. F. E. **Smith** said that the captain of a vessel would certainly want to make the observations himself. It might not be generally known that, backward as the wireless equipment of this country may be, some thousands of locations are given annually to ships by the D/F shore stations. In spite of the greater accuracy of R/A, the D/F stations will probably hold the field in this service on account of their relatively small cost. Their accuracy is greatest where it is most needed, namely, in the proximity of the stations themselves. The speaker paid a tribute to the naval officers and ratings who had worked at the method and had shown great keenness as to both its scientific and its technical aspects.

Mr. T. **Smith**, referring to the question of the distribution of sound intensity raised by Mr. Twyman, said that on general physical grounds one would assume that air and water are not dispersive media for sound. If, however, on a closer approximation to the facts they should be regarded as dispersive, could the existence of " silent zones " be explained on such lines ?

Major W. S. **Tucker** said that there are two kinds of silent zones. The first is of the type exemplified in the recent explosion experiment in Holland. The other type is really illusory, and is due simply to screening by acoustic clouds. The illusion arises from the fact that the observations have been made upon foghorn blasts emitted at one-minute intervals. If during such an interval the cloud has moved, the observing vessel having meanwhile also moved along its course, an impression is created that the vessel has steamed out of a silent zone. In reply to a question by Dr. Rayner, the speaker said that an acoustic cloud consists in the surface of separation between two bodies of air in different physical states.

Dr. A. **Russell** pointed out that the parabolic formula given by the authors for the variation of the velocity of subaqueous sound with temperature showed a maximum. Was the formula a purely empirical one ?

Mr. R. S. H. **Boulding** (communicated) : The authors are to be congratulated on having carried out some very interesting experiments which should prove extremely useful to the Mercantile Marine. In regard to recording, would it not be preferable, if possible, to operate the galvanometer by the W.T. signals without the aid of a Brown microphone relay and a Weston relay ? The statement that a rough position can be given in ten minutes seems to suggest that there is room for improvement. Admittedly, accurate determinations require careful calculation, but rough locations could, one would think, be worked out graphically or instrumentally in a negligible time. I should very much like to have any information the authors can give as to the time lag introduced by a sound wave passing round the edge of a sand bank. I gather that this rarely occurs ; but if it does, it constitutes a somewhat serious objection to the R/A method, unless it is possible for the observer to ascertain without doubt exactly what is happening. Although quite a small explosive charge is sufficient, it is feared that the use of an explosive of any sort may be objected to in connection with the Mercantile Marine. I have obtained satisfactory results with a submarine bell up to distances of 10 to 15 miles. The comparison between the R/A position of the Ruytingen L/V and that given by the French hydrographer is interesting. The difference between these two positions appears to be some 500 yards. What degree of accuracy is claimed for the R/A method ? I should like to ask whether any allowance has been made for tidal streams which, in the Straits of Dover, reach at times velocities 8 ft. to 10 ft. per second.

Reply to the Discussion by Dr. A. B. **Wood** : The method mentioned by Col. Fowler for the prevention of collisions at sea by the simultaneous emission of foghorn and W/T signals has already received attention by the authors. On account of the great variability of atmospheric conditions, however, it is considered more satisfactory for the lightship to emit simultaneous *under-water acoustic signals* and W/T signals. A proposal on these lines has already been submitted to the Admiralty by the authors* in which the light-vessel emits a series of W/T dots (the first of which is accompanied by the acoustic signal) at intervals corresponding to half a mile distance from the light-vessel (*i.e.*, every 3/5 second app.). A ship can thus obtain her distance from the light-vessel by counting the number of W/T dots received up to the arrival of the acoustic signal. The direction can, of course, be obtained by W/T.

Light-vessels are liable to shift position in stormy weather, however, and this might have serious consequences under certain conditions.

The R/A method has one great advantage over the directional W/T method in that it gives not only bearings, but also ranges. An error of 2 or 3 deg. in bearing might result in an error of several miles in range of location, especially so when the position of the vessel lies on a line making only a small angle with the base-line (the transmitters in W/T and receiving hydrophones in R/A). A knowledge of the range as well as the bearing is essential under such circumstances.

The time of 10 minutes estimated for giving a location is largely occupied by the W/T procedure of calling up, &c., which is common to all systems of this sort and cannot be reduced appreciably. One or two minutes might, however, be gained by the use of more automatic methods of reading off the time-intervals and plotting the positions.

(To Mr. F. Twyman): The method of directional acoustic transmission in air as suggested is certainly very interesting, but experience has shown that the transmission of sound in air over great distances is very unsatisfactory both as regards distortion of wave-front and erratic variation of intensity. Unless extremely powerful sources are employed the range is limited to a few miles only. Under the best conditions it is considered that an accuracy of ±5 deg. in direction could hardly be obtained by this method. The consequent errors of location would under such circumstances become very serious. Directional reception of under-water signals has already been considered and abandoned mainly on account of such serious errors in direction-finding.

As we have already stated above, the main feature of the R/A method is that *it gives an accurate value of the range in addition to accurate bearings.*

(To Dr. C. V. Drysdale): There is much to be said in favour of the "converse" R/A method. Owing to the closing down of the St. Margaret's Bay station, however, it was not possible to continue experiments on these lines.

(To Mr. F. E. Smith): It should be noted that in W/T direction-finding the captain of a vessel does not make the observations personally any more than in the R/A method. In the latter case it should be noted the record is taken in permanent form so that locations can be checked, complaints can be investigated and reasons ascertained. This is not possible in the W/T D.F. method. In the R/A method there are no "night-errors" or any such errors corresponding. The difference of cost in the two systems worked on a commercial basis need only be very trifling.

(To Mr. T. Smith and Major W. S. Tucker): The dispersive and distortional effects of a non-homogeneous atmosphere is the most serious objection to the use of acoustic signals in air as a means of direction-finding. The sea is a much more homogeneous medium in which sound waves are propagated for very great distances without serious distortion of wave-front. No "silent zones" have ever been observed in the sea, except perhaps in the very shallow water above a sand-bank such as the Goodwins.

(To Dr. A. Russell): The velocity formula referred to is an empirical one which applies only within the range of temperatures 6°C. and 17°C., between which limits the velocity measurements were made.

(To Mr. R. S. H. Boulding): In recording W/T signals both methods have been employed, but for certain experimental reasons it was found preferable to use the relay arrangement.

With regard to the time of 10 minutes required to obtain a navigational "fix," *see* reply to Col. Fowler.

On only one occasion has any distortion effect by a sand-bank been observed, and on that occasion the experiment was deliberately arranged to determine the extent of such distortion under extreme conditions. The record obtained was quite normal as regards the three hydrophones clear of the sand-bank, but in the case of the fourth hydrophone the record was only just readable and was obviously different in character from the usual sharp "break" produced by an explosion. Under service conditions such a record would have been rejected without hesitation and the "fix" given to the ship would have been based entirely on the information supplied by the three perfectly readable breaks. In *all* cases the ranges were good, but the bearing obtained by using the fourth record was clearly in error. This point was only mentioned in the Paper as being of academic interest—even under the worst conditions such an effect could certainly not be regarded as serious in the service application of the R/A method.

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* *See* also Joly, loc. cit.

It has been definitely decided by the Board of Trade that, subject to proper precautions, no objection will be raised to the carrying of the necessary explosive bombs by merchant vessels.

The accuracy of the R/A method as applied to survey purposes is very great. Since the velocity of the explosion wave is known with an accuracy of 1 or 2 ft. per second at all seasons of the year,* it is considered that the error in range at a distance of 50 miles does not exceed ±250 ft. as an outside estimate. The difference in position of the Ruytingen Light Vessel obtained by R/A and by the French Hydrographer might possibly be ascribed to the fact that the observations were made at different times and the light-vessel had shifted in the meantime, or what is more probable, the position given by the French Hydrographer (obtained by ordinary laborious survey methods) is in error by the amount stated.

In accurate survey work it is recommended that all observations be made as far as possible at times of slack water (neaps preferred) when no tide is running. Otherwise a correction for tidal velocity must be applied to the explosion wave-velocity.

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* *See* Proc. Roy. Soc. loc. cit.

DEMONSTRATION of an Electro-Capillary Relay for Wired Wireless. By Major C. E. PRINCE, *O.B.E.*

THE relay is intended for use with a calling device in connection with high-frequency currents acting as carrier waves for telephony over power-mains. The high-frequency current is rectified and passed through a thread of mercury which is contained in a capillary tube, and is in contact at each end with a quantity of acid, platinum wires by which the current enters and leaves being immersed in the latter. The passage of the current causes the mercury thread to move, and it was suggested many years ago by Wheatstone that this thread might be made to close an electric circuit; but practical difficulties prevented the successful application of the idea. In the present invention the capillary tube is arranged horizontally on a beam which, as soon as the mercury moves, over-balances in consequence of the weight of the latter and closes the circuit of a call bell or lamp. To prevent the evaporation of the acid the cups containing it are connected by a further capillary tube which serves to equalise the pressure, and the tube in which the mercury thread lies is widened at its middle point to form a chamber for the reception of bubbles and other obstructions which may have got into the tube before sealing.

The circuit arrangements make provision for the restoration of the mercury to its original position whether the call be answered or not. In series with the thread and with a rectifier is arranged a condenser in which the charge that has passed round the circuit is stored, and after the call this charge is sent through the mercury and acid in the reverse direction. The required discharge is effected by contacts which short-circuit the rectifier when the switch-hook is raised, but if the call be unanswered the same result is produced more slowly by a high-resistance leak.

The instrument will respond to currents of 4 or 5 or even 2 microamperes, and will function during long periods without attention. Its sensibility was demonstrated by passing through it the voice-current from the secondary of a microphone transformer, and subsequently the current from a single dry cell in series with the body of the demonstrator. If the leak resistance be small enough Morse signals will fail to effect a call in consequence of the leakage which takes place during the intervals between the signal elements, while a series of long dashes will nevertheless be effective and can be used as a call signal.

The cause of the movement of the mercury is obscure, and the phenomenon is of some interest to physicists. The total movement appears to be proportional to the coulombs which pass, so that the arrangement might be used as a quantity meter for small currents, such as voice currents.

DEMONSTRATION of the Flamephone (Scientific & Projections, Ltd.) By Mr. H. W. HEATH, *B.Sc.*

THE apparatus constitutes an improved gramophone, and employs a gas flame to improve the quality and intensity of reproduction.

The sound-box is divided by its diaphragm into two chambers, one of which communicates with a small horn while the other communicates with a supply pipe through which coal gas passes to a pair of vertical thin burner tubes. The tubes are perforated with a series of holes from which the gas issues, the jets projecting over the mouth of the horn. On lighting the gas an increase in the volume of sound can be observed, and also a marked improvement in quality, the notes of lower pitch being accentuated. This effect has not been completely explained.

XIX. *The Analysis of Bubbles in Glass.*

By the RESEARCH STAFF *of the General Electric Co., Ltd., London.* (*Work conducted by* J. W. RYDE *and* R. HUDDART.)

RECEIVED MARCH 17, 1923.
(COMMUNICATED BY C. C. PATERSON, O.B.E., M.I.E.E.)

ABSTRACT.

In order to distinguish bubbles generated by chemical action in glass from those introduced by mechanical processes spectroscopic tests are made for the presence of nitrogen. In order to liberate the gas from the bubbles a specimen of the glass is placed in one limb of a quartz U-tube and mercury in the other, and the glass is heated and then disintegrated by sudden cooling, the tube being plunged into cold water at the same time that the mercury is thrown on to the glass.

THE presence of bubbles is one of the most serious defects of glass. It has long been recognised that bubbles might arise in two ways, first by the generation of gas in the glass itself, second by the introduction of air or furnace gases during ladling, pouring or stirring. Bubbles due to these two causes could be distinguished easily if the nature of the gas contained in them could be determined, for bubbles due to the second cause should contain nitrogen, while in general those due to the first should not.

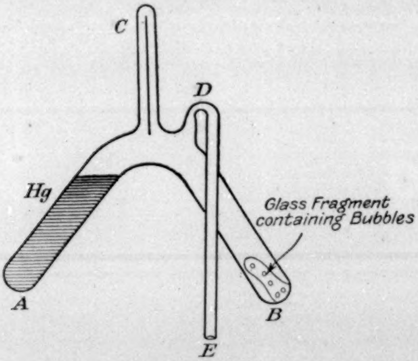

FIG. 1.

The problem of determining the nature of the gas in a bubble may not appear at first sight easy to solve. We have found that it is an extremely simple matter. A piece of the glass containing bubbles is placed in one limb of a small quartz U-tube (Fig. 1); the other is three-quarters filled with mercury. The U-tube is evacuated with all the usual precautions against the presence of gas absorbed on the walls and sealed-off from the pump. The limb containing the glass is then heated till the glass is just soft; it is then plunged into water, the cold mercury from the other limb being thrown on the hot glass at the same time. This process is repeated several times, if necessary. By this process the glass is broken into small fragments and the gas contained in the bubbles liberated. A discharge is then excited in the capillary tube *C* by a Tesla apparatus and the spectrum examined for nitrogen. It has been found possible to distinguish quite definitely between bubbles which contain nitrogen and those which do not.

DISCUSSION.

Mr. J. Guild expressed surprise that the spectrum of nitrogen should be distinguishable in the presence of mercury vapour, which might be expected to carry most of the discharge and to mask the effect of the nitrogen.

Dr. E. A. Owen inquired what precautions were taken to prevent atmospheric gases from entering the tube along with the mercury.

Dr. R. T. Beatty suggested that if the method could be made quantitative it might be used to find out what becomes of the gas which disappears from the interior of a discharge tube during the life of the latter.

Mr. J. W. Ryde, in reply to the discussion, said that the nitrogen spectrum would be completely masked by that of mercury if the tube were warm, and would be only just distinguishable at ordinary temperatures. In practice, therefore, one limb of the tube is immersed in liquid air whereby satisfactory results can be obtained. In filling the tube with mercury every precaution is taken, by drying and boiling the mercury and so on, to prevent the admission of gaseous impurities. Greater difficulty is experienced, however in getting rid of the gases adsorbed on the silica walls of the tube, which have to be heated repeatedly with a blow-pipe. The method does not readily lend itself to quantitative applications, as the line intensities are much affected by the fact that the discharge takes place in a mixture of gases.

XX. *A Simple Regenerative Vacuum Device and Some of its Applications.* By H. P. Waran, *M.A., Ph.D.* (*Cantab.*), *F.Inst.P.*

Received March 24, 1923.

ABSTRACT.

The Paper deals with the difficulties arising from residual traces of air fouling the vacuum above the mercury column in syphon gauges and other devices. A remedy is suggested in the form of a small attachment taking the shape of bent capillary tubing ending in a bulb. This enables the air to be repeatedly pushed into the vacuum of this bulb, the mercury at the bottom of the capillary preventing the subsequent return of the air. With such an arrangement fitted to the top of a syphon gauge the height of the column of mercury becomes a true measure of the gas pressure, subject only to the definite correction for the vapour pressure of mercury and temperature. The device is regenerative in the sense that, irrespective of any progressive fouling of the vacuum, a fresh air-free vacuum is automatically created by it every time it is brought into action. As typical examples of the wide field of utility of the device its application to mercury barometers and mercury vapour lamps is also briefly discussed.

One of the devices finding constant use in vacuum work is the simple syphon gauge illustrated in Fig. 1 (*a*). The readings of such a gauge are made on the assumption that the space above the mercury in the closed limb is a perfect torricellian vacuum containing nothing but a trace of mercury vapour which, at the average temperature of about 20°C. has a negligible vapour pressure of about 0·001 mm. of mercury. For convenience in measuring the difference of mercury level in the two limbs of the gauge, it is customary to have them side by side. This arrangement introduces difficulties in the way of filling the gauge with mercury without locking any small bubbles of air in the closed limb. In practice even with the best of care small bubbles of air do get locked in the closed limb of the gauge, especially when a vacuum process of filling the gauge with boiling mercury is not adopted. At the first opportunity when the gauge comes under operation these bubbles expand into the torricellian vacuum above the mercury in the closed limb. Considering that the volume available for expansion is generally very small, and that the bubbles to start with are very nearly at atmospheric pressure, it is easy to see that quite small bubbles are enough to cause an error in reading of about 1 mm. When the actual pressure in the system is only a millimetre or two, this is a serious error of about 50 per cent.

Further, the extent of the error due to this is uncertain, and it can be determined only by comparison with another standard connected to the system or by measuring the gas pressure accurately by some other means. Apart from this uncertainty, the error is also variable with the pressure, since the volume of the vacuous space above the mercury varies with the pressure. In addition to these troubles, if the gauge comes into operation frequently and is to be used for a long time connected to an apparatus, there is a progressive deterioration of the vacuum in the closed limb. This is due to the film of air carried along the walls by the mercury flowing in and out of the closed limb periodically. For such gauges it is useful to fit a simple air-trap, as shown in Fig. 1 (*c*), and this would arrest the forward progress of the air film to a great extent.

Because of such inherent defects the syphon gauge has always remained a rough indicator for low pressures, and its readings have never been relied upon to an accuracy greater than a few tenths of a millimetre of mercury. If means can be devised to deal successfully with such accumulations of air fouling the vacuum and ensure the vacuum above the mercury being practically perfect, there is no reason why it could not be used to measure the pressures directly with an accuracy of about 0·001 mm. Its directness and simplicity are greatly in its favour and the limit is set only by the accuracy of the kathetometer used to measure the difference in level.

The simplest way of dealing with such residual gases seems to be that of pushing them out into a subsidiary chamber and preventing their return back into the system. In a syphon gauge this can easily be achieved by attaching the small regenerative extension *R*, as indicated in Fig. 1(*b*). This device consists of a small length of capillary tubing *T*, bent to the shape shown and ending in the auxiliary bulb *B*.

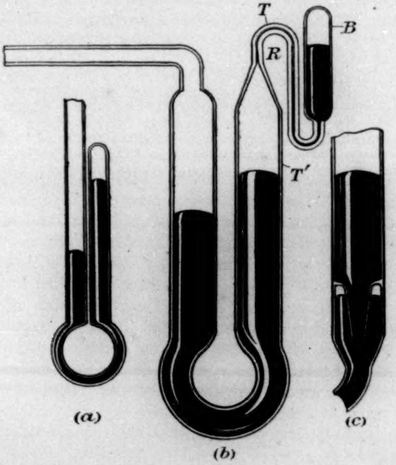

FIG. 1.

To start with, the gauge, including the regenerative attachment, is completely filled with mercury as usual. When it is connected to a low-pressure system mercury level goes down in *B* and *T′*, forming torricellian vacua in the two chambers. The little mercury that remains in the lower portion of *B* serves to cut off communication between *B* and *T′*. Into this vacuum the residual bubbles of gas locked in the tubes during the filling operations can rise. Now, if the open limb is opened to the atmosphere, the mercury rises up in *T′* and passes into *B*, forcing the residual gases above it into *B*. As the pressure goes down again the mercury level goes down in *T′*, sucking a fresh air-free torricellian vacuum above it. Thus the vacuum above the mercury is practically perfect, and the difference in heights of the mercury in the two limbs is an exact measure of the gas pressure within an accuracy of about 0·001 mm. mercury. The arrangement described is regenerative in the sense that this action is repeated automatically every time the gauge comes into action.

Such a regenerative device is applicable to many laboratory devices in one form or other.

One such principal field of application is supplied by the mercury barometers which, in essentials, are elaborated syphon gauges. As prominently pointed out in the Meteorological Office observers' handbook,* the chief defect of mercury barometers is their gradually increasing error, due to the progressive accumulation of air in the space above the mercury. Considering that a barometer to be of any use for meteorological purposes must be capable of maintaining an accuracy greater than 0·01 mm. for periods reckoned in years, it is easy to see how small and slow a leakage is sufficient to make the instrument of no use. A small regenerative attachment of the type described is easily fitted on to a barometer of any pattern, and then it can deal with such accumulations of air quite easily. All that one has to do is to periodically tilt the barometer a little, as shown in Fig. 2, in the case of portable instruments of the Kew pattern, or screw up the base screw in the case of the Fortin type of standard barometers. Especially is this device of value for standard barometers. The device has the great advantage that it is of small dimensions, and can readily be fitted to existing types of barometers without necessitating any serious structural alterations.

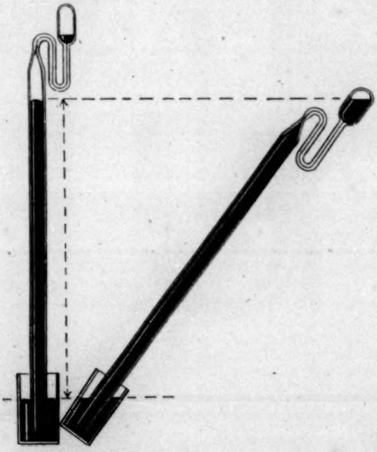

FIG. 2.

Another instance of its application is in the case of a simple type of mercury vapour lamp. In a lamp of the design shown in Fig. 3 the vacuum in the arc space is automatically renewed every time a little air is let in, and sucked back by a filter pump, or the lamp may be tilted suitably. In fact, the lamp exhausts itself to start with, without the aid of any high vacuum pump. As such it ought to be very useful in many laboratories where the requisite facilities for the production of high vacunm do not exist.

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* 1919 Edition, p. 22.

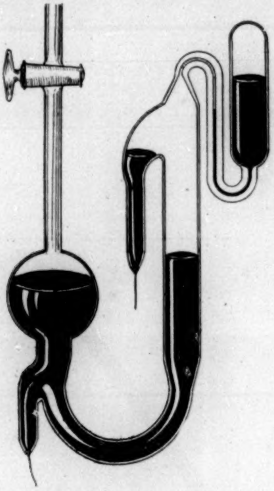


FIG. 3.

Doubtless there are many other similar instances where the principle of the device could be used in one form or other.

In conclusion, I must thank Prof. A. W. Porter, D.Sc., F.R.S., for his sympathetic interest in this device.

DISCUSSION.

Dr. R. T. Beatty said that the device is a remarkably simple and effective one. If it could be put on the market in combination with an oil trap and a drying tube it would fill a long-felt want.

Dr. G. B. Bryan said that if the capillary tube were too small there might be difficulty in getting the mercury to move air bubbles along it, as there is a tendency for the mercury to slip past a bubble. He inquired as to the kind of glass employed.

Dr. E. A. Owen mentioned a somewhat similar device which has been in use for some time at the National Physical Laboratory.

Mr. F. A. Gould (communicated): The author has drawn attention to a device which has been known for many years to users of precision barometers.

In the "Travaux et Memoires du Bureau International," Tome II (published in 1883), there is a description of a barometer in which practically the same device is used. It is recorded there that the object of the device was not entirely fulfilled to the satisfaction of the Bureau

International, as it was found that " the complete expulsion of the air from the vacuum-space of the barometer required the passage of a considerable quantity of mercury upwards through the instrument, a procedure which probably introduced fresh small bubbles of air from the lower end of the barometer."

The device for renovating the vacuum has also been used in this country. A reference standard mercury barometer incorporating this design was constructed by Dr. Aston, formerly of the Royal Aircraft Factory, Farnborough, during the war, and is still in use at Kidbrooke, while a similar principle was adopted in an experimental standard barometer which, though designed at the National Physical Laboratory in 1913, was not filled until 1922. The efficiency of the device has not yet been tested at the N.P.L.

The reference made by the author to the possibility of securing an accuracy of 0·001 mm. of mercury in the use of a syphon gauge requires some qualification. It is well known that the capillary action of mercury is not negligible, even in tubes of comparatively large diameter, e.g., 1 in. Exact equality in the amount of the capillary depression in each limb of the gauge cannot be relied upon, and it would be necessary to select a syphon tube of internal diameter at least 1·3 in. in order that the differential capillary error should not exceed 0·001 mm. Other causes would render this degree of accuracy difficult to secure.

The reference to the meteorological barometer is misleading, and an estimate of 0·1 mm. would be far nearer the truth than 0·01 mm. for all ordinary purposes. It is usual to insert an air trap in the tubes of all Kew-pattern barometers (i.e., those barometers with uniform, contracted scales and no fiducial point), but it is doubtful whether it would be practicable to fit the " vacuum-renewal " device to the ordinary portable types of mercury barometers.

Mr. J. GUILD said that he was particularly interested in the application to mercury lamps and would like to know more of the author's experiences in this connection. The models shown were on a small scale and suitable for low voltages, but where a bright discharge is exacted the vapour pressure of the mercury becomes considerable and it might be expected that in these circumstances the mercury seal would cease to be effective.

Dr. D. OWEN asked if the author could furnish some experimental results. Had he, for instance, data of inter-comparison of two syphon barometers constructed as described and subjected to the same conditions ?

The AUTHOR, in reply to the discussion, said that the device could very readily be combined with an oil trap and drying tube. Various difficulties have to be overcome in practice, but there is none in getting the mercury to move the bubbles provided a capillary tube of 0·5 mm. diameter be used. It was not surprising that the idea should have occurred to others, but he was not previously aware of the fact and had thought that other workers might find the device useful. He had not had an opportunity to obtain experimental data, but he hardly thought a syphon barometer would be capable of a total accuracy of 1μ. That figure represented the maximum error due to imperfection of the vacuum. The device would not be effective for an indefinite period, but might increase the life of a barometer by some years. The mercury lamps of which he had had experience were some taking 2 amp. at 110 volts which he had made for laboratory use. In such lamps when hot the mercury rises about 1 cm. The construction of the apparatus is such that there is no tendency for the rise in vapour pressure to break the mercury seal.

In regard to the previous work on the subject in France, he concluded from his own inquiries that this had produced but little impression here, and he trusted his Paper would serve to advance the matter.

XXI. *Application of the Eötvös Torsion Balance to the Investigation of Local Gravitational Fields.* *By* Capt. H. Shaw, *M.Sc., F.Inst.P., and* E. Lancaster-Jones, *B.A.* (*Cantab.*).

RECEIVED FEBRUARY 24, 1923.

ABSTRACT.

In a Paper read before this Society on February 9, the Eötvös Torsion Balance was described in detail, and the theory of its operation considered, the preparation of suitable torsion wires was mentioned, but no practical tests with the instrument were discussed.

In view of the sensitivity of the balance, which, as previously stated, measures derivatives of gravity of the order of 10^{-9} C.G.S. units, it was anticipated that a gravitational survey of the laboratory would disclose the varying effects of the neighbouring masses of the walls, pillars, &c. The balance was therefore set up at different stations in the laboratory, and the derivatives at each station measured. The measurements at several stations were checked by repetition, and the results compared with the theoretical values.

The mutual consistency of the results obtained at each station, and their general agreement with the calculated effects exceeded expectations, as the local gravitational field varied so rapidly that the theoretical assumption of a uniformly varying field in the neighbourhood of a station was obviously vitiated.

I.

THE experiments were conducted in a basement room of the new Science Museum building, South Kensington, which was the only room available for the purpose. The temperature variation from day to day, which is an important factor, never exceeded about 2°, except in the change over from winter to summer conditions in the heating arrangements of the laboratory. In order to avoid disturbances due to radiation, the laboratory was kept in total darkness, except when an observation was being made, the radiation from the electric light and the presence of the observer, for about 30 seconds, being insufficient to disturb the reading in that period.

This basement room, measuring 7·3 metres by 6·6 metres (*see* Figs. 1 and 2), was adjacent to a solid bank of earth, while on the remaining sides it was bounded by a 9 in. brick wall, connecting massive columns of re-inforced concrete, which, considerably affect the local gravitational conditions.

Other considerations also rendered the room to some extent unsuited to our purpose, it being situated in the basement of a massive building which was constantly undergoing important structural alterations, while in addition objects of considerable weight were frequently being moved to and from adjacent rooms.

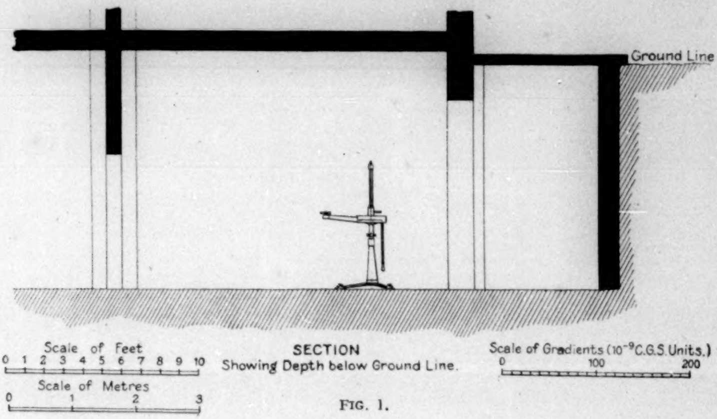

Ground Line
Scale of Feet
0 1 2 3 4 5 6 7 8 9 10
Scale of Metres
0 1 2 3
SECTION
Showing Depth below Ground Line.
Scale of Gradients (10^{-9} C.G.S. Units.)
0 100 200

FIG. 1.

II.

From a consideration of the theory of the instrument in the Paper referred to above, we obtained the formula

$$n_a - n = A \sin 2a + B \cos 2a + C \sin a + D \cos a \quad \ldots\ldots \quad (1)$$

where

$$\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) = \frac{A\tau}{D'K}$$

$$\frac{\partial^2 U}{\partial x \partial y} = \frac{B\tau}{2D'K}$$

$$\frac{\partial^2 U}{\partial x \partial z} = -\frac{C\tau}{2D'mhl}$$

$$\frac{\partial^2 U}{\partial y \partial z} = \frac{D\tau}{2D'mhl}$$

$\frac{\tau}{D'K}$ and $\frac{\tau}{2D'mhl}$ being instrumental constants.

In his investigations Eötvös made observations at intervals of 72° requiring only five readings to complete a set, but by increasing the number of observations, and taking the readings at each 60°, we are able to simplify the calculations.

The six equations resulting from putting $a=0°, 60°, 120°, 180°, 240°, 300°$ in equation (1) give us

$$A=\frac{1}{\sqrt{3}}\left(n_{60}-n_{120}-\frac{n_0-n_{180}}{2}\right)$$

$$B=\frac{n_0+n_{180}}{2}-n$$

$$C=\frac{1}{\sqrt{3}}\left(n_{60}-n_{240}-\frac{n_0-n_{180}}{2}\right)$$

$$D=\frac{n_0-n_{180}}{2}$$

while in addition we find that

$$n_0+n_{120}+n_{240}=n_{60}+n_{180}+n_{300} \quad \ldots\ldots\ldots \quad (2)$$

Equation (2) thus provides us with a simple and convenient check on the observations, and any error of sensible magnitude can be detected immediately, and the observations repeated before the instrument is moved.

During this investigation the values of the instrumental constants $\frac{\tau}{D'K}$ and $\frac{\tau}{2D'mhl}$ were $134{\cdot}48\times10^{-9}$ and $44{\cdot}17\times10^{-9}$ respectively, so that we get

$$\left.\begin{aligned}
\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)&=77{\cdot}64\left(n_{60}-n_{120}-\frac{n_0-n_{180}}{2}\right)\times10^{-9}\\
\frac{\partial^2 U}{\partial x\partial y}&=67{\cdot}24\left(\frac{n_0+n_{180}}{2}-n\right)\times10^{-9}\\
\frac{\partial^2 U}{\partial x\partial z}&=25{\cdot}50\left(n_{240}-n_{60}+\frac{n_0-n_{180}}{2}\right)\times10^{-9}\\
\frac{\partial^2 U}{\partial y\partial z}&=44{\cdot}17\left(\frac{n_0-n_{180}}{2}\right)\times10^{-9}
\end{aligned}\right\} \quad \ldots\ldots \quad (3)$$

III. VERIFICATION OF RESULTS.

(*a*) As the local gravitational field varied rapidly, it appeared desirable to check the results at a few stations by repeating observations with new initial azimuths and subsequent azimuths at intervals of 60°. The results of such a repetition at one station Y with initial azimuths 0°, 10°, 20° . . . . 50° are given in Table I., and the derivatives calculated for each set and reduced to common axes in and perpendicular to the magnetic meridian.

TABLE I.

Initial azimuth	$\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)$	$\frac{\partial^2 U}{\partial x\partial y}$	$\frac{\partial^2 U}{\partial x\partial z}$	$\frac{\partial^2 U}{\partial y\partial z}$	$\sqrt{\left(\frac{\partial^2 U}{\partial x\partial z}\right)^2+\left(\frac{\partial^2 U}{\partial y\partial z}\right)^2}$	Gradient Direction, from initial azimuth.	Gradient direction (magnetic)
	10^{-9}	10^{-9}	10^{-9}	10^{-9}	10^{-9}		
0°	−308·4	− 34·96	−123·5	− 53·0	134·35	23°·10′	23°·10′
10°	−253·0	− 82·40	−133·1	− 24·6	136·35	12°·30′	22°·30′
20°	−278·8	−131·0	−137·0	− 7·12	137·2	3°·00′	23°·00′
30°	− 73·48	−147·6	−136·4	+ 15·36	137·3	− 6°·30′	23°·30′
40°	+ 33·76	−151·6	−132·6	+ 33·96	138·2	−16°·20′	23°·40′
50°	+133·1	−141·5	−122·0	+ 60·3	136·1	−26°·20′	23°·40′
Means	...	...	...	...	136·6	...	23°·15′
Max. error	...	...	...	...	2·2	...	0°·45′

REMARKS.—The maximum error of 1·6 per cent. is smaller than was anticipated in view of the non-uniformity of the local field.

(*b*) An additional check is provided by repeating the observations at one station with constant initial and subsequent azimuths, *e.g.*, results for station Z are shown below for a series of readings which were repeated daily for a week.

TABLE II.

Day.	Azimuth.						$\frac{\partial^2 U}{\partial x\partial z}$	$\frac{\partial^2 U}{\partial y\partial z}$	$\sqrt{\left(\frac{\partial^2 U}{\partial x\partial z}\right)^2+\left(\frac{\partial^2 U}{\partial y\partial z}\right)^2}$	Gradient Direction Magnetic.
	50°	110°	170°	230°	290°	350°				
Mon.	16·93	18·73	19·01	16·49	17·04	17·75	−37·49	9·71	38·74	215°·30′
Tues.	16·92	18·73	19·00	16·50	17·04	17·77	−37·74	9·28	38·86	216°·10′
Wed.	16·93	18·72	19·01	16·49	17·05	17·75	−36·98	9·71	38·23	215°·20′
Thurs.	16·92	18·73	19·00	16·48	17·03	17·75	−37·74	9·71	38·78	215°·30′
Friday	16·92	18·72	19·00	16·48	17·04	17·75	−37·23	9·71	38·48	215°·20′
Sat.	16·90	18·71	19·00	16·47	17·01	17·73	−37·87	9·50	39·03	215°·55′

(*c*) In order to compare the results obtained at one station after an interval of three months, observations were taken at Station Z in November, 1922, and again in February, 1923. During the interval between these observations the balance had been set up at several stations and transported to the Imperial College of Science for demonstration before this Society, whilst the lower suspension wire had been removed on a few occasions.

TABLE III.—*Observed Results.*

Date	Initial Azimuth.	$\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)$	$\frac{\partial^2 U}{\partial x\partial y}$	$\frac{\partial^2 U}{\partial x\partial z}$	$\frac{\partial^2 U}{\partial y\partial z}$	$\sqrt{\left(\frac{\partial^2 U}{\partial x\partial z}\right)^2+\left(\frac{\partial^2 U}{\partial y\partial z}\right)^2}$	Direction of Maximum Gradient Magnetic.
		10^{-9}	10^{-9}	10^{-9}	10^{-9}	10^{-9}	
14/11/22	0°	−120·7	29·5	−31·62	−21·20	38·07	213°·50′
10/2/23	0°	−119·5	28·24	−31·625	−21·42	38·18	214°·30′

IV. COMPARISON OF RESULTS AT DIFFERENT STATIONS.

Table IV. gives the values of the four derivatives at all the stations in the laboratory at which observations were made, and illustrates the rapidity with which these derivatives vary, especially in the neighbourhood of the attracting masses. In Fig. 2 the stations are plotted and the resultant horizontal gradients represented in magnitude and direction by arrows. The gradient is the resultant of $\frac{\partial^2 U}{\partial x \partial z}=\frac{\partial g}{\partial x}$ and $\frac{\partial^2 U}{\partial y \partial z}=\frac{\partial g}{\partial y}$, and therefore indicates the direction in which gravity varies most rapidly, and the amount of variation per centimetre in that direction. If the attraction g at any point (say Z) is known by pendulum methods, the attraction at any other point (say Y) can be calculated by integrating the resultant gradient along any line joining Z and Y.

TABLE IV.

Station.	$\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)$	$\frac{{}^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$	$\sqrt{\left(\frac{\partial^2 U}{\partial x \partial z}\right)^2+\left(\frac{\partial^2 U}{\partial y \partial z}\right)^2}$	Maximum Gradient Direction Magnetic.
	10^{-9}	10^{-9}	10^{-9}	10^{-9}	10^{-9}	
A ...	...	...	33·82	− 37·77	50·70	19°·50′
B ...	...	...	− 25·25	− 26·95	36·93	22°·50′
C ...	...	...	+ 30·62	− 2·65	30·74	155°·3′
D ...	...	...	+ 3·1	− 19·43	19·63	79°·10′
E ...	−177·15	+ 30·93	− 62·75	− 17·67	65·19	196°
F ...	−180·33	34·29	− 55·00	− 33·57	64·44	221°·20′
G ...	− 61·56	16·47	13·88	− 14·35	19·94	313°· 0′
H ...	− 61·96	− 2·69	32·25	− 15·57	36·21	334°·15′
J ...	− 94·53	6·72	− 15·25	0·88	15·30	176°·20′
K ...	−111·22	16·81	− 10·75	− 11·48	15·72	226°·55′
L ...	− 66·33	203·72	14·50	3·97	15·04	15°·20′
M ...	− 50·44	11·43	15·88	− 0·66	15·89	357°·35′
N ...	— 62·36	15·46	3·125	1·99	3·706	32°·30′
O ...	− 85·00	13·45	− 2·00	− 8·39	8·627	256°·10′
P ...	− 89·35	20·51	2·125	− 15·68	15·82	277°·50′
Q ...	− 90·16	28·24	2·375	− 22·30	22·43	276°·50′
R ...	− 90·55	31·61	6·00	− 39·31	39·77	278°·50′
S ...	−107·64	10·42	+ 36·59	− 21·42	42·40	329°·40′
T ...	−106·84	56·84	− 32·26	− 43·06	53·80	223°· 0′
U ...	− 32·18	16·81	26·52	− 9·28	28·09	19°·20′
V ...	−135·04	8·07	− 31·1	2·67	31·22	184°·50′
W ...	−319·36	91·48	128·52	12·02	129·1	185°·20′
X ...	−388·20	61·2	126	30·25	129·80	193°·40′
Y ...	−308·40	− 34·96	123·5	53·0	134·35	203°·10′
Z ...	−119·5	28·24	− 31·62	− 21·42	38·18	214°·10′
W_1 ...	...	...	+102·25	−170·9	199·2	187°·50′
Y_1 ...	...	...	+125·50	−148·8	194·6	194°·10′
T_1 ...	...	...	− 15·9	− 66·05	68·0	237°·30′

The curves in Fig. 2 denote the lines of equal resultant gradient, and are numbered to correspond with the magnitude of the gradient, whilst their direction is always normal to the curve.

The arrows and curves show very markedly the gravitational attraction of the neighbouring masses. Although it was impossible to set up the balance very near the walls and pillars, their effects can clearly be traced, particularly that of the north wall (which is a solid bank of earth extending several metres beyond the laboratory on each side). The line of stations R Q P O N shows where the gradient of the attraction of this wall is counterbalanced by that of the opposite walls and pillars. Near the point N the gradient vanishes completely and the gravitational attraction is a maximum.

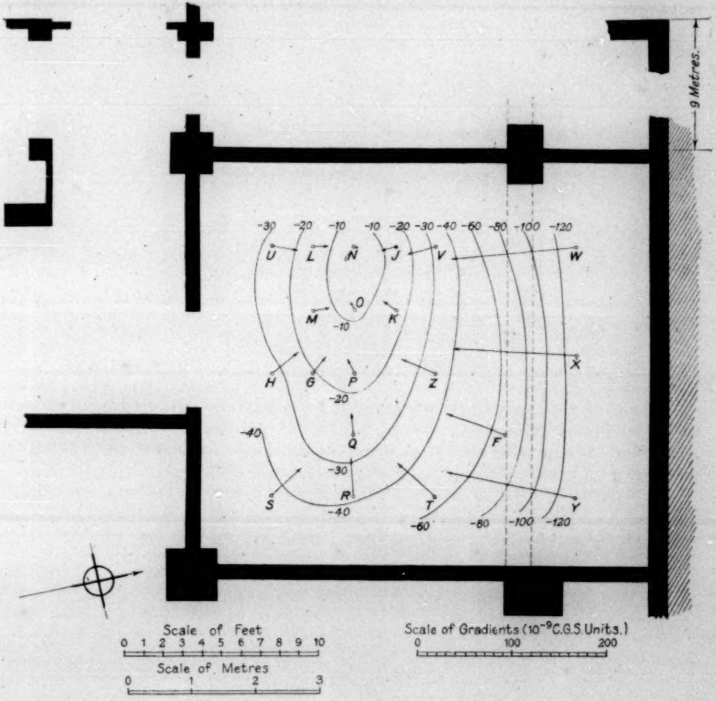

9 Metres.
-30
-20
-10
-10
-20
-30
-40
-60
-80
-100
-120
U
L
N
J
V
W
O
M
K
-10
H
G
P
Z
X
-20
-40
Q
F
-30
S
R
T
Y
-40
-60
-80
-100
-120
Scale of Feet
0 1 2 3 4 5 6 7 8 9 10
Scale of Metres
0 1 2 3
Scale of Gradients (10^{-9} C.G.S. Units.)
0 100 200

FIG. 2.

The point of maximum gravity, near N, is more on the west side of the room than would be expected from the approximate symmetry of the laboratory. This appears to be due to masses not represented in the figure, the laboratory being nearly in the north-west corner of the basement of the building.

V. COMPARISON OF OBSERVED RESULTS WITH THEORETICAL VALUES.

Owing to the large number of disturbing masses in the neighbourhood of the laboratory, and the impossibility of calculating the effect of each, only a rough comparison of the observed results with the theoretical effects was possible. It is, however, fairly easy to demonstrate that the observed magnitudes are of the same order and act in the same direction as the gradients due to the more important local masses. The gravity potential at the origin due to a mass, m grams, at the point (a, b, c) is

$$U_0=\frac{mG}{r}$$

where $r^2 = a^2+b^2+c^2$

and G = gravity constant$=66{\cdot}3\times10^{-9}$ C.G.S.

The gradients $\frac{\partial^2U}{\partial x\partial z}$ and $\frac{\partial^2U}{\partial y\partial z}$ at the origin due to this mass are easily shown to be

$$\left(\frac{\partial^2U}{\partial x\partial z}\right)_0=\left(\frac{\partial^2U}{\partial a\partial c}\right)_0=mG.\frac{3ac}{r^5}$$

$$\left(\frac{\partial^2U}{\partial y\partial z}\right)_0=\left(\frac{\partial^2U}{\partial b\partial c}\right)_0=mG.\frac{3bc}{r^5}$$

In the case of an attracting mass of finite dimensions, these expressions have to be integrated throughout the mass. For a rectangular block of density ρ having its sides parallel to the co-ordinate planes, the bounding sides being the planes $(x=a_1,\ x=a_2,\ y=b_1,\ y=b_2,\ z=c_1,\ z=c_2)$, we have

$$\left(\frac{\partial^2U}{\partial x\partial z}\right)_0=\left(\frac{\partial^2U}{\partial a\partial c}\right)_0=\rho G\int_{b_1}^{b_2}db\int_{a_1}^{a_2}da\int_{c_1}^{c_2}\frac{\partial}{\partial c}\left\{\frac{\partial}{\partial a}\left(\frac{1}{r}\right)\right\}dc=\rho G\int_{b_1}^{b_2}db\int_{a_1}^{a_2}\frac{\partial}{\partial a}\left[\frac{1}{r}\right]_{c_1}^{c_2}dc$$

$$=\rho G\ log_e\frac{s_{222}\,s_{211}\,s_{121}\,s_{112}}{s_{111}\,s_{122}\,s_{212}\,s_{221}}\quad\text{where } s_{121}=\sqrt{a_1^2+b_2^2+c_1^2}+b_2,\ \&c.$$

Similarly, $$\left(\frac{\partial^2U}{\partial y\partial z}\right)_0=\rho G\ log_e\frac{t_{222}\,t_{211}\,t_{121}\,t_{112}}{t_{111}\,t_{122}\,t_{212}\,t_{221}}$$

where $t_{121}=\sqrt{a_1^2+b_2^2+c_1^2}+a_1$, &c.

As an example, the gradients due to the north wall density $\rho=1{\cdot}92$ may be calculated for the point X.

In this case, X being the origin and the axes parallel to the walls, we have

$a_1=1{\cdot}15\ m$	$b_1=-13$	$c_1=-2{\cdot}5$
$a_2=+\infty$	$b_2=\infty$	$c_2=+1{\cdot}07$

Owing to the infinite values of a_2 and b_2, it will be found that the formulæ for $\frac{\partial^2 U}{\partial x \partial z}$ and $\frac{\partial^2 U}{\partial y \partial z}$ reduce to

$$\frac{\partial^2 U}{\partial y \partial z} = \rho G \log_e \frac{s_{112}}{s_{111}},$$

and we have

$$s_{112} = \sqrt{(1\cdot 15)^2 + 13^2 + (1\cdot 07)^2} - 13 \qquad s_{111} = \sqrt{(1\cdot 15)^2 + 13^2 + (2\cdot 5)^2} - 13$$

$$= 0\cdot 095 \qquad\qquad = 0\cdot 288$$

$$\log_{10} s_{112} = \bar{2}\cdot 9777 \qquad \log_{10} s_{111} = \bar{1}\cdot 4594$$

$$\log s_{112} = \underline{\bar{2}\cdot 9777}$$

$$\log \frac{s_{111}}{s_{112}} = 0\cdot 4817$$

$$\therefore \frac{\partial^2 U}{\partial x \partial z} = -\rho G \times 2\cdot 3026 \times 0\cdot 4817$$

$$= -141\cdot 1 \times 10^{-9} \text{ C.G.S. units.}$$

Similarly,

$$\frac{\partial^2 U}{\partial y \partial z} = -\rho G \times 2\cdot 3026 \times 0\cdot 0059$$

$$= -1\cdot 7 \times 10^{-9} \text{ C.G.S. units.}$$

By similar methods, Table V. has been calculated for stations Z, Y and S, allowing only for the more important local disturbing masses. The results are in reasonable agreement with the observed magnitudes at these stations.

TABLE V.—*Theoretical Gravitational Effects of Laboratory.*

Disturbing Mass.	σ	Station Z.		Station S.		Station Y.	
		$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
North Wall ...	1·92	−40·3	− 1·5	−14·8	− 1·0	−145·6	− 1·6
South Wall ...	1·8	+ 8·0	...	+17·3	+10·0	+ 3·0	+ 1·0
East Wall ...	1·8	...	−10·9	−10·8	−17·9	+ 10·8	−17·9
West Wall ...	1·8	...	+ 9·6	− 3·0	+ 4·0	+ 1·8	+ 4·3
S.E. Pillar ...	2·3	+ 2·7	− 2·5	+15·4	−15·4	+ 2·6	− 0·6
S.W. Pillar ...	2·3	+ 1·3	+ 1·2	...	+ 2·2	+ 1·5	+ 1·2
E. Pillar	2·3	− 1·8	− 4·2	− 3·3	− 1·3	+ 5·2	−15·8
W. Pillar... ...	2·3	− 1·3	+ 3·7	+ 1·0	− 1·0	+ 0·3	+ 1·8
E. Outer Wall ...	1·8	+ 3·5	− 1·0	+ 7·5	+ 3·0	+ 1·7	+ 0·2
Normal Values ...	...	+ 8·0	...	+ 8·0	...	+ 8·0	...
Resultant ...	...	−19·9	− 5·6	+ 9·0	−17·0	−119	− 27

Referred to axes parallel to the walls.

VI. CONCLUSIONS.

Although the present results afford no guarantee as to the behaviour of the balance under field conditions, they fully substantiate the claims made for it as regards sensitivity and consistency, and it seems reasonable to suppose that, if due precautions are taken to avoid interference from temperature and radiation disturbing influences, the instrument would function equally well in field operations.

In conclusion we desire to express our thanks to Col. H. G. Lyons, Director of the Science Museum, for the generous way in which he has facilitated this work.

DISCUSSION.

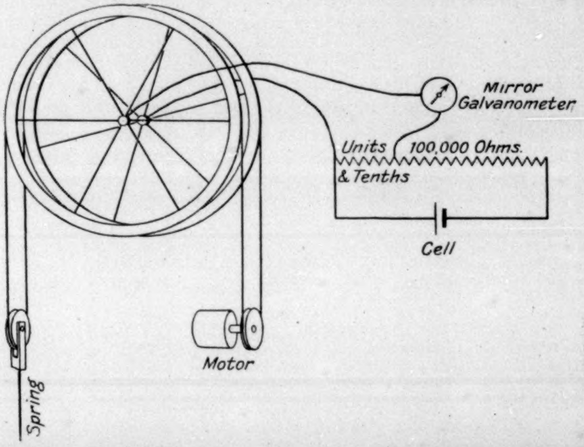
Dr. ALEXANDER RUSSELL said that the Authors' previous Paper on the Eötvös balance had excited considerable interest amongst scientists. The sensibility of the instrument is wonderful and the agreement between theoretical expectations and practical measurements extremely satisfactory.

Mr. E. LANCASTER-JONES said that the calculations were based on the assumption that third order derivatives of the gravitational potential could be neglected, but in the experiments described this condition was by no means fulfilled. The close agreement between theoretical and practical results was all the more surprising in view of this fact.

DEMONSTRATION of an Electromagnetic Inductor. *By* L. F. RICHARDSON, *F.Inst.P.*

THE apparatus shown is intended for teaching purposes, and serves to introduce students at an earlier stage than usual to a quantitative conception of electromagnetic E.M.F. Like the Lorentz apparatus, the present instrument involves a rotating conductor and produces a steady electro-motive force.

Two bicycle wheels are mounted vertically and co-axially, and are driven in opposite directions by a 4-volt motor, the driving band being constituted by an endless wire (stranded picture wire was used), which passes from the motor-pulley over one wheel, under an idle pulley, and back over the other wheel to the motor-pulley. The electro-motive forces generated by the revolution of the wheels in the earth's field are thus added (the rims of the wheels being electrically connected through the driving wire), and their sum can be tapped from terminals connected to the hubs of the wheels. The speed of the wheels is found by counting the revolutions against a stop-watch, one of the spokes being marked for this purpose, and from this speed and the length of a spoke the E.M.F. can be found in terms of H. It can be compared with that of a Daniell's cell by arranging the latter as the source of current in a potentiometer. The cell is connected in series with a fixed resistance of 100,000 ohms, and with a variable resistance of ohms and tenths, the inductor in series with a galvanometer being shunted across the variable resistance.



Results accurate to 4 per cent. can be obtained with care. Errors due to thermal E.M.F. are eliminated by turning the apparatus through 180 degrees and repeating the observations.

DEMONSTRATION of an Experiment Demonstrating Time-lag in Vision. By F. Ll. Hopwood, *D.Sc., A.R.C.Sc., F.Inst.P.*

The experiment* depends upon the fact that the time-lag in visual perception is greater for dimly than for brightly illuminated objects. A pendulum swinging in a plane carries a glow lamp at its lower end. A second glow lamp is fixed immediately below the mid-position of the swinging lamp, and both are viewed with one eye in the ordinary way, while in front of the other eye a metal disc perforated at its centre is so placed as to cut off most of the light reaching that eye. The pendulum then appears to be a conical instead of a plane pendulum, the apparent direction of rotation changing when the disc is transferred from one eye to the other.

This phenomenon is explained by the fact that the image seen by the obstructed eye corresponds to an earlier position of the swinging lamp than does the image seen by the free eye, in consequence of the greater time-lag in the former case. The images seen by the two eyes thus fail to coincide, and as they are combined by the binocular process into a single object, this object appears to be in front of or behind its true position according to the direction of swing and to whether the right or left eye is obstructed.

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* *See* C. Pulfrich, Die Naturwissenschaften, Heft 25-27 und 33-38 (1922).

DEMONSTRATION entitled Experiments on the Production of Electromotive Forces by Heating Junctions of Single Metals. By CHAS. R. DARLING, *F.I.C., F.Inst.P., and the Hon.* CHAS. W. STOPFORD

THE experiments shown were devised in order to explain certain effects observed during the progress of a thermo-electric investigation, in which contact had to be made occasionally by a cold metal with a hot piece of the same metal. Large E.M.F.s were noted, which could not be accounted for by the existence of strain in one or other of the pieces joined. In the first experiment a piece of bare copper wire was connected across the terminals of a galvanometer and cut at the middle. One of the cut ends was then heated in a burner, and touched with the cold end, when a large deflection was shown on the galvanometer, which fell to zero on the two ends attaining the same temperature. The "hot-to-cold" E.M.F. was thus shown to depend upon the existence of a temperature gradient at the junction, and was found in iron, nickel, zinc, graphite, and various alloys. In order to maintain such a gradient, two rods were taken, a V-groove being made in the end of one, and a chisel-edge filed on one end of the other. The V-groove was heated by placing a burner near the end of the rod, and the chisel-edge pressed into the groove. Connections were made to the galvanometer by wires of the same material as the rods, and by water-cooling just beyond the chisel-edge a steady gradient was obtained. A constant E.M.F. was generally obtained, which could be measured by a potentiometer. The following results were observed :—

Material.	Groove Temperature (approx.).	Direction of Flow.	E.M.F. (volts).	Resistance of Junction (approx.).
Copper	700°C.	Hot to cold through junction	0·25	40 ohms
Graphite ...	700	Ditto	0·015	50 ,,
Constantan ...	850	Ditto	0·3 (max.)	Variable
Iron	700	Opposite to above	0·024	120 ohms

The E.M.F. in all cases increased progressively with the groove temperature. The high voltages obtained with copper and constantan were noteworthy, and were due to some extent to the coatings of oxide which formed, and this might apply to other oxidisable materials. The result with graphite, however, showed that a coating of oxide was not essential. It was suggested that the magnitude of the E.M.F.s observed rendered the matter of importance, as ordinary thermo-electric effects might easily be overpowered in a circuit where a temperature gradient existed at a joint. It was added that a copper "hot-to-cold" junction served as a wireless detector.

DISCUSSION.

Dr. J. S. G. **THOMAS** called attention to an account of experiments somewhat similar given by Prof. Carl Benedicks in a lecture at the Institute of Metals in 1920.

Dr. H. **BORNS** said he hardly understood why Mr. Darling was surprised at his strong thermo-electric forces. When he broke his copper wire with a pair of pincers the two ends of the wire would not be strained equally. The one end of his iron rod (and of his graphite rod) was grooved, the other filed to chisel shape. The one end was heated in a gas flame to about 700°C. ; the temperature gradients were very steep, and the conditions were favourable to oxidation and other chemical effects, expulsion of gases (from the graphite), and to pronounced heterogeneity.

Mr. F. E. **Smith** commented on the extreme difficulty of treating metals in any way without affecting their constitution. In trying to make a junction with E.M.F. as low as 10^{-7} volts he had found it impossible even to cut silver and platinum wires without straining them, and by twisting the wires he obtained a much greater effect. One inference from the omnipresence of thermo-electric voltages is that in the Wheatstone Bridge the galvanometer key ought to be closed before the battery key, contrary to the usual practice.

Dr. D. **Owen** said that the voltages obtained appeared to be of the order to be expected from thermo-electric currents at contacts of oxides or sulphides of the metals. Had the authors tried the effect with platinum ?

Dr. J. A. **Harker** said that in using the Callendar-Griffiths Bridge some years ago to measure the resistance of copper wire, he had found that if the wire were asymmetrically heated two different values of its resistance were obtained for the two directions of the current through the bridge. The two values drifted further apart with lapse of time. Lord Kelvin regarded the phenomenon as a modified form of the Kelvin effect.

Mr. R. S. **Whipple** asked whether the E.M.F. obtained is affected by the pressure at the junction. If the effect is chemical, the compression would affect the thickness of any film formed.

Dr. E. H. **Rayner** said that thermo-electric currents were formerly dealt with very briefly in the text-books. He would like to know how the subject is now handled in the class-room.

Dr. F. Ll. **Hopwood** said that information bearing on the subject of the Demonstration could be found in the published writings of Richardson and Benedicks.

Mr. A. **Campbell** (communicated) : As the result of experiments on iron and mercury, J. M. Benade (Phys. Rev., p. 199, Vol. 18, 1921) concluded that the effects shown by unsymmetrical heating in solids are due to lack of homogeneity. He found that the effect was absent in mercury.

DEMONSTRATION of the Double Refraction due to Motion of a Vanadium Pentoxide Sol, and Some Applications. By R. H. HUMPHRY, *M.Sc.*

A COLLOIDAL solution of vanadium pentoxide in water has been found by Freundlich, Diesselhorst and Leonhardt (1915) to show double refraction on stirring when it has been aged either artificially or by being allowed to stand. The phenomenon is due to the formation of rod-shaped particles which set themselves in a definite direction when the liquid is caused to move. In linear flow the liquid behaves in the same way as a plate of uniaxal crystal cut parallel to the axis, and placed with the axis parallel to the direction of flow. In convergent light, with the flow along the line of sight, the familiar cross and rings are produced.

The following applications of this double refraction were demonstrated:

(1) Deviation from rectilinear flow owing to the presence of an obstacle. For this purpose the nicol prisms are set in directions perpendicular to and parallel to the direction of the stream.

(2) The thermal convection stream from a wire heated electrically; the nicol prisms are set at 45° to the direction in which it is desired to investigate the stream. These are also the positions of the nicols for (3) and (4).

(3) Efflux of liquid from a jet below the surface.

(4) Disturbance caused by the fall of a liquid globule of nearly the same density through the solution.

(5) Disturbances below the surface due to surface forces when a drop of alcohol is placed on the surface. Only the effect in certain directions could be observed owing to the positions of the nicols.

The colloidal solution of vanadium pentoxide is the most sensitive of all those which show a similar effect. It was kindly prepared by Mr. E. Hatschek in the following way:

Concentrated nitric acid is diluted to 10 times its volume. 0·5 grm. of ammonium vanadate is placed in a mortar and 2 cc. of the dilute acid are added and the mixture stirred. A further 2 cc. of the acid are added and the pentoxide is washed on a filter until it begins to pass through. It is then taken from the filter paper, placed in 200 cc. of distilled water and allowed to stand for 12 hours in a warm place. In 14 days the solution shows excellent striæ between crossed nicols.

DISCUSSION.

Mr. E. HATSCHEK said that the experiments called for exceptional cleanliness as the sol s very sensitive to electrolytes. Aniline blue is interesting in that it gives colour effects; it has a well-marked absorption band, and behaves like a positive crystal to wave-lengths on one side of the band, but like a negative crystal to wave-lengths on the other side.

Mr. R. W. PAUL asked whether the effect of subjecting the liquid to an electrostatic field had been tried.

Mr. J. GUILD said he was not satisfied as to the cause of the double refraction. The colloidal particles are too large to have the effect of molecules, and too small to be regarded as transmitting light after the manner of glass dust. In his reading of ultra-microscopic literature he had come across nothing which suggested a satisfactory explanation. In interpreting the appearances obtained by Prof. Coker's method the greatest caution is necessary. The intensity of the effect of a strain or a streamline depends on its direction, and for certain directions it is impossible to detect a strain at all.

In reply to the discussion Mr. **Humphry** said that the experiments which were shown were only to be regarded as preliminary, and no interpretation of the observed effects had yet been attempted. It seemed possible by use of this effect to throw more light on streamline problems, and with better apparatus, designed to overcome any tendency towards gelation, useful results might be obtained. The effect of an electric field on the arrangement of the particles had been worked out by Freundlich (Kapillar Chemie, 1922), who also explained the production of the double refraction. It is true that with stationary nicol prisms certain stream lines are accentuated, while others may not appear at all; but this difficulty would be overcome by having rotating nicols. The more delicate means of examining polarised light might show effects which were missed with nicol prisms.

THE PHYSICAL SOCIETY OF LONDON
AND
THE RÖNTGEN SOCIETY.

A DISCUSSION
ON
X-RAY MEASUREMENTS.

Held on February 23, 1923.

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DISCUSSION ON X-RAY MEASUREMENTS.

Dr. Alexander Russell (in the Chair) said that all present would receive with regret the announcement just received of the death of Prof. W. K. von Röntgen. He spoke of the beneficent services to humanity of the discoverer of X-rays, and asked the meeting to signalise their respect for his memory by standing.

The Chairman then called on Sir William Bragg to introduce the discussion on "X-Ray Measurements."

Sir **William Bragg**, K.B.E., F.R.S., said that the subject of X-ray measurements was a wide one, but the discussion at this meeting would be mainly confined to the measurement of intensity for the practical purposes of radio-therapy. Methods evolved in recent years make it possible to analyse a heterogeneous beam and measure the wave-length and intensity of its components with extreme accuracy; but although valuable in connection with pure physics, such methods are not suitable for radio-therapy. Here the intensity of the beam as a whole is required, whereas crystal methods introduce effects dependent on the nature of the crystal, such that it would be difficult to synthesise the composition of the original beam from observations made upon its components. The radio-therapeutist must make some assumption as to the distribution of energy amongst these components, and by absorption or otherwise determine the general intensity. In addition to the main beam the scattered radiation has to be considered. It is probably far more considerable than is generally realised, and needs to be taken carefully into account.

THE MEASUREMENT OF X-RAY INTENSITY AND THE NECESSITY FOR AN INTERNATIONAL METHOD.

BY

S. Russ, *D.Sc., F.Inst.P., Physics Dept., Middlesex Hospital.*

Physicists rely mainly upon ionisation methods for the measurement of X-ray intensity, and a very large variety of devices is now available by which the intensity of a beam of X-rays can be gauged.

It is important that the instrument should be equally reliable for the measurement of X-rays of different wave-lengths. This is not at all an easy matter because of the complicated phenomena which appear when a primary beam of X-rays enters an absorbing medium. Let us consider a beam of X-rays of intensity I_0 and wave-length λ_0 entering an air space of length x (Fig. 1). The intensity is defined as the amount of energy incident per second upon unit area placed normal to the rays.

On entering this space some of the radiation is scattered, some is absorbed, and the remainder is transmitted. We may write this as follows :—

$$I_0 = sI_0 + s'I_0 + q_0 + I_0 e^{-\mu_0 x},$$

where s and s' are the scattering coefficients in the backward and forward directions, q_0 is the quantity of energy quenched per second in the air space and causing ionisation, and the last term is the fraction transmitted, μ_0 being the absorption coefficient of the medium for the wave-length specified ; heating effects are neglected here.

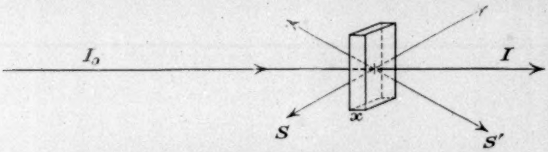

Fig. 1.—Intensity Effect Meter.

Consider now a beam of different wave-length λ_1 and of intensity I_1, then, as before, we have—

$$I_1 = s_1 I_1 + s_1' I_1 + q_1 + I_1 e^{-\mu_1 x}.$$

Now it is possible experimentally to arrange that an equal intensity of ionisation in the meter is produced by these two beams, so that $q_0 = q_1$.

Hence, for this particular case we may say

$$\frac{I_1}{I_0} = \frac{1 - e^{-\mu_0 x} - (s + s').}{1 - e^{-\mu_1 x} - (s_1 + s_1').}$$

Now if the terms involving the absorption coefficients are more significant than those involving the scattering coefficients—and this can be arranged experimentally —we may write

$$\frac{I_1}{I_0} = \frac{1 - e^{-\mu_0 x}}{1 - e^{-\mu_1 x}}.$$

The above expression shows that if we wish to compare the intensities of two beams of different wave-lengths, then this can only be done if we know the coefficients of absorption of the medium.

It may be noted here that even if measurements are restricted to beams of X-rays of a single wave-length, the ordinary ionisation chambers can hardly be said to measure the intensity because the bulk of the radiation is transmitted through them, and so they should perhaps be more correctly looked upon as intensity-effect meters.

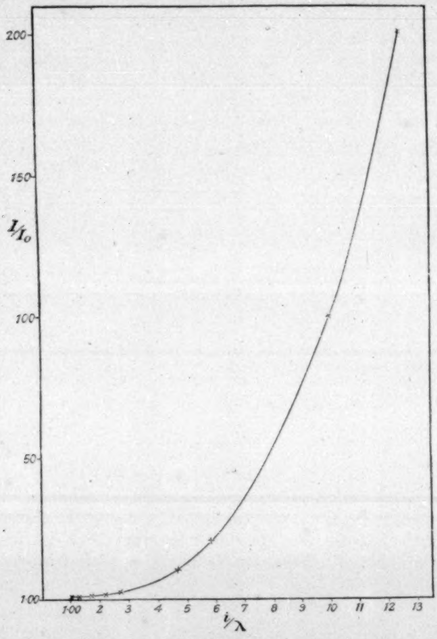

FIG. 2.

E. A. Owen has shown that a simple relation holds between the wave-length of a beam of X-rays and the absorption coefficient in a medium, provided that no "characteristic" radiation plays a part in the process of absorption. This relation is $\mu \propto \lambda^{5/2}$, and subsequent work has shown that $\mu \propto \lambda^3$ is of even more general application. By means of this last expression we can express the ratio I_1/I_0 in terms of the wave-lengths, and the graph of Fig. 2 shows how this ratio increases as the wave-length is shortened. The range of wave-length includes practically the whole extent of the radiation which is used in medical radiology.

The simple relationship outlined is only true under certain experimental conditions. They are, however, not too exacting to be practical. Perhaps the most important thing to bear in mind in designing an ionisation chamber suitable for comparisons of the intensity of different wave-lengths is that any secondary radiation produced in the instrument should not vitiate the results. There seems to be very little doubt that the simple air gap is the best for this purpose, for directly the rays are allowed to strike the solid walls of the chamber or the electrodes, secondary radiation may be produced by one wave-length but not by another, and so in this way true measurements are not obtained.

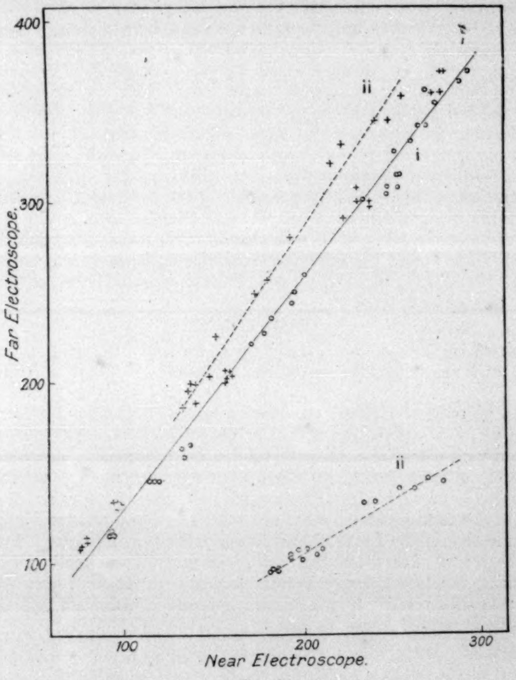

FIG. 3.

As an example of this, Fig. 3 shows two sets of experimental results, which were obtained in the following way :—

An X-ray tube was set up in such a way that it could send a vertical beam to a near electroscope (40 cm.) and a horizontal one to a far electroscope (4 metres). In experiment No. 1 the near electroscope consisted of a simple air gap cut transversely in a lead cylinder, the electrodes being set so that the vertical beam of X-rays just missed them. The ionisation in this gap was measured by means of a gold leaf connected electrically to one of the electrodes. The far electroscope consisted of an ordinary gold leaf electroscope made of lead fitted with a central brass stalk holding the leaf and a window of thin aluminium to allow the X-rays to enter the instrument.

Simultaneous readings were obtained upon the two electroscopes with (1) a beam of " soft " X-rays and (2) a beam of " hard " rays.

It was easy to arrange that the near electroscope gave identical readings for the " hard " and " soft " rays—but when this was the case the reading of the far electroscope was considerably greater with " hard " than with " soft " rays until a diaphragm was used to cut down the radiation so that identical readings were also obtained on the far electroscope with " hard " and " soft " rays. This is shewn by the single full line *I* going through the crosses (" soft " X-rays) and circles (" hard " X-rays) (Fig. 3). In experiment No. 2 the only change made was in the near electroscope. A paper cylinder 3·5 cm. long and 1·5 cm. diameter, fitted with a central electrode of aluminium, was substituted for the simple air-gap. Exactly the same thing was now carried out with this arrangement, and the two sets of experimental values show that the composition of this ionisation chamber has greatly favoured the soft type of radiation. It will be seen that in order to get an equality of readings with " hard " and " soft " X-rays on this ionisation vessel it was necessary to use a beam of " hard " rays about 2½ times as intense as would have been sufficient had the simple air-gap been used.

CONCLUSION.

Little remains for me to say about the necessity which exists for a method of comparing and measuring the intensities of beams of X-rays of different wavelengths.

In the practice of medical radiology a variety of indicators is used. Each has its use within restricted limits, but no one of them can at the present time be looked upon with complete confidence to give the information desired.

In purely scientific work, each one puts up some method which serves the purpose in hand, but it is almost impossible at present for any research centre to make use of the data given by other centres. For these reasons I hope that this combined meeting of the Physical and Röntgen Societies (February 23, 1923) may mark the beginning of an attempt to set up an unequivocal method of comparing and measuring the intensities of beams of X-rays of different wave-lengths, and that it may then be possible to specify some quantity which shall be looked upon as an international unit of X-ray energy.

THE QUALITY OF X-RAYS PRODUCED BY VARIOUS HIGH-TENSION GENERATORS AND AN INCANDESCENT CATHODE TUBE.

BY

F. J. HARLOW, *B.Sc., F.Inst.P., A.R.C.Sc., and* E. J. EVANS, *B.Sc.*

ABSTRACT.

1. Measurements of the absorption in aluminium have been made of the X-radiation emitted by an incandescent cathode X-ray tube excited by various forms of high-tension generator. The results show that after traversing 1 cm. of aluminium the coefficient of absorption in aluminium becomes approximately constant.

2. The radiation transmitted through 1 cm. of aluminium is defined as the "end-radiation," and it is shown that for the same value of equivalent spark-gap different machines give different qualities of "end-radiation."

The induction coil is shown to give different results at the same equivalent spark-gap with different lengths of interrupter contact.

3. Curves showing the observed coefficient of absorption of the "end-radiation" plotted against peak voltage deduced from the equivalent spark-gap are different for different machines, and for the induction coil operated with various lengths of interrupter contact. It is consequently deduced that measurements of equivalent spark-gap do not provide a measure of the hardness of the "end-radiation."

4. The important result is obtained that the absorption curves for the various machines under conditions of the same quality of "end-radiation" are indistinguishable, from which it follows that under these conditions X-radiation is produced in which the intensity distribution in the spectrum is the same for all the machines. Somewhat similar results have been obtained by A. Dauvillier using the X-ray spectrometer. The same result obtains with the induction coil operated with various lengths of interrupter contact.

5. A practical method of measuring the general quality of X-radiation is suggested.

6. The discrepancy between the result obtained with the Wimshurst machine and that obtained by A. Dauvillier with an arrangement producing constant potential led to the suggestion that the output of the former might be of an intermittent character. Experiments are described showing that intermittent X-radiation is produced when the Wimshurst machine is employed as the high-tension generator. A further investigation of this phenomenon is in progress.

INTRODUCTION.

THE relative advantages of various types of high-tension generator in the production of X-rays have been the subject of considerable discussion by radiologists and others during recent years. The following is an account of experiments carried out by the authors in an attempt to elucidate some of the phenomena hitherto unexplained. Reference to the work has already been made by one of the authors in the discussion on the subject* at the joint meeting of the Institution of Electrical Engineers, the Royal Society of Medicine, and the Röntgen Society, held in February, 1920. Publication of the complete results has been delayed pending further investigation with the Wimshurst machine which appeared desirable.

The high-tension generators available for the experiments were the induction coil, two types of closed magnetic circuit high-tension transformer and the Wimshurst machine. These machines are largely used in the production of X-rays in medical and industrial practice.

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* *See* Journal Inst. Elect. Eng., Vol. 58, No. 294, August (1920).

At the time the work was undertaken the opinion was generally held among radiologists that high-tension interrupterless transformers, although possessing many advantages in practice, were, when operated under similar conditions as to spark-gap and milli-amperage, less productive than the induction coil, of rays of the quality most useful in radiography.

The authors of the present Paper considered that this might be due to a marked difference in the distribution of intensity among the various wave-lengths due to the difference in potential and current wave forms given by the two types of machine.

Ulrey* has shown that the X-ray output is proportional to the square of the voltage through which the generating cathode rays fall ; and the higher the voltage the harder is the radiation produced. The distribution of intensity among the various wave-lengths of the spectrum should, therefore, it seems, depend upon the potential and current wave forms, and also upon the phase difference, if such exists, between current and potential.† A complete investigation of the problem requires the taking of oscillograms of potential and current and the correlation of these with the observed distribution of X-ray intensity.

In the absence of a suitable oscillograph and of an X-ray spectroscope this could not be undertaken, but it appeared that an examination of curves of absorption in aluminium for the radiation produced by the various machines might contribute towards the solution of the ploblem.

APPARATUS AND EXPERIMENTAL ARRANGEMENTS.

Absorption curves in aluminium were obtained by means of the apparatus shown in Fig. 1 (*a*).

A is a heavily protected tube box containing the X-ray tube with an adjustable rectangular diaphragm at *D*.

B is a lead plate having a rectangular opening and also a shelf fixed at (*F*) to hold aluminium filters.

C is an ionisation chamber, the plate *P* being connected to one pair of quadrants of a Dolezalek electrometer.

All the points marked *E* were connected to earth, while the other wire connections were insulated by pieces of ebonite marked *I*.

The X-rays were passed through different thicknesses of aluminium and the ionisation currents produced in the ionisation chamber by the transmitted rays were measured.

In order to reduce to a minimum the effects of secondary radiation, either scattered or fluorescent, the distances between the diaphragm, aluminium strips and ionisation chamber were made large.

Different methods of measuring ionisation currents were tried, and the most suitable for such large intensities was found to be the steady deflection method as used by Bronson.‡

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* Physical Review, p. 407, May (1918).

† Dauvillier, Revue Générale de l'Electricité Mars (1917).

‡ Phil. Mag., pp. 143-146, January (1906).

This method consists in connecting to earth through a high resistance the pair of electrometer quadrants receiving the ionisation current. The quadrants thus become charged to a potential such that the rate of leak to earth is equal to the charging current ; and a steady deflection of the needle is obtained dependent upon and proportional to the ionisation current. The resistance employed was about 10^{10} ohms, and consisted in a mixture of xylol and alcohol contained in a glass U-tube (see Fig. 1 (*b*)).

The steady deflection method of using the electrometer has the advantage in work covering a wide range of ionisation current in that the sensitiveness can be adjusted by varying the value of the high resistance ; provision for which was made as indicated in the diagram by a number of platinum terminals at various points along the limbs of the U-tube.

The X-ray tube employed was a Coolidge tube of the self-rectifying radiator type, the cathode filament of which was heated by an insulated battery in the usual manner.

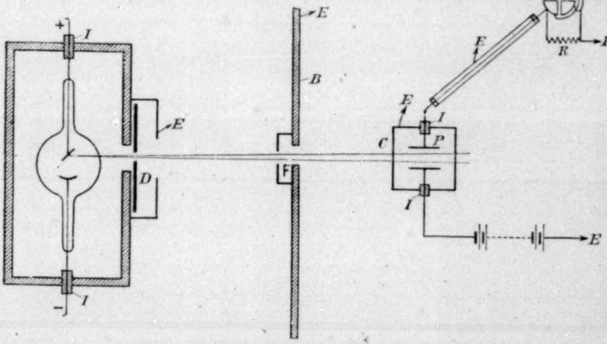

FIG. 1(*a*).

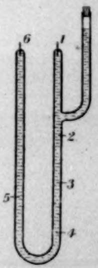

FIG. 1(*b*).

The high-tension generators available for the purpose of the investigation were as follows :—

(*a*) Induction coil, 16 inch bi-sectional, with mercury jet interrupter.
(*b*) Small high-tension oil transformer, U.S. Army pattern.
(*c*) Large high-tension transformer, Snook pattern.
(*d*) Large 20 plate sectorless Wimshurst machine, plates 22 ins. diameter.

Our thanks are due to Sir Archibald Reid, K.B.E., to the Army Medical Department, War Office, to the Medical Supply Association and to Messrs. Newton & Wright, Ltd., for their kind assistance in placing apparatus at our disposal.

In order to excite the X-ray tube by the various machines under similar conditions of equivalent spark-gap, spheres of diameter 5 cm. were employed, and in every case it was found possible to arrange that throughout an experiment the equivalent spark-gap should be maintained sufficiently constant to obtain steady and consistent readings of the electrometer. The conditions were adjusted so that during a run an occasional spark passed between the spheres.

EXPERIMENTAL RESULTS.

A typical set of logarithmic absorption curves as obtained with the various types of high-tension generator for different spark-gap values is shown in Fig. 2. For each curve the initial intensity was made the same by adjustment of the diaphragm, other intensities being expressed as percentages of the initial intensity; the slope of the logarithmic absorption curve is a measure of the coefficient of absorption, and it will be seen that the radiation after passing through 1 cm. of aluminium is approximately of a homogeneous character as measured by its coefficient of absorption (μ), a fact already established by other workers—*e.g.*, by Russ* and by Rutherford† and others.

The radiation remaining after constancy of coefficient of absorption is reached is usually spoken of as the "end-radiation,' and for the sake of clearness of description it is convenient to define the "end-radiation" as that remaining after absorption by 1 cm. of aluminium.

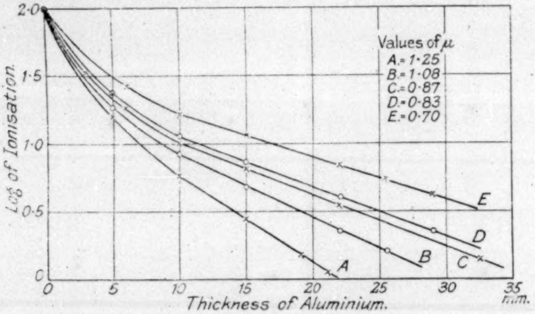

FIG. 2.

COMPARISON OF ABSORPTION CURVES FOR THE DIFFERENT MACHINES UNDER CONDITIONS OF SIMILAR EQUIVALENT SPARK-GAP.

In Figs. 3, 4 and 5 are shown three different sets of logarithmic absorption curves selected from a large number. Each set of curves corresponds to a definite length of equivalent spark-gap, the different curves being obtained with the various machines. The stated voltages are calculated from the lengths of equivalent spark-gap by means of the tables prepared by Kaye.‡ Fig. 3 shows two curves at 67 k.v.

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* J. R. S., April (1915).

† P. M., September (1915).

‡ *See* Cantor Lectures, R. S. A., March (1921).

for the induction coil and Snook transformer. Fig. 4 shows five curves at 78 k.v. one for each of all the machines employed. In this case the curves for the induction coil (used with a long contact in the interrupter), and the Snook transformer coincide. Fig. 5 shows three curves at 86 k.v. taken with the Snook transformer, the Wimshurst machine and the induction coil.

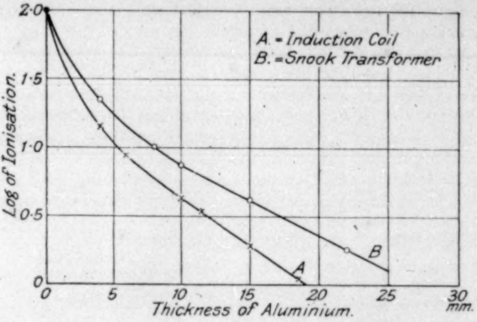

FIG. 3.

It appears to be fairly generally assumed in X-ray practice that whatever type of machine is employed or under whatever conditions any one type of machine may be employed, if the equivalent spark-gap is the same the same quality of radiation is produced. In radiography as a rule, it is the "end-radiation" which reaches the photographic plate and which consequently is the effective radiation so that it will be the "end-radiation" which is assumed to be the same for the same equivalent

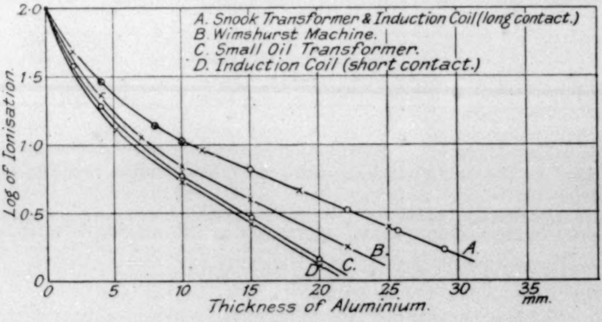

FIG. 4.

spark-gap. An examination of the curves in Figs. 3, 4 and 5 shows that for the same equivalent spark-gap the different machines give very different qualities of "end-radiation." Further, curves A and D (Fig. 4) were both obtained with the induction coil under similar conditions except for length of interrupter contact. With the longer contact the rays were more penetrating.

It was anticipated when the research was undertaken that a set of curves obtained at the same equivalent spark-gap would show considerable differences in their initial portions, but that the "end-radiation" would be of the same quality (same slope of logarithmic absorption curve). Had this been the case we purposed comparing the suitability of the various machines for radiographic purposes by ascertaining in each case the ratio of the intensity of the "end-radiation" to that of the initial radiation, and comparing the ratios obtained with the various machines. Those machines yielding small percentages of "end-radiation" would obviously not be so useful for radiographic purposes as those yielding larger percentages, as the softer radiation, being absorbed in the object to be radiographed, is of no radiographic value. However, as the various machines gave different qualities of "end-radiation" for the same equivalent spark-gap no useful purpose would be served by comparing the relative percentage amounts of "end-radiation" present in the initial heterogeneous beams.

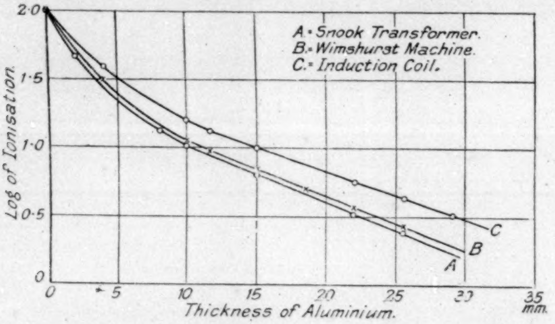

FIG. 5.

EQUIVALENT SPARK-GAP AND HARDNESS OF "END-RADIATION."

The curves in Fig. 6 show for each machine the way in which the coefficient of absorption in aluminium of the "end-radiation" varies with the peak voltage as calculated from the equivalent spark-gap. The curves for the various machines are distinctive, variations in hardness being as a rule very considerable for the same spark-gap, as has already been pointed out in connection with the curves of Figs. 3, 4 and 5.

It will be seen that the curve "a" for the Snook transformer crosses one of those for the induction coil curve "b"; this means that for the lower values of equivalent spark-gap the induction coil gave softer radiation than the transformer, the reverse being true for the larger values of gap. Results at one value of equivalent spark-gap only, namely, 78 k.v., represented by point "e," were obtained with the small oil transformer without rectifier. The hardness in this case is less than for the Snook transformer employing a rectifier.

It is possible that this latter result is due to the inverse voltage being greater than the direct through the suppression of the inverse current, and when the tube acts as rectifier the spark-gap will be that equivalent to the inverse voltage rather than to the direct.

By varying the conditions of excitation of the coil, viz., by shortening the interrupter contact a different curve "*d*" was obtained. Curve "*c*" shows the results obtained with the Wimshurst machine.

The experiments appear therefore to establish quite clearly and definitely that, when operated under conditions of similar equivalent spark-gap, different forms of high-tension generator, when employed to excite an incandescent cathode X-ray tube, produce different qualities of effective or "end-radiation"; similarly, if the length of contact in the interrupter of an induction coil be varied and the current adjusted to produce the same equivalent spark-gap a different quality of "end-radiation" results. The equivalent spark-gap does not therefore give a measure of the relative hardness of "end-radiation" produced by different machines, neither does it give a measure of the relative hardness of "end-radiation" produced by the induction coil under different conditions of excitation.

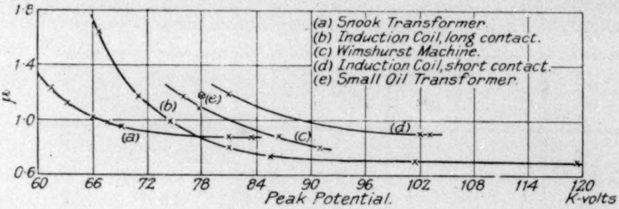

FIG. 6.

COMPARISON OF ABSORPTION CURVES FOR THE DIFFERENT MACHINES UNDER CONDITIONS OF SIMILAR HARDNESS OF "END-RADIATION."

In view of the above procedure failing to provide a satisfactory basis for comparing the performance of the machines, it appeared desirable to examine the results with a view to comparing the percentage intensities of "end-radiation" under conditions such that all the machines were producing the same quality of "end-radiation" (*i.e.*, giving the same slope of logarithmic absorption curve after absorption by 1 cm. of aluminium).

In Table I. are shown under conditions of similar hardness of "end-radiation" the intensities of "end-radiation" expressed as percentages of initial radiation. This condition may be attained by a suitable choice of equivalent spark-gap. The results are shown for several different hardnesses of "end-radiation," the hardness being indicated by the coefficient of absorption in aluminium.

TABLE I.

Coefficient of absorption in aluminium.	Per cent. Intensity of "End-Radiation."				
	Small oil transformer.	Snook transformer.	Induction coil (long contact).	Induction coil (short contact).	Wimshurst machine.
1·11 to 1·18	6·1	5·9	5·8	5·7	6·3
1·03	...	8·3	8·2	...	7·8
0·86	...	11·7	11·0	...	11·5
0·83	...	11·9	12·0	...	...
0·81	...	12·3	12·4	...	...
0·77	...	...	16·6	...	16·9

It will be observed that within the limits of experimental error the percentage intensity of "end-radiation" is the same for all machines. Further when the absorption curves are plotted for the same hardness of "end-radiation," they are indistinguishable.

A typical curve is shown in Fig. 7, the experimental data for which are given in Table II.(A). Other experimental data giving indistinguishable curves are set out in Table II.(B), (C) and (D).

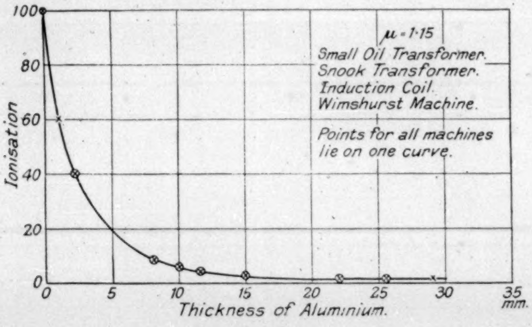

FIG. 7.

The result given on following page is of considerable technical importance. It shows that by a suitable adjustment of the conditions of equivalent spark-gap it is possible to obtain the same quality, not only of the "end-radiation," but of the initial radiation whichever machine is employed; or, in the case of the induction coil the same quality of radiation with different lengths of interrupter contact.

In addition the result appears to indicate that the distribution of X-ray intensity in the heterogeneous beam of radiation is characteristic of the X-ray tube rather than of the potential wave-form of the high-tension machine.

TABLE II.—*Radiation Intensities after Filtration expressed as percentages of Intensities of Unabsorbed Radiation.*
(Same hardness of " end-radiation " for all machines.)

Set No.	Type of machine.	μ	P.D. in k.v.	Percentage intensity of radiation after filtration. Thickness of aluminium in mm. 0	1	2	4	6	8	10	11·5	15	18·5	22	25·5	29	32·5	34·25
A	Oil transformer ...	1·13	78	100	60	41	22·1	13	8·6	6·1	50	2·9	...	1·2	0·8	...	0·5	0·3
	Snook transformer	1·15	62	100	...	...	22	...	...	5·9	...	2·9	...	1·2	...	0·6	...	...
	Induction coil (long contact) ...	1·18	71	100	...	...	...	...	...	5·8	4·8	2·7	...	1·2	0·9	...	...	...
	Induction coil (short contact) ...	1·18	80	100	...	...	...	...	...	5·7	4·8	2·65	...	1·15	0·85	...	...	...
	Wimshurst	1·11	76	100	...	41	27	...	8·7	6·3	5·2	3·2	2·4	1·3	...	...	...	...

Set No.	Type of machine.	μ	P.D. in k.v.	Thickness of aluminium in mm. 0	4	5	8	10	11·5	15	18·5	22	25·5
B	Snook transformer	1·03	66	100	...	21	...	8·3	...	4·3	3·0	2·1	1·4
	Induction coil (long contact)	1·06	73	100	...	...	...	8·2	6·1	4·1	3·0	1·9	1·3
	Wimshurst	1·07	77	100	27	...	14	7·8	...	3·6	2·8	1·8	1·3

Set No.	Type of machine.	μ	P.D. in k.v.	Thickness of aluminium in mm. 0	2	4	6	8	10	11·5	15	18·5	22	25·5	29	32·5
C	Snook transformer	0·86	79·5	100	47·3	28	18	14	11·7	9·2	6·3	...	3·2	2·3	...	1·4
	Induction coil (long contact) ...	0·87	77·5	100	...	...	...	...	11	...	6·5	4·8	3·4	2·5	1·9	...
	Wimshurst	0·85	85	100	49	29	...	14	11·5	8·9	6·6	4·9	3·5	2·6	1·9	...

Set No.	Type of machine.	μ	P.D. in k.v.	Thickness of aluminium in mm. 0	2	4	8	10	11·5	15	18·5	22
D	Induction coil (long contact) ...	0·78	82	100	...	...	...	16·6	...	12	...	6·9
	Snook transformer	0·76	90	100	60	41	21	16·9	13	10	7·9	...

QUALITY OF X-RADIATION AND CURRENT THROUGH X-RAY TUBE.

If absorption curves are plotted for a given machine at constant equivalent spark-gap but for various initial intensities of radiation (produced by varying the filament heating current of the X-Ray tube), the general quality of the radiation remains the same. Fig. 8 shows four logarithmic absorption curves taken for the same equivalent spark-gap and with various currents passing through the X-Ray tube.

It will be observed that the curves are parallel indicating similarity of quality. It follows, therefore, that not only can the same quality of radiation be obtained for the various machines by a suitable adjustment of the conditions, but that by adjustment of the current passing through the tube the same intensity may also be obtained.

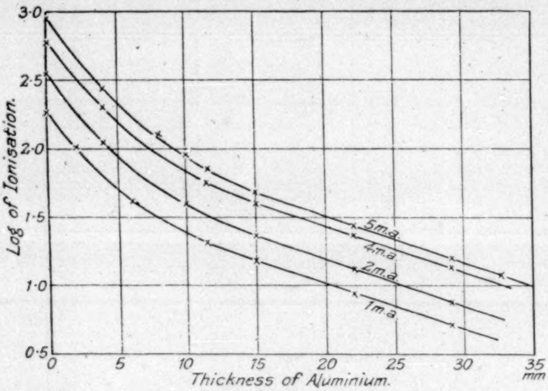

FIG. 8.

DISCUSSION OF EXPERIMENTAL RESULTS AND COMPARISON WITH THOSE OF OTHER INVESTIGATORS.

After the completion of the experiments described in the present Paper, results of an investigation along somewhat similar lines were published by A. Dauvillier,* who employed the X-ray spectrometer, which permits of a much more thorough analysis of the quality of radiation emitted. As sources of high-tension current he employed the induction coil, the closed-magnetic circuit high-tension transformer, and for constant potential a special arrangement consisting of an induction coil, a valve and a condenser charged by the induction coil through the valve. To secure equality of peak potential, Dauvillier employed the method of arranging that the shortest wave length in the spectrum is the same in each case. By reason of the quantum relationship $hc/\lambda = Ve$ this would appear definitely to provide for equality of peak and constant potentials. Dauvillier found that the induction coil and the transformer gave, for the same peak potential, similar spectral curves, *i.e.*, curves showing similar distributions of intensity among the various wave lengths. This result was obtained at 46 kv. and also at 95 kv. On the other hand, with the constant potential the spectral curve was relatively richer in intensity for the rays of shorter wave-length.

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* Ann. de Physique, Tome 21, Mars-Avril (1920).

Comparison of Results for Coil and Transformer.

Dauvillier's procedure with the coil and the transformer of adjusting the conditions until the shortest wave-length was the same in each case appears to correspond precisely to our selection of conditions giving the same quality of " end-radiation." Dauvillier's observations thus agree with ours in showing that under suitable conditions both the induction coil and the transformer produce radiation giving similar spectral curves. Further investigations, however, appear to be necessary to ascertain the reason why the values of the equivalent spark-gap, which were measured with care under ordinary practical conditions, are different for the two machines.

Dauvillier* recommends the measurement of the shortest wave-length in the X-ray spectrum as the most accurate method of measuring peak potential, and has in consequence, by an extended series of observations, calibrated both sphere and point spark-gaps for peak potentials produced in various ways. The obtaining of concordant results appears to have been a matter of considerable difficulty. When brass spheres were used special precautions had to be taken to maintain them in a highly polished state and, in addition, it was found necessary to include in the spark-gap circuit a resistance of distilled water of a million ohms to prevent the formation of arcs. With these precautions the calibration curves obtained with the coil, high-tension transformer, and apparatus for constant potential showed a fair agreement. They differed, however, considerably above 75 kv. from the curve obtained with the unrectified transformer, the peak voltages for which were not obtained by the X-ray method, but by calculation from the R.M.S. values measured by an electrostatic voltmeter. This divergence is attributed by Dauvillier to the effect upon the value of the spark-gap of ultra-violet rays produced by brush discharges from the high-tension leads of the X-ray tube circuit. A similar effect probably operated in our experiments. It was not found necessary, however, in our experiments to take such special precautions as those described by Dauvillier in order to obtain the reasonably smooth curves shown in Fig. 6.

The considerable differences obtained in our experiments with the various machines can scarcely be attributed simply to variations arising from contamination of the surfaces of the spheres, and it is considered that some other explanation must be looked for. In this connection it is of interest to note that in the earlier publication† of Dauvillier's reference is made to the fact that some of the oscillographic records indicate a small phase difference between current and potential arising presumably through the electrostatic capacity of the tube which, although small, would be sufficient with a peaked wave form to modify considerably the character of the radiation produced.

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* Loc. cit.

† Dauvillier, Revue Générale de l'Electricité, Mars (1917).

It may be mentioned that manufacturers of X-ray apparatus have observed that tube boxes fitted with earth-connected metal modify the X-ray output. It is conceivable that this may be due to the increased capacity causing a phase difference between the potential and the current. Such a capacity effect would obviously be different with different machines and might account for the variable hardness of "end-radiation" obtained in our experiments when the different machines are run under conditions of similar equivalent spark-gap. It should be observed that the tube-box employed was heavily protected with earth-connected sheet lead and in addition a large metal diaphragm was employed (Fig. 1). Observations to test this suggested explanation would require careful simultaneous oscillographic records of current and potential. These, however, we have been unable to make.

Results at Constant Potential.

In a brief account of researches on the spectral curves obtained with a generator at constant potential, A. W. Hull* points out that, in his opinion, it is possible to find a voltage and current for constant potential which will give a spectrum practically the same as that given by the transformer at a different definite voltage and current. The two similar spectral curves reproduced in the Paper are for constant and peak voltages of the same value, viz., 70 kv., the currents being 2 and 3 milliamperes respectively. Assuming the Wimshurst machine to generate current at constant potential, the obtaining in our experiments of similar absorption curves with the Wimshurst machine and the transformer lends support to the view expressed by Hull (which, however, does not appear to be supported by appreciable experimental evidence). As already mentioned, Dauvillier's experiments at constant potential have given a different result, viz., a spectral curve relatively richer in rays of shorter wave-length. It appeared, therefore, to the authors of the present Paper desirable to make further observations with the Wimshurst machine, and, as will be seen in a later section, experimental evidence has been obtained that the X-ray emission with the Wimshurst machine is intermittent in character. Under these circumstances the results obtained by Hull and Dauvillier are scarcely comparable with those obtained by us with the Wimhurst machine.

In connection with the above results it is interesting to notice a further result obtained by Dauvillier, who extended his observations to the gas form of X-ray tube. With a constant potential applied to the gas tube Dauvillier has shown that the discharge is intermittent.† In his later Paper‡ he shows that this gives rise to spectral curves practically indistinguishable from those obtained with the induction coil or with the transformer.

There is therefore both in our researches and in those of Dauvillier considerable experimental evidence that the application of an intermittent potential to an X-ray tube produces similar spectral curves under suitable conditions, whatever the wave form of the discharge.

It is not clear why this should be so, and, as already explained, it was expected when the present research was undertaken that differences in the spectral curves would be shown for the various machines.

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* Amer. Jour. Roert (1915).

† Dauvillier, Revue Générale de l'Electricité, Mars (1917).

‡ Ann. de Physique, Tome 21, Mars-Avril (1920).

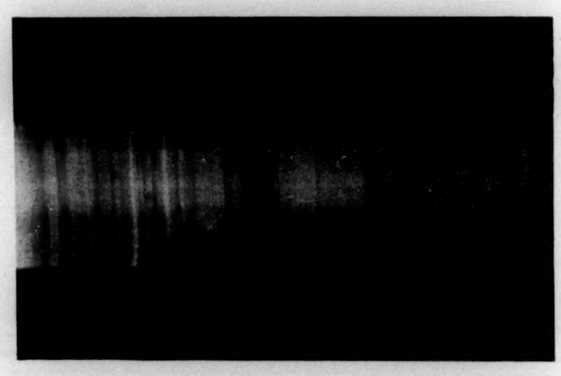

FIG. 10.—INTERMITTENCY OF DISCHARGE. (WIMSHURST MACHINE, COOLIDGE TUBE.)

[To face page 21D.

Criticism of the Absorption Method.

Mention should be made of the criticism of the absorption method given in Dauvillier's Paper (loc. cit.). The X-ray spectrometer method is admittedly a much more powerful method of analysis, capable of giving more detailed information concerning the quality of radiation emitted. It is, however, justifiable to make deductions of a general character from the results obtained by absorption method, as the pioneer work of Barkla and others has shown. In our experience the statement made by Dauvillier that a small experimental error makes a considerable alteration in the position of a point when plotted on the graph is not in accordance with fact.

It is disproved by the facility with which a smooth curve can be obtained, in spite of the several factors contributing to the intensity of radiation. We consider rather that for the harder radiations, owing to a high proportion of the absorption being due to scattering, the co-efficient for which varies little with wave length, the method lacks somewhat in sensitiveness as compared with the method of the X-ray spectrometer.

The curves of Fig. 2 show, however, that in spite of this consideration, the change with voltage in the coefficient of absorption of the "end-radiation" is considerable, and on this account the deductions we have made when similar absorption curves have been obtained under various conditions are considered to be justifiable.

Further Investigations with the Wimshurst Machine.

As already mentioned, the discrepancy between the results obtained with the Wimshurst machine and those obtained by Dauvillier at constant potential, rendered desirable an investigation into the character of the discharge from the Wimshurst machine. In the absence of an oscillograph to test the constancy of potential, the intermittency of X-ray emission was tested by a photographic method.

The apparatus is shown in Fig. 9, and consisted in a tunnelled photographic plate carrier *C* arranged so that the latter passed, at a rate which could be regulated, in front of a narrow slit (*S*) cut in the lead face *A* of the tunnel. The tunnel was arranged vertically, the carrier being counter-weighted, so that its speed in front of the slit could be regulated by adjustment of the weight *W*. For the purpose of ascertaining the speed of the plate a vibrating strip of known frequency fitted with a camel hair brush was arranged at the back of the tunnel. From the curve traced out on a paper strip gummed to the plate the speed could be determined. The remarkable result shown in Fig. 10 was obtained confirming the predicted intermittency, the frequency of which was found to be of the order of 15 per second. An experimental investigation of the cause of this intermittency is in progress by one of us (E. J. Evans). The discrepancy between the results obtained by Dauvillier at constant potential and those obtained in our experiments with the Wimshurst machine may thus be due to the fact that the current given by the Wimshurst machine is not generated at constant potential. A possible explanation is that the intermittency results from the collection of the charges from the plates of the machine by the phenomenon of the brush discharge for, as already pointed out, Dauvillier has shown that under certain conditions discharge through gases is of an intermittent character, even when a constant potential is applied. If the discharge at the collectors is of this character the potential applied to the X-ray tube will be intermittent.

SUGGESTED METHOD OF MEASURING QUALITY OF X-RADIATION.

The results obtained in the present Paper indicate at least this fact—namely, that the equivalent spark-gap as measured under conditions obtaining in X-ray practice is not a satisfactory measure, for purposes of standardisation, of the quality of the general radiation emitted by an X-ray tube where variable conditions of excitation obtain. A complete analysis of the radiation by the X-ray spectrometer is ideal, but scarcely within the range of practical realisation outside a well-equipped physics laboratory.

The results obtained with the various machines under conditions of constancy of quality of "end-radiation" show that standardisation is possible. Seeing that the same relative distribution of intensity can be obtained by a suitable adjustment of the conditions, whichever machine as used in our investigation is employed, the following standard method of measurement is suggested—viz., the percentage reduction of intensity by 1 cm. of aluminium. This reduction of intensity is unique for a definite intensity distribution, and corresponds to a definite quality of "end-radiation."

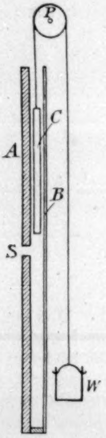

FIG. 9.

E. A. Owen* and P. K. Bowes have shown that the pastille method of measuring intensity agrees with the ionisation method; consequently the ratio of the time required for a pastille dose with an aluminium filter of 1 cm., to that without the filter would give a measure of the "hardness" of the radiation emitted by the tube. The procedure affords a method of measurement easily applied in practical radiology. For any given outfit the indications by this method could be calibrated against equivalent spark-gap.

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* Journ. Röntgen Soc., July (1921).

In conclusion, we wish to thank the Governors and Principal of the Sir John Cass Technical Institute, Aldgate, for the facilities granted in the carrying out of this research; also to express our indebtedness to Dr. D. Owen for valuable suggestions during the preparation of the Paper.

DISCUSSION.

Dr. G. W. C. **Kaye** (communicated): The main feature of the authors' Paper is, perhaps, their deduction that the sphere gap does not provide a trustworthy measure of the peak voltage operating an X-ray tube, or alternatively, of the hardness of the "end radiation." The work of Peek and others in America on the sphere gap has been generally accepted by electrical engineers, and the belief is current that the sphere gap is in general capable of an accuracy of a few per cent., and that it is unaffected by either periodicity or steepness of wave front. It is true that the added factor of an X-ray tube was not present in Peek's experiments, but, nevertheless, the work of Dauvillier (to which the authors refer) seems to show that no very striking anomalies, at any rate up to 100 kv., attend the use of the sphere gap when used in parallel with an X-ray tube.

The authors in the present Paper use peak voltages up to 86 kv., and one is led to examine their results. Some ambiguity arises from their use of the term "end radiation." Rutherford, who I believe coined the term, meant it to refer to the X-ray of shortest wave-length present—that is, the quantum limit of the X-ray spectrum, the frequency of which is directly proportional to the maximum potential employed A glance at a typical spectral curve shows that the intensity of this shortest wave in a spectrum of rays is very small, and thus to filter out the rest of the radiation with any approximation to completeness, the initial radiation must be cut down to a very small fraction.

Now if in Figs. 3, 4 and 5 of the authors' Paper we produce back to the "y" axis the straight-line portion of the curve, we readily ascertain that their "end radiation" refers to what is left after only 70 or 80 per cent. of the initial radiation has been absorbed. Their values of the absorption coefficients (μ) are average figures for the residual 20 or 30 per cent. It is perhaps, therefore, not surprising that no simple relation is found between μ and the peak voltage as measured by the sphere gap.

From experience of similar absorption measurements (*see* Kaye, Proc. Roy. Soc., Series A, Vol. 93, p. 427, 1917), one is aware how slowly the slope of the log absorption curves changes with the greater thicknesses, and how tempting it is to run a straight line through the observational points. But, nevertheless, one is really only drawing a tangent, and if one continues the observations not to a residual intensity of the order of 1 per cent., but far beyond, say, 1 in 10,000, one sees that the absorption curve is not really straight, but slowly flattening out all the time. Rutherford's observations (Phil. Mag., Sept., 1915) show this clearly for voltages from 22 to 96 kilovolts.

The above refers to "general" X-rays, but the presence of characteristic radiation may have to be reckoned with, as the presence of an appreciable amount of it tends to produce a straight region in the log absorption curve. Now it so happens that the critical voltage for the K-radiation of tungsten is about 70 kv. and the optimum about 100 kv. It seems likely then that the residual 20 or 30 per cent. on which μ is based contains an admixture of K-radiation to an extent which increases with the voltage.

It appears then that the authors' μ refers to no definite radiation, but may be associated with a wave-length which is, so to speak, the centre of gravity of the spectral curve of the group of residual rays containing both general and characteristic rays. It is probable that this mean wave-length approximates to that of the peak or maximum intensity of the spectral curve. On the other hand, the sphere gap refers (or so we have grown to believe) to the quantum limit the wave-length of which bears no certain relation to the mean wave-length with which the authors are dealing.

There is evidence to show, and it seems to me to be very probable, that the spectral curves of X-rays excited, *ceteris paribus*, by different types of potential generators may differ slightly if not appreciably in form. Such differences would be emphasised by filtering, and we therefore have an explanation why μ as determined by the authors is not the same for different machines.

The shape of spectral curves is altered by change of exciting voltage, and as their area is a measure of the intensity, it is not difficult by adjusting one or the other variable or both, to make differently excited spectral curve "fit" approximately and so have the same μ. As the authors' results show, the "fit" is exact enough to satisfy absorption measurements, which at best are not a very sensitive test.

For the above reasons I do not agree that the results of the authors throw discredit on the sphere gap as a measurer of peak voltage on an X-ray tube. It is true that the sphere gap requires precautions—for example, in the absence of a high resistance in series, it will, if the gap is left near the sparking value as described by the authors, usually measure the occasional surges which are of higher voltage than the normal.

I have only one comment to make on the X-ray photograph showing intermittent discharge with a Wimshurst machine. Similar results have been obtained by other workers with coil and transformer discharges, and it should be cleared up as to whether or not such photographs are produced by higher frequency currents superposed on the main potential curve. The existence of such currents is well known, and their frequency and indeed their existence are controllable by capacity, inductance or resistance changes.

The AUTHORS (in reply): Much of Dr. Kaye's criticism is directed against an alleged deduction which we do not make, viz., "that the sphere-gap does not provide a trustworthy measure of the peak voltage operating an X-ray tube." We do not claim this; on the contrary, we agree that there seems to be little doubt that with adequate precautions the sphere-gap is a reliable measure of peak potential. We do, however, conclude that the quality of the "end-radiation," viz., that left after transmission through 1 cm. of aluminium (which corresponds approximately to that used in medical radiology), is not satisfactorily measured by the equivalent spark-gap. There should be no ambiguity about the term "end-radiation," which we clearly define in the Paper. Reference to Rutherford's Paper mentioned by Dr. Kaye shows that he understood it to be the radiation which after filtration shows constancy of absorption coefficient in aluminium.

Since radiation corresponding to the "end-radiation" is that frequently employed in practice, and equivalent spark-gap is taken as a measure of its quality, it becomes important, if possible, to correlate the two. Our experimental results show that this cannot be done satisfactorily, at any rate, under conditions obtaining in practice. As Dr. Kaye points out, the intensity of the radiation of shortest wave-length (corresponding to peak potential) is vanishingly small. It therefore contributes to a negligible extent to the useful radiation, and unless there is some correlation between the average quality of the "end-radiation" and the shortest wave-length in the spectrum, this is an additional argument against the use of the equivalent spark-gap as a measure of the general quality of radiation employed in practice.

The presence of characteristic radiation, if appreciable, is a disturbing factor which, however, seeing that its wave-length is independent of the applied voltage, would give added weight to the conclusion that the quality of the residual or end-radiation bears no simple relationship to the equivalent spark-gap. Examination of the curves of Fig. 6 reveals a tendency for the coefficient of absorption of the end-radiation to become independent of applied voltage for the higher values. This is due no doubt partly to a greater proportion of the absorption being due to scattered radiation, but probably also to an increasing proportion of the radiation being characteristic *K*-radiation.

Dr. Kaye points out that the coefficient of absorption which we measure refers to no definite radiation. In his "Practical Applications of X-rays" (Chapman & Hall, 1923) he reproduces in Fig. 29 spectral curves by Hull showing the effect of filtering the radiation through 3 mm. of aluminium, from which it is seen that filtration narrows down considerably the range of wave-lengths transmitted, and that the point of maximum intensity is shifted towards the shorter wave-lengths. This narrowing down of the range will be considerably more pronounced with 1 cm. of *Al* and, as absorption experiments show, is sufficient to give rise to constancy of absorption coefficient within the limits of experimental accuracy. It is therefore sufficiently definite for the purpose in view. It is quite true, as Dr. Kaye suggests, that the straight lines give average results and that in reality the lines are not strictly straight. This is obvious from the spectral curves, but it does not in our opinion modify the conclusions to be drawn from the absorption curves.

We fail to see how the results we have obtained admit of the explanation that different spectral curves can be made to fit approximately. We have selected the same value of μ for the different machines, i.e., the same average wave-length of end-radiation after filtering out from 80 to 95 per cent. of the radiation of longer wave-length. When this is done, as we have demonstrated in the Paper, there is a very close " fit " of the absorption curves for the whole of the initial radiation, which we think can only be obtained by a close similarity of spectral curves for the selected conditions. If, as Dr. Kaye proposes, differences in spectral curves are emphasised by filtering, there is all the more reason to conclude that the similar absorption curves obtained indicate similarity of spectral curves.

There appears to be no doubt from our results and from those of Dauvillier that spectral curves obtained with the different machines display a close similarity when the conditions are suitably adjusted. Since each instantaneous potential gives rise to a spectrum of radiation, it is quite conceivable that, within limits, differences in the original wave-form are relatively unimportant in the resulting X-ray spectrum.

There appears to be no advantage in adopting the procedure of producing the straight portion of the curve back to the log-intensity axis. This gives the intensity remaining after absorption expressed as a percentage, not of the intensity of the original heterogeneous beam, but of that radiation in the initial beam, which has the same coefficient of absorption as the " end-radiation." The residual intensity expressed as a percentage of the total initial intensity is obtainable from our experimental results direct.

With regard to Dr. Kaye's suggestion that the intermittent effect with the Wimshurst machine may be due to high-frequency currents superposed upon the main discharge, this possibility has not been overlooked. The low frequency of intermittence seems to rule out this explanation, but the question is being fully investigated by E. J. Evans.

PRACTICAL X-RAY MEASUREMENTS FOR MEDICAL PURPOSES.

BY

DR. MARTIN BERRY.

§ 1. SINCE the science of radiology as a whole has now become an integral portion of many other sciences and arts, and since advances in the science can only result from physical investigations, no single sphere in which its utility has been manifested can claim the whole attention of those whose duty it is to investigate the general phenomena.

We who employ rays in the diagnosis and treatment of diseased conditions are apt to consider this as their chief application, but we must not overlook the other claims that are made on the attention of the physicist. To physics we owe the discovery of the rays, and to physics we have looked, and not in vain, for further advances. At the same time it behoves every medical man who practises radiology to understand not only the biological effects of radiations, but also the physical conditions under which they are produced, and the various devices by which the characteristics of the rays can be altered.

Medicine has particular limitations of its own to impose which may possibly be lost sight of during general physical investigations. When dealing with inanimate objects it is immaterial whether an exposure lasts three hours, three days or three weeks, but this is not so in the case of patients. An even more important limitation is the possibility, or rather probability, in inexpert hands of causing active damage. Whilst it is true that journalistic enterprise has resulted in awakening needless alarm in the minds of the public, yet we must remember that the margin between inefficient and excessive dosage is narrow, and it is our bounden duty to make this margin as wide as we possibly can. This has been the purpose of the various measurements I have made, some of which are referred to in the present Paper.

One of the chief dangers which confronts us, and which imposes a limit on the dosage we can apply, is that of inflicting irreparable damage on the skin. In a very large proportion of cases we are dealing with lesions situated deeply in the human economy, which lesions are known to require a certain dosage for their destruction. To fall far short of this dosage may result in actual stimulation of the growth, so that we are left to steer between the Scylla of under-dosage and the Charybdis of over-dosage.

The first and most important consideration for us is the depth dose—that is, the proportion of radiation received by a growth in the interior of the body as compared with that received by the skin and tissues covering it. The first and most obvious device to increase the depth dose is the method of cross-fire, by which the lesion is attacked through several ports of entry ; but this method is not always available.

When we speak of practical measurements for medical purposes the subject really divides itself into measurements of the rays which are actually applied to the patient, so that we know what dose has been given, and measurements of the changes in the quality and quantity of radiation resulting from adjustments of the apparatus. It is chiefly the latter part with which I am dealing in an attempt to show how a medical man who has to work with a particular set of apparatus may get the best out of it.

§ 2. The apparatus which was used for all the experiments is a twin coil working in conjunction with a mercury interrupter of the centrifugal jet type. Various makes of tubes have been used, all of them of the self-hardening, boiling-water variety.

The adjustments and variations possible on such apparatus are the amount of self-induction, the primary condenser capacity, the speed of the interrupter, and the duration of contact in the interrupter for each individual impulse ; of these, the interrupter speed is the most easily controlled and will be dealt with first. That considerable variations in the penetrating power of the radiation may result from simple changes of interrupter speed is shown by Table I., which is a single example of a large series giving similar results.

TABLE I.—*Variation of Speed of Interruption.*

Tube P. Filter 0·5 mm. zinc. Field 6 × 8 cm. Measurements on iontoquantimeter. Figures are times of discharge in seconds. Focus to centre of ionisation chamber 37 cm. Water tank 5 cm. thick behind ionisation chamber in all tests. Penetration taken through 10 cm. of water. Readings taken alternately without switching tube off. Boiling water tube.

					Mean.
3,080 interruptions per minute.					
Through 10 cm. water	25·4	24·2	24·4	24·4	24·6 sec.
Without front tank	7·8	8·0	8·8	8·0	8·2 ,,
2,640 interruptions per minute.					
Through 10 cm. water	23·2	22·8	23·0	23·6	23·2 sec.
Without front tank	8·6	9·0	7·8	8·4	8·5 ,,

Result.—12·8 per cent. penetration at 3,080 interruptions per minute and 16·6 per cent. penetration at 2,640 interruptions per minute.

Same tube and general arrangement as above.

Comparison of Penetration by Kienbock Strips.

Results.—15 per cent. penetration at 2,640 interruptions per minute.
17 per cent. penetration at 2,320 interruptions per minute.

The penetrations mentioned in these two tests are those which relate to the dose received by a lesion lying 10 cm. under the skin surface as compared with that received by the skin itself. The tube was in rather too soft a condition for deep treatment of malignant conditions.

If the secondary current pass through an oscilloscope tube which is viewed in a rotating mirror, or is itself revolved, as in the ondoscope demonstrated by Dr. Hopwood, it will be seen that the phenomena are those of intermittent discharges through the tube with much longer intervening periods when no discharge is taking place, each wave of current representing a single interruption. In a gas X-ray tube these intervals are important, not only since they allow the heat generated at the focal spot to be distributed through the mass of the target, but even more since it is in these intervals that the air space between the cathode and target becomes de-ionised, and thus allows the rise of voltage on the tube terminals which gives to the electrons the velocity necessary to produce hard radiation. If we knew all the factors we should be able to say from the appearance of the oscilloscope discharge whether the resting period was too long, too short or just correct. But we can obtain the necessary information, without all these factors, by examining the radiation produced. Table I. shows such an examination at different speeds of interruption, carried out by readings on an iontoquantimeter and checked by another experiment with Kienbock strips. By taking a series of such measurements it is possible to obtain the optimum speed for each given set of working conditions. In the experiments in question readings were taken of the intensity of radiation received by an ionisation chamber, both with and without the interposition of 10 cm. of water between the source of rays and the chamber, the rays having already passed through 0·5 mm. zinc before reaching the water tank. In the experiments of Table I. the depth of dosage is given under the conditions laid down by the Erlangen technique.

Another point in connection with the interrupter which should be capable of variation is the duration of contact, and it is to be regretted that so many interrupters are constructed in which this adjustment is not possible. For any given set of working conditions there is one length of contact which is correct ; a shorter time of contact than this gives insufficient saturation, and a longer one passes an unnecessary amount of current through the primary, in addition to encroaching on the de-ionisation interval of the tube. My experiments on this point are not sufficiently advanced for publication.

With regard to varying self-induction, as a rule this is only possible in very few steps, and on my own apparatus one degree is so obviously better than the others that it is the only one I have used.

Turning now to variations of the condenser. Much work has been done on this subject, but some of it is not applicable to apparatus already purchased, though it may be of great value to the manufacturer of such apparatus. In particular, the work of Taylor Jones was carried out under conditions different from those which obtain in a modern radio-therapeutic department. Seitz and Wintz have published the results of tests with their own apparatus, showing a very marked optimum capacity. This optimum depends on so many factors that it cannot be transferred from one apparatus to another ; the results of tests on my own apparatus are shown in Table II. In this series of measurements the only factor altered was the condenser capacity, and the influence of this variation is exceedingly marked. The figures show a steady rise in the primary current, whilst the secondary current only varies slightly, the extra amount of energy intake being expended largely in the form of increased voltage.

The lower part of the table shows an optimum at 3·97 microfarads, which is approximately the same as that obtained by various other series of tests. The columns headed " Air " and " Water " represent the time taken in seconds for the discharge of the iontoquantimeter, respectively without and with the interposition of 10 cm. of water in the path of the rays between the filter and the ionisation chamber. The field of irradiation in these experiments was larger than that used in Table I., and the distance from focus to ionisation chamber was greater, hence the percentage penetration is greater. These experiments were carried out more nearly under my practical working conditions than those of Table I. As a general rule, I prefer a smaller number of fields each of larger area and a comparatively long focus-skin distance to the technique of a larger number of smaller fields with a shorter distance.

TABLE II.—*Variation of Primary Condenser.*

Experimental Details:—Boiling water tube. Measured on iontoquantimeter. Focus to ionisation chamber 46 cm. Diaphragm 6×8 cm. midway. Water tank 20 cm. square and 10 cm. deep. Wax block behind ionisation chamber. Filter 0·5 mm. zinc. Spark gap 40 cm. between points. Tube rheostat 12. Motor rheostat 7. Auxiliary rheostat 7·5.

Micro-farad.	Penetro-meter.	Amps.	M.A.	Time in secs.	
				Air.	Water.
2·0	75	5·5	1·9	12·8	30·8
2·93	80	5·5	1·8	12·4	31·6
3·06	81	6·5	1·9	13·0	32·6
3·2	80	6·5	1·9	12·8	31·0
3·97	88	8·5	1·9	10·8	22·0
4·26	91	9·0	1·9	11·0	25·0
2·0	76	5·5	1·9	14·0	33·0
2·93	81	6·0	1·9	12·0	30·0
3·06	83	6·5	1·8	12·2	28·2
3·2	85	6·5	1·9	12·0	28·6
3·97	94	9·0	2·0	10·0	23·0
4·26	95	9·0	1·9	10·2	22·6
2·0	74	5·0	2·0	15·0	35·0
2·93	82	6·0	1·9	12·4	27·2
3·06	83	6·5	1·9	10·8	27·4
3·2	85	7·0	1·8	11·8	28·6
3·97	94	9·0	1·9	11·6	22·6
4·26	94	9·0	1·9	10·4	24·6

Above figures arranged under condenser capacity.

Capacity, microfarad	2·0	2·93	3·06	3·2	3·97	4·26
No water ...	12·8 14·0 15·0	12·4 12·0 12·4	13·0 12·2 10·8	12·8 12·0 11·8	10·8 10·0 11·6	11·0 10·2 10·4
Mean	13·9	12·3	12·0	12·2	10·8	10·5
10 cm. water	30·8 33·0 35·0	31·6 30·0 27·2	32·6 28·2 27·4	31·0 28·6 28·6	22·0 23·0 22·6	25·0 22·6 24·6
Mean	32·9	29·6	29·4	29·4	22·5	24·1
Per cent. through water	42·3	41·6	40·9	41·5	48·0	43·6

In addition to the figures in Table II. it may be stated that with a condenser of only 2 microfarads the working of the tube was most irregular and the conditions not suitable for treatment of a patient. The alternate spark-gap (between blunt points) was set at 40 cm., and as the condenser capacity was raised sparks bridged this gap with increasing frequency until, at the two highest capacities used, they were almost continuous. I did not wish to alter the gap, but had this been done the figures for the larger capacities would have shown still greater superiority over the other figures. The readings were taken in the order given in the upper part of the table in order to eliminate as far as possible any error due to variation of tube condition.

§ 3. Let us assume now that we have produced a quality of radiation which we regard as suitable for application to a patient, and turn to some consideration regarding its application. Let us also remember that our primary object is to make the depth dose as large as possible compared with the skin dose. Though consideration of the patient's feelings and comfort causes us to shorten the time of dosage as much as possible, yet we must not sacrifice efficiency for this purpose.

One method of increasing the depth dose is to cover the surface of the body with a layer of some material which has approximately the same coefficient of absorption as the tissues. Rays are absorbed according to an exponential law, and therefore the actual amount of energy absorbed in the superficial layers is greater than that in the deeper layers. Add to this the fact that the superficial layers lie nearest to the source of origin of the radiation, and it is obvious that the amount of radiation affecting them is much greater than that affecting a corresponding layer in the depths. Since there is very good ground for believing that the biological effect is proportional to the amount of energy absorbed, we are in the position that the greatest absorption and biological effect occurs in the skin ; and this limits the depth dose which we can apply. It is with the purpose of reducing this disparity as far as possible that the body surface may be covered with a layer of some centimetres of material to take up this strong absorption.

In most of the experiments designed to examine the distribution of ray intensity water has been used as an absorbing and scattering material, since it behaves in this respect similarly to body tissues. It is, however, rather inconvenient to cover the body surface with a layer of water, and other materials have been used. Paraffin wax has been employed extensively for this purpose. From Table III. it will be seen that paraffin wax is wholly unsuitable, and that its use for this purpose would give entirely fallacious ideas about actual dosage. Various compounds were tried as substitutes for wax, but the result of tests of the latest composition is not entirely satisfactory for two reasons. Firstly, because it still allows a rather greater percentage of radiation to pass through than water does ; and, secondly, because it is not sufficiently plastic to be moulded easily to the contours of the patient. A formula for a mixture which would satisfy these two conditions would be a boon. It is fairly easy to evolve a formula to satisfy either condition, but it is essential that both should be complied with. If we are building up the surface of the body with a mass which we intend to consider as forming a homogeneous whole with the body for the purposes of calculation and measurement, it really must adapt itself to the surface without leaving air spaces ; some portions of the body present great difficulties in this respect. At the same time, the material should be capable of retaining its shape without the necessity of enclosing it in bags or other containers. Possibly dough, which is the next material it is proposed to try, may prove suitable. In Germany a pulp of cellulose is used, but this does not retain its shape satisfactorily.

§ 4. One final point on which comment should be made is the importance of back-scattered radiation. Table IV. gives in its upper part the results of some measurements made with a water phantom, and, below, a trial with Kienbock strips on a patient during an actual treatment. The importance of back-scattered radiation in practical therapy becomes most evident in cases where it may be diminished in the depths, such, for example, as in the treatment of a case of uterine cancer, where a gas-filled rectum lies behind the uterus. The skin always receives radiation scattered back from the subcutaneous tissues, and the amount of such radiation is well shown in the curves published by Prof. Dessauer, of Frankfort, to whom I am indebted for much useful information and assistance. This effect of back-scattering calls for caution when radiating two skin fields close to each other.

TABLE III.—*Comparison of Penetrations through Water, Paraffin Wax and Compo Wax.*

Tube M. boiling water. Filter 0·5 mm. zinc. Focus to ionisation chamber 33 cm. Diaphragm 6 × 8 cm. at 23 cm. from focus. Alternate spark gap 40 cm. Readings taken alternately.

Times of discharge of iontoquantimeter in seconds.					Mean.
10 cm. water in front and 5 cm. behind ...	22·4	22·6	22·2	22·0	22·3 sec.
10 cm. P. wax in front and 5 cm. behind	15·4	15·6	17·0	17·2	16·3 ,,
No front tank, wax behind	7·0	7·8	7·8	8·0	7·6 ,,

Result.—34·1 per cent. transmitted through water and 46·6 per cent. through paraffin wax.

Experiment to Compare Paraffin Wax and Compo Wax with Water.

Tube P. boiling water. Other experimental arrangements as above. Dimensions of all tanks and wax blocks 20 cm. square. Front tanks and blocks 10 cm. thick, back tanks and blocks 5 cm. thick. Times of discharge of iontoquantimeter in seconds. In all the experiments the back tank was water.

				Mean.
10 cm. of water in front	23·9	23·3	24·0	23·7 sec.
10 cm. of compo in front	22·6	22·5	23·0	22·7 ,,
10 cm. of P. wax in front	19·4	19·6	18·8	19·3 ,,
No block in front	7·6	7·2	7·2	7·3 ,,

Result.—Radiation transmitted through 10 cm. water, 30·6 per cent.
Radiation transmitted through 10 cm. compo, 32·1 per cent.
Radiation transmitted through 10 cm. P. wax, 38·0 per cent.

TABLE IV.—*Importance of Back-Scattered Radiation.*

Tube P. boiling water. Filter 0·5 mm. zinc. Focus to centre of ionisation chamber 37 cm. 2,640 interruptions per minute. 6·5 amps. in primary, 2·2 M.A. in secondary. Front water tank 20 cm. square and 10 cm. thick. Tank behind ionisation chamber 20 cm. square and 5 cm. thick.

Times of discharge of iontoquantimeter in seconds.					Mean.
Tanks front and back	19·4	20·4	20·0	21·0	20·2 sec.
Tank in front only	25·8	26·4	25·4	25·8	25·8 ,,
Tank at back only	6·6	7·8	7·0	7·4	7·2 ,,
No tanks	10·0	9·2	10·2	10·2	9·9 ,,

Result.—The intensity in the ionisation chamber becomes increased by 27 per cent. if the rays have passed through 10 cm. of water, and by 37 per cent. if they have only passed through air between the filter and the ionisation chamber, these percentages being the amounts added by the radiation scattered back from the tank behind the ionisation chamber.

Experiment to show amount of back-scattered radiation under actual conditions of treatment. A field 6 × 8 cm. was being treated, and a Kienbock strip was placed with the sensitive surface downwards on the patient's skin 4 cm. distant from the edge of the field. Its back was protected by thick lead, so that the only radiation it received was that back-scattered in the patient. Whilst the field received a dose of 26 X, the strip placed outside it showed 1·75 X.

A similar experiment under the same conditions showed 6·5 X on the field and 0·5 X on the strip.

The figures of Table IV. speak for themselves. They have been selected from a large number of similar experiments. The trial by Kienbock strips on the skin of the patient shows that this effect actually does occur in practical work, and is not merely a theoretical consideration.

In conclusion, I should like to say that only the fringe of a very large subject has been touched, and that there are many other devices which may be used to help us towards our goal—the cure of disease.

DISCUSSION.

Mr. F. Harrison Glew, referring to the problem of protecting the skin of the patient, stated that long experience had shown him that ordinary table jelly satisfies all requirements in this connection. Its absorptive qualities are approximately the same as those of water, and it can either be moulded to any required shape or applied in the form of overlapping wedges for convenience in adjusting the thickness of the absorbing layer.

X-RAY PROTECTIVE MATERIALS.

BY

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ABSTRACT.

The protective values expressed in terms of the equivalent thickness of lead have been measured for a variety of materials. Numerically, 1 mm. of the material in question is equivalent to the following thicknesses of sheet lead in millimetres :—

Lead glass	0·12 to 0·20
Lead rubber	0·25 to 0·45
Bricks and concrete	About 0·01
Woods	0·001 or less
Baryta plaster	0·05 to 0·13
Steel	0·15

The above figures relate to tungsten X-rays generated by 100,000 volts.

INTRODUCTORY.

THE steady addition to the already lengthy list of casualties to hospital and other X-ray workers led to the formation nearly two years ago of an X-ray and Radium Protection Committee (under the chairmanship of Sir Humphry Rolleston), which drew up a series of recommendations for the better protection and general improvement of the working conditions of the X-ray operator. These recommendations have resulted in a large number of X-ray protective materials being submitted for test to the National Physical Laboratory, which from the outset agreed to work in co-operation with the Committee, and has inspected the X-ray departments of many hospitals from the point of view of the Committee's recommendations.

It is well known that the absorption of an atom is greatest for those X-rays which have wave-lengths slightly shorter than one or other of its characteristic radiations, and partly with this in mind it was thought that it would be of value to ascertain the protective efficiencies of the various materials which are commercially available for the purpose of affording protection to the radiologist. Among these are sheet lead, lead impregnated rubber, lead glass, and various wall compositions such as barium-sulphate plasters, &c. The choice of one or other is normally dictated by considerations such as dielectric strength, portability, electrical and thermal conductivity, and not least by price.

Protection may be afforded in a variety of ways in practice. For example, by mounting the tube in a surrounding tube box suitably designed to permit the freedom of movement desired, or where this is impracticable the operator and his controls are situated behind a screen or wall which is constructed to give the necessary protection. Less frequently and conveniently the operator is " armoured " and wears protective aprons, gloves, face mask or goggles.

To the inexperienced the practice of speaking of percentage absorption may be misleading and calculated to give a wrong impression of the value of a protective material. It is not always realised how rapid the rate of absorption is with thin layers, and how slow with thicker layers. For example, in the case of tungsten X-rays generated at 120,000 volts, a sheet of lead only 0·1 mm. thick absorbs as much as 80 per cent. of the radiation, whereas a sheet 2 mm. thick only increases this figure to 99·4 per cent.

For simplicity, therefore, the Protection Committee recommended that the protection afforded by a material should always be referred to in terms of the equivalent thickness of lead. This use of lead as a standard of reference, while not wholly free from objection, is very convenient for the approximate accuracy which suffices in practice, especially in dealing with composite or laminated materials as the lead values of the several layers can be regarded as additive. The Committee laid down certain thicknesses of lead which it regarded as the minima which should be employed under specified conditions; and, in view of the very unsatisfactory conditions which prevail in the majority of X-ray installations in this country, we have been led to put on record some of our measurements in this connection.

APPARATUS.

The method of test adopted by the Laboratory is the very simple one of measuring by an ionisation method the absorption of a beam of X-rays by a material of given thickness, and then ascertaining the thickness of lead sheet which produces the same degree of absorption. A Coolidge tube with a tungsten target is usually employed for the purpose and, to avoid any possible anomaly arising from selective absorption, comparisons are conducted with at least two different exciting voltages. The size of the pencil of rays utilised is dictated by the absorptive qualities of the specimen to be measured. An obvious convenience is the provision of a series of lead sheets graded in progressive order of thickness which can be rapidly piled at will.

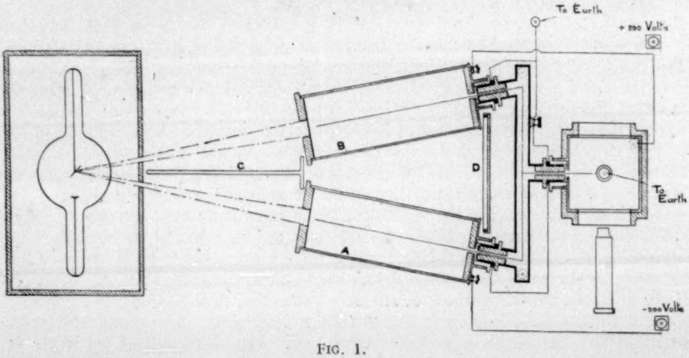

FIG. 1.

Fig. 1 shows in plan the details of an apparatus employing a balance method, which proves a useful alternative to the direct method referred to above. The diagram is self-explanatory, but one or two points may be mentioned. The two ionisation chambers *A* and *B* are exactly alike and placed as symmetrically as possible with respect to the target of the X-ray bulb. The near ends are closed with aluminium sheet 0 5 mm. thick. The various insulating plugs are of ebonite with sulphur ring insertions and earthed guard rings. The sides of the electroscope are of lead 3 mm. thick. The gold-leaf system is initially earthed. The outer case of the electroscope is raised to 200 volts, whilst the outer cases of the two ionisation chambers are raised to +200 and – 200 volts respectively. Suitably disposed lead screens *C* and *D* prevent anomalies rising out of scattered or stray radiation.

The whole system is mounted on a wooden stand provided with levelling screens and means of lateral adjustment, so that the position of exact balance of ionisation between the two chambers can readily be found. When this has been secured, the test sample is placed in front of one of the ionisation chambers, and the thickness of lead in front of the other chamber rapidly altered until balance once again results.

The above two methods serve in general, but there are certain protective articles, the shape of which makes it difficult to test the protection afforded in all directions. To cope with these exceptional cases, a third apparatus has been constructed consisting of a small exploring ionisation chamber connected by a flexible conductor to a geld leaf electroscope.

RESULTS.

The table given on the following page contains the " lead equivalents " for a variety of materials, *i.e.*, the thickness of lead equivalent in absorbing power to unit thickness of the material. The results are mostly for X-rays generated by 100,000 volts, but the values hold over a considerable range of some 20,000 or 30,000 volts on either side of this figure. The values are ordinarily diminished by from 5 to 10 per cent., when the voltage is 50,000. In the case of those materials which have feebly absorptive properties it was found convenient to compare them with lead through the intermediary of aluminium.

Provision is now being made for the measurement of lead equivalents at voltages ranging up to 200,000. It is intended also to extend the investigation to include gamma rays from radium.

Walls and Screens.

We have included a number of samples of building and other materials, as a knowledge of the various protective figures is not without value when buildings intended for radiological purposes are being erected. It will be remarked that the lead values of bricks and concrete are mostly of the order of 0·01. Thus, the lead equivalent of a 300 mm. (12 inch) solid concrete floor is about 3 mm. ; rather more for ferroconcrete. The majority of the woods have lead values of 0·001 or less.

With reference to iron, it will be noticed that its lead value is 0·15, and thus 20 mm. (about ¾ inch) of steel plate would be required to give protection equal to 3 mm. of lead, the figure recommended by the Protection Committee for exciting voltages over 100,000. The densities of steel and lead being about 7·8 and 11·4, the steel sheet would be four times as heavy as the lead. It so happens that this ratio is roughly that of the present cost by weight of lead and steel sheet,* so that there is nothing in it on the score of cost. It may be noted, however, that lead would increase its protective ratio somewhat at deep therapy voltages. An objection to iron is its rusting tendencies.

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* About 40s. per cwt. for lead ; 10s. per cwt. for steel.

Material.	Density.	Lead Equivalent.
Lead Rubber—		
(60 different samples)	3·7 to 6·5	0·25 to 0·45
Lead Glass—		
(40 different samples)	3·2 to 4·1	0·12 to 0·20
Metals—		
Aluminium	2·7	0·011
Brass	8·4	0·25
Steel	7·5	0·15
Miscellaneous—		
Water	1	0·004
50% red lead, 50% litharge	—	0·3
Ordinary rubber	1·2 to 1·7	0·02 to 0·05
Concrete—		
4 stone chippings, 2 washed sand, 1 cement	2·1	0·012
4 clinker, 1 cement	1·5	0·010
4 granite, 1 cement	2·1	0·013
Roman mortar	1·5	0·009
20 chalk, 1 cement	1·6	0·011
6 sand, 1 lime	1·8	0·009
Coke breeze	1·0	0·004
Bricks—		
Fletton brick (red)	1·6	0·010
Stock brick (yellow)	1·4	0·008
Diatomaceous brick	0·6	0·003
Protective Wall Plasters—		
(1) 55% Native $\mathrm{Ba\ Co_3}$, 45% $\mathrm{Ca\ So_4}$	2·0	0·091
(2) 33% $\mathrm{Ba\ So_4}$, 33% sand, 33% cement	1·5	0·048
(3) 33% coarse $\mathrm{Ba\ So_4}$, 33% fine $\mathrm{Ba\ So_4}$, 33% cement*	2·3	0·12
Woods—		
Ash	0·73	0·0013
Balsa	0·096	0·0001
Bass	0·48	0·0005
Fir	0·52	0·0008
Mahogany	0·49 to 0·68	0·0006 to 0·0011
Oak	0·65	0·0008
White Pine	0·47	0·0006
Pitch Pine	0·53 to 0·56	0·0008
Spruce	0·41 to 0·43	0·0004 to 0·0006
Teak	0·58 to 0·76	0·0006 to 0·0011
Walnut	0·64	0·0008

* Mixture suggested by Mr. P. J. Neate.

Incidentally, sheet lead is commercially described and sold by its weight per superficial foot, " 2 lb. lead " referring to sheet lead weighing 2 lb. per sq. foot. In this connection the following table may be useful :—

Weight.	*Thickness.*	*Weight.*	*Thickness.*
2 lb. lead	0·8 mm.	7 lb. lead	**3·05 mm.**
3 ,,	1·25 ,,	8 ,,	**3·5 ,,**
4 ,,	1·7 ,,	9 ,,	**3·95 ,,**
5 ,,	2·15 ,,	10 ,,	**4·3 ,,**
6 ,,	2·6 ,,	11 ,,	**4·75 ,,**
		12 ,,	**5·2 ,,**

There is some tendency at the present time to employ baryta and other plasters in lieu of lead. We give the lead equivalents of three of these plasters. It may be remarked, to take the case of mixture No. 2, that 60 mm. (nearly 2½ inches) thickness will be required to give the protection of 3 mm. of lead, and that the weight of the plaster is nearly three times that of the lead. These figures would be more favourable for plaster No. 3. We have no knowledge of the costs of these plasters as compared with lead, nor of the cost of the labour involved in erection.

Certain plasters have been devised containing iron turnings. They possess no special merit as regards absorption, and the tendency to rust is a disadvantage.

Tube Boxes.

Protective tube boxes, if constructed of lead, have to be of dimensions sufficiently generous to prevent sparking between the bulb and the box. If, however, the tube box is required to be in close proximity to the bulb, the former commonly takes the form of a lead glass bowl, or, alternatively, a wooden box with several layers of lead-rubber lining or wrapping. The open glass bowl is open to grave criticism : it is constructed of lead glass, usually affording, as our measurements show, a total protection of from 0·5 to 0·7 mm. lead ; but in many directions, owing to its design, it affords no protection whatever. In our experience, where this type of shield is alone employed, it is often possible to take a radiograph of the hand in any part of a room where a tube is working. The bowl should be provided with some kind of cover and the material increased in effectiveness.

The type of double hemispherical shield supplied for radiator Coolidge tubes is much better, in that it completely encloses the tube. The different shields (whether tinted blue or yellow) which we have tested have a lead equivalent of about 0·2, the total protection provided ranging between 1·2 and 1·5 mm. lead. The Protection Committee recommends not less than 2 mm. for exciting voltages below 100,000. We refer again to the question of lead glass in connection with protective windows, etc.

Different makes of lead rubber differ in protective value by 100 per cent., and the importance of this fact should be stressed. Care should be taken that the lead rubber is not cut away locally when the box is being constructed.

The choice of lead rubber or lead glass in preference to lead sheet is usually dictated by the fact that the former are electrical insulators, at any rate to a limited extent. The resistivity of lead rubber is usually of the order of several thousand megohm centimetres. The insulating value may be greatly improved by including a layer of micanite, ordinary rubber or other suitable insulator among the layers of lead rubber.

Measurements of the dielectric strength have been made in the Electrical Department of the Laboratory on a considerable number of samples. Two circular electrodes 1½ in. in diameter with rounded edges were placed in contact with opposite sides of the material, an alternating potential of approximately sine wave form being applied and steadily increased from zero until the material punctured. For good specimens of lead rubber the breakdown voltage was about 5,000 to 12,000 R.M.S. volts per millimetre thickness. In other cases the values were a good deal less, and on occasion the material acted as a conductor. For lead glass the values ranged from about 5,000 to 9,000 R.M.S. volts per millimetre.

One other point may be referred to. In a prolonged run, especially with a Coolidge tube, a great deal of heat has to be got rid of, and it is usually advisable to provide for ventilation of the tube box; in that event the openings should be properly safeguarded from a protection standpoint. A knowledge of the thermal conductivity of the several protective materials may be useful in this connection. Measurements at the Laboratory show that the values of the thermal conductivity at room temperature are as follows in c.g.s. Centigrade units :—

Woods (various)	About	0·0003
Ordinary rubber	,,	0·0008 to 0·0011*
Lead rubber	,,	0·0003 to 0·0007
Lead glass	,,	0·002
Lead	,,	0·08

If considerations of weight are of first importance, then lead sheet is almost always the lightest among the protective materials commonly employed for tube boxes. For example, the protective values of lead rubber range between about 0·25 and 0·55, and the densities between about 3·7 and 6·5. On the average, lead rubber is about 10 per cent. heavier than sheet lead, affording the same protection, though it may be as much as 25 per cent. and as little as 5 per cent.

Similarly with lead glass the protective values commonly lie between 0·12 and 0·20, and the densities between 3·2 and 4·1. On the average, lead glass is about twice as heavy as sheet lead, affording the same protection, the figures ranging from about 1·75 to 2·5.

These results (as with those for the screen materials) are, of course, predictable, as it is known that a heavy atom is ordinarily much more absorbent than a light. For the same reason in the case of two composite materials, each containing a mixture of atoms and having the same density, the one containing a higher proportion of heavy atoms will usually have a higher absorption factor. With lead rubber the lack of proportionality between density and lead value is sometimes pronounced, and of two samples the one with the lower density may have the higher lead equivalent.

Diaphragms of the iris type are often fitted to tube boxes. The leaves are usually much too transparent. They should provide protection equal to not less than 3 mm. of lead, and the simpler rectangular diaphragm with two motions is a more practical job.

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* **Depending on the mineral content.**

Protective Glass Screens or Windows.

As already mentioned, the lead values of lead glass range between about 0·12 and 0·20 per mm. But glass with the highest values, while perfectly mouldable into protective bowls and the like, is inadmissible for fluorescent screens and windows on account of the impossibility of producing it in flat sheets free from streakiness and colour. Such sheets are rarely more than 4 to 6 mm. thick, and to get the minimum protection recommended by the Protection Committee for fluorescent screens or windows it may be necessary to pile several sheets, a proceeding which is impracticable, as it becomes impossible to see through them.

One has to turn to glass of lower lead content, and this can be obtained in sheets, perfectly flat, of great clarity and almost colourless. The lead value is about 0·12 per mm., but as sheets up to 18 mm. thick can be obtained, the requisite protection is readily afforded, though at the cost of greater weight.

Miscellaneous Materials.

Among these are included aprons, gloves, face masks and goggles, of which the minimum protective value recommended by the Protection Committee is ½ mm. lead. The Committee were deterred from recommending a higher figure by considerations of weight and convenience. For obvious reasons lead rubber and lead glass are almost solely utilised in the manufacture of the above articles. The linings of chamois leather or stockingette often employed, while having no appreciable lead value for the primary rays, are doubtless useful in absorbing the softer characteristic rays of lead.

DISCUSSION.

Dr. H. B. **Keene** (comunicated) : I should like to support very strongly the views expressed by Dr. Kaye regarding the inadequacy of X-ray protection in many existing equipments. I have had similar experiences when inspecting hospital and clinic installations in other parts of the country. One or two examples may serve to show how unsatisfactory the conditions may be.

I have seen protective goggles in use in which one window was of lead and the other of soda glass. I have seen a modern Coolidge outfit for radioscopy in which the protection has been so totally inadequate that a fluorescent screen held in the hand in practically any part of the room showed an excellent picture of the bones of the forearm.

I recall a vertical screening stand of modern manufacture which, although beautiful to look upon, leaked X-rays in such a direction that they fell full on the operator at his control table and over the full length of his body. This condition had continued for some eighteen months. A similar leakage below the vertical lead rubber screen provided a bath of scattered radiation for the legs and feet of the doctor who sat and examined the patients.

Recently an outfit has come to my notice in which the entire tube protection consists of a portion of a hemisphere of lead rubber sheet 0·8 mm. thick.

It is very evident from what Dr. Kaye has said that such undesirable conditions are not uncommon. The X-Ray and Radium Protection Committee and their collaborators at the National Physical Laboratory are doing an excellent and very necessary work. But the rules which they have drawn up and the valuable protective data which Dr. Kaye and Dr. Owen have made available need to be brought very directly—perhaps by a succession of further printed pamphlets—to the notice of the *authorities* responsible for the various X-ray equipments throughout the country, as well as to the notice of the operators themselves.

There seems to me to be a very strong case for the compulsory inspection of existing installations and of those which are being daily produced by the makers, if only on the grounds that those who operate them exercise a " dangerous occupation."

GENERAL DISCUSSION.

In addition to the discussion already recorded in connection with the various Papers, the following contributions were made :—

Major C. E. S. **Phillips** said that radio-therapeutic methods seemed to be acquiring a nutritive as well as a curative value, since Dr. Berry proposed to coat the patient with dough and Mr. Glew with jelly. As a member of the Röntgen Society, he desired to express his sense of the value of the present meeting, which was the first joint meeting held by the two societies, and his hope that it would not be the last. It is important that physicists should know what are the chief practical difficulties of medical men in order that they may be able to concentrate attention on these, and it is equally important for medical men to become familiar with the latest methods devised by pure scientists.

Dr. G. B. **Batten**, speaking as one of the oldest medical members of the Röntgen Society, said that a meeting like the present must be valuable to the future practice of radiologists. Every advance in knowledge can sooner or later be turned to advantage for the well-being of humanity. Even the thermionic valve, which was invented for a purely commercial purpose, has recently been used in electro-medical treatment for measuring the current passing through the tissues of a patient. It is important that investigators working in separate departments of science should cultivate a readiness both to get and to give knowledge.

Dr. N. S. **Finzi** said that he considered Major Phillips had hit the nail on the head when he mentioned the difficulties surrounding radiologists. Most of the work brought before them that evening was concerned with voltages much lower than those which radiologists used in modern work. The communications dealt with pressures of about 100 K.V. and 120 K.V., whereas for modern treatment pressures of 200 K.V. to 250 K.V. were employed ; it was likely that they would soon employ higher pressures still. It seemed therefore desirable that British physicists should give more attention to this more difficult part of the spectrum and the difficulties and dangers which arose from its use. He used a Sabouraud pastille for measurement and found it quite satisfactory if one remembered that the dose was not the same as with rays corresponding to rays of lower pressures. With rays produced by pressures of 220 K.V. to 260 K.V. and using filters of 0·8 mm. of copper and 3 mm. of aluminium the dose was three times the Sabouraud B. Tint.

He had recently been on the Continent and seen some of the newer work in France and Germany. In Germany there is a new instrument made by Siemens and Halske in which the ionisation current through a small ionisation chamber, made, he believed, of horn, was measured by means of a galvanometer after amplification by a triode valve and the apparatus used was so arranged that a continuous reading was obtained without the necessity of intermittent charging and discharging of an electrometer, valuable time being thus saved. Wintz and Rump have shown by means of an arrangement of tanks containing water and air that the amount of rays received at a certain point varies considerably according to where the air space is placed, the total proportions of water and air remaining the same ; the least radiation was received when the air space was nearest to the point being measured. Consequently the effect of gas in the intestines and similar organs is of the greatest importance. An attempt was made to deal with this by increasing the pressure of the applicator on the patient's abdomen. They were also getting greater accuracy by localising the exact position of their beam of rays by means of a lead pellet used under the guidance of a fluorescent screen. In France he saw a new apparatus for producing high-tension current, comprising two transformers connected through four kenotron rectifiers with four condensers, this instrument producing continuous current up to 250 K.V. He only saw it working up to 150 K.V., as they had not a tube which would stand up to a higher voltage. All measurements of voltage, current, the output of rays and everything else were carried out on a switchboard under the control of the operator. The X-ray output was measured directly by means of a current from a 500-volt dry battery passed through a very large ionisation chamber and then through a galvanometer.

Mr. F. E. **Smith** referred to Prof. Russ's suggestion that an International Committee on Standards of Measurement in Radiology should be set up. The present situation with regard to the latter is seriously unsatisfactory, and if a lead is to be given, the Physical and Röntgen Societies would seem to be the appropriate bodies to take the initiative. He invited all who are interested in the matter to communicate with the Secretary of one or the other of the Societies.

Mr. M. A. **Codd** said that a committee of the kind referred to had been appointed in 1905. Perhaps Major Phillips could say whether it had done anything ? At present every medical practitioner has his own empirical standard of measurement, but some uniformity of practice is very desirable.

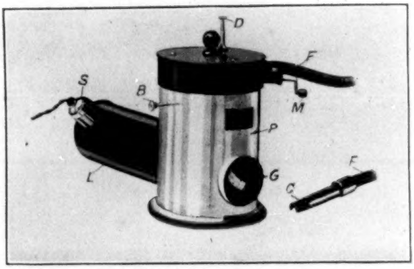
D
F
S
B
M
P
G
L
C
F

FIG. 1.

[*To face page* 41D.

DEMONSTRATIONS.

The following demonstrations were shown at the afternoon meeting:—

1. "A Method of Measuring X-Ray Intensity," by Major C. E. S. **Phillips.** An apparatus employing an improved form of the Self-charging Electroscope, described in the "Proceedings" of the Physical Society, Vol. 34, Part 5, Page 213. Various types of ionisation chambers can be used, and readings can be taken by an observer at a safe distance from the apparatus.

2. "Intermittent Discharge from a Sectorless Static Machine," by E. J. **Evans**, B.Sc. A static machine, although affording a constant voltage, has been found by Mr. Evans to give an intermittent discharge with both Coolidge and Geissler tubes. For a helium tube the intermittency was demonstrated by means of a rotating mirror, and it was shown that the frequency changes when the speed of the machine or the resistance in series with the tube is altered, or when the tube is shunted by a condenser.

3. "An X-Ray Balance," by L. H. **Clark**, M.Sc. The radiation to be measured and that from a fixed quantity of radium are caused simultaneously to affect a double ionisation-chamber, the effects being balanced against one another so as to produce a steady deflection of a gold leaf. The method has been described in the Philosophical Magazine, Vol. 44, December, 1922.

4. "Dr. Solomon's Ionometer," by H. B. **Gough**, Sunic Research Laboratories. The principle of this apparatus is based upon the fact that when an electrified body, for example, the leaf of an electroscope, is exposed to the influence of a beam of X-rays, it loses its charge owing to the ionisation of the surrounding air.

If the ionisation current is measured by timing the rate of fall of the leaf of the electroscope, we have a measurement of the intensity of the radiation.

The value of this ionisation current is directly proportional to the amount of energy absorbed by the surrounding air. In the case of this instrument the air is contained in a small chamber, which will be described in detail later in this Paper.

Further, it has been proved experimentally that the ratio of the coefficients of absorption of air and of tissue does not vary to any great extent with the wave-length of the radiation.

It therefore follows that if the ionisation current is measured as previously described, we shall know how much radiation is being absorbed by the air of the chamber and consequently the amount being absorbed by the tissue, as these two values are directly proportional to each other.

The apparatus for this purpose here described was designed by Dr. Solomon and consists of three main parts, viz., the measuring instrument or electrometer, the conductor and the ionisation chamber.

Fig. 1 shows the apparatus with the exception of the rigid part of the connecting rod. P is the electrometer, C the ionisation chamber and FF the flexible parts of the connecting rod. An optical system L is used for projecting upon a ground-glass screen G a sharp image of the gold leaf. Engraved upon the glass is a graduated scale.

The source of light is a 25 watt gas filled lamp of special design, having a straight filament enabling the scale to be evenly illuminated. An adjustable lamp-holder *S* is fitted for focusing purposes, and in front of the scale a magnifying lens is arranged to enable the operator to read the scale easily.

Fig. 2 shows the apparatus in section. The walls of the electrometer *P* are made of lead 4 mm. thick and serve to protect the apparatus from stray ionising radiation and stray electrostatic fields. The walls are "earthed" through the terminal *T*. The gold leaf *L* is supported upon a brass stem *H* which is insulated from the walls by means of ebonite and sulphur. The stem is connected to the connecting wire *K* by means of a small spring *C*. The connecting wire is carefully insulated by means of amber olives in the two flexible ends F_1 and F_2 of the rod. At the end of the rod is the ionisation chamber *A*. An air condenser or additional capacity is connected if desired to the leaf system by means of the small spring *S*. This extra capacity reduces the sensitivity of the apparatus about three and a half times, thus making measurements of long duration possible. This condenser consists of the walls of the chamber and a brass tube *J* supported by small ebonite insulating blocks. The leaf is charged by a small friction machine fitted inside the top of the electrometer (not shown in the diagram).

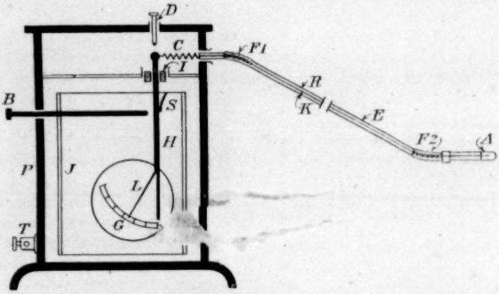

FIG. 2.

Doubtless many will appreciate this device after having used an ordinary electroscope charged by means of an electrified ebonite rod, with the result that at times the leaf receives too great a charge, in which case the leaf may be damaged, or, on the other hand, the leaf may not be charged sufficiently and time is thus wasted in arranging the leaf as required. With this instrument, as the small handle of the friction machine is slowly rotated so the leaf slowly rises over the scale. As the handle is released a spring carries the device away from contact with the leaf system and the apparatus is then ready for work. Should the leaf be charged to a higher point on the scale than is required, the leaf may be slowly discharged by a semi-conducting discharger *D*.

Fig. 3 shows the ionisation chamber in detail. The chamber itself consists of a small cylindrical tube of graphite *I* containing a volume of air *J*. In the centre of the chamber supported by an amber block *G* is a stem of graphite *H*, which is connected by means of a wire F and a small spring D to the wire of the conducting rod. The chamber is fitted to the conducting rod by the bayonet fitting A. The dimensions of the chamber are such that it may be placed upon the skin or even introduced into the natural cavities of the body.

The apparatus is standardised with a known quantity of radium.

The ionisation produced by 1 gramme of radium element in one second placed at a distance of 20 mm. and screened by 0·5 mm. of platinum is denoted by the letter R. The ionisation produced by this gramme of radium will cause the leaf to fall through the full number of divisions of the scale, viz., 50, in a certain time, which time is a function of the capacity of the apparatus. This time expressed in seconds represents the number of R units which correspond to the total graduation of the scale and is known as the constant of the apparatus. There are two values of this constant for the apparatus, K_1 corresponding to the large capacity, that is with the additional air condenser connected to the leaf system, and K_2 corresponding to the small capacity.

Dr. Solomon, after considerable experimental work, states that $1{,}000R = 5\ H$ or tint B of the Sabouraud Noire radiometer. As the constant K_1 is roughly equivalent to 500 R, it will be seen that the full scale corresponds to approximately 2·5 H. To predetermine the time of exposure for a skin dose equal to tint B Sabouraud Noire radiometer, the ionisation chamber is placed at an equal distance from the anticathode as the skin of the patient. The X-ray plant is switched on and the leaf previously charged is timed by means of a stop watch over 50 small divisions of the scale. It is necessary for the purposes of this calculation to assume that the electrical conditions of the generating plant and the consequent tube emission are constant. Reference must now be made to the constant K_2 of the instrument. In the case of the instrument shown here this value is 130 R. If this value is divided by the time of fall in seconds, the value of the incident radiation is known in terms of R per second. Suppose the time of fall was found to be 90 seconds, then the intensity of radiation or the value of the incident dose equals $\frac{130}{90} = 1{\cdot}44\ R$ per second. As $1{,}000\ R = 5H$, or tint B Sabouraud Noire radiometer, the total time for the exposure will be $\frac{1{,}000}{1{\cdot}44} = 694$ seconds. Should electrical conditions be unstable the *total* X-ray dose may be measured by means of the ionometer. The ionisation chamber is placed upon the skin of the patient and the leaf connected up to the large capacity. The number of divisions through which the leaf falls is carefully noted at the end of the treatment. Reference to constant K_1 will show that each small division of the scale in the case of the instrument shown here equals 10 R. Suppose the number of small divisions through which the leaf has fallen is 40, the total dose applied will equal $10 \times 40 = 400\ R$, or 2 H.

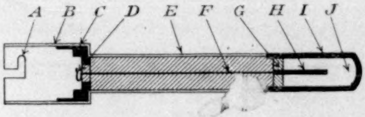

FIG. 3.

For deep therapy measurements, if the ionisation chamber is not introduced into the natural cavities of the body, it is necessary to discover the ratio of the depth dose to the incident dose. For this purpose the "Ionometric Standardiser" (Fig. 4) is used, which consists of a vessel containing water into which the ionisation chamber may be introduced at a depth of 10 cm.; the dimensions of the beam, the height of the tube and the conditions of filtration being predetermined.

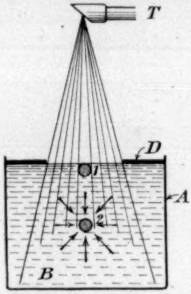

FIG. 4.

The coefficient of absorption of water being very nearly equal to that of the tissues, an effective value of the radiation is thus obtained, having taken into account all diffused secondary radiation.

To discover the ratio of the depth dose to the incident dose is a simple matter. The leaf is charged and its fall timed over the full scale with the ionisation chamber placed at 10 cm. depth in the water, this time being represented by T_1. The experiment is repeated, except that the ionisation chamber is this time placed at the surface of the water. The time of fall is noted and represented by T_2. Then T_2/T_1 is the value required. Any measurements taken at skin distance may be converted into depth measurements by multiplying by T_2/T_1.

Another use to which the instrument may be put is to measure the coefficient of transmission of a filter. By two successive measurements with the same electrical conditions the intensity after filtration I_1 and the intensity before filtration I_2 are found. Then the ratio $\frac{I_1}{I_2}$ gives the coefficient of transmission of the filter.

The instrument is simple to operate and measurements made with it are reliable.

5. "A Spectrometer for Measuring End Radiation," by W. E. SCHALL, B.Sc. This instrument, due to Drs. Staunig, March and Fritz, of Innsbruck, is an X-Ray spectrometer which combines the methods of Laue and the Braggs. The beam of X-rays is passed through a crytsal, and after reflection has taken place at the internal atomic planes of the crystal the reflected beam falls on a fluorescent screen.

Fig. 1 is a diagram of the instrument. D_1 and D_2 are two slits in thick lead plates, which serve to cut down the incident beam to a narrow pencil. C is the crystal, which can be rotated. S is the fluorescent screen. As the crystal is rotated a point is reached when the reflected beam flashes up suddenly on the screen. A similar point may, by rotating the crystal, be found on the other side of the central undeflected ray. These points, represented in the figure by λ_0 and λ'_0, are quite sharply defined. A scale is provided whereby the wave-length of the radiation can be read off directly in Ångstrom units, with an error not exceeding 0·005 Å.

The instrument may be applied to the calibration of X-ray tubes for purposes of radiography and radiotherapy. (For further description, *see* Jour. Rönt. Soc., April, 1923.)

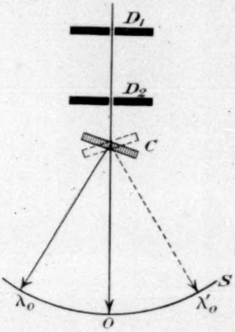

FIG. 1.

6. "The Ondoscope," by F. L. HOPWOOD, D.Sc. Designed for exhibiting the wave form of generators used with X-ray tubes. A vertical neon or air tube is rotated about a parallel axis while connected in series with the machine to be tested. As the length of the glow is proportional to the current passing, a luminous graph of the wave form is thus obtained. The device is described in the Journal of the Röntgen Society, Vol. 19, January, 1923.

DISCUSSION ON THE DEMONSTRATIONS.

Mr. M. A. CODD : With regard to Mr. Evans's demonstration, he suggested that the current from a Wimshurst machine is itself intermittent. There can be no doubt that the current in an X-ray tube is so.

Dr. D. OWEN : If, as appears to be the case, the X-rays come off in intermittent spurts, it seems possible that there is an optimum group frequency. As this frequency can be controlled by varying the capacity shunted across the tube or otherwise, the matter may repay the attention of X-ray workers.

Mr. EVANS (in reply) said that work on the characteristics of the static machine, on which he was at the present time engaged, favoured the view that the cause of the intermittency lay in the X-ray tube, and not in the static machine. It had been observed by Dauvillier that a gas bulb gave an intermittent discharge even when supplied by a secondary battery. He (the speaker) was not aware that the like effect had been observed in the case of the Coolidge tube prior to the work by Mr. Harlow and himself as recorded in their Paper contributed to the present discussion.

DUDDELL MEMORIAL

William Du Bois Duddell was born in London in 1872, and was educated privately and at the College Stanislas, Cannes. He served his apprenticeship as an engineer with Messrs. Davey, Paxman & Co., of Colchester, from 1890 to 1893. He then studied at the City and Guilds Central Technical College at South Kensington from 1893 to 1900 under the late Professor Ayrton, obtaining a Whitworth Exhibition in 1896 and a Whitworth Scholarship in 1897 ; subsequently (in 1913) the Fellowship of the City and Guilds of London Institute was conferred upon him. He remained on at the College engaged in original experimental work, and his unusual talents as an experimentalist soon became apparent. During this period he developed his oscillograph for the study of alternate current wave forms. In 1898, in conjunction with Mr. (now Professor) E. W. Marchant he read a Paper before the Institution of Electrical Engineers, entitled " Experiments on Alternate-current Arcs by the Aid of Oscillographs," and it was on this occasion that he gave the first of many brilliant demonstrations on the application of this instrument. Our present knowledge of the theoretical side of alternators, switchgears, surges in cables and telephonic phenomena undoubtedly owes much to the Duddell Oscillograph. In 1900, in a Paper read before the Institution of Electrical Engineers, entitled " On Rapid Variations of Current through the Direct-current Arc," Duddell described the " musical arc " as a generator of high-frequency currents. This discovery was of fundamental importance, and the musical arc in the hands of Poulsen became the means of producing continuous electric waves for telegraphic purposes. Duddell later turned his attention to wireless telegraphy, and by his thermo-galvanometer was able to measure the currents received in the antenna circuit under various conditions. In 1909 he described before the Physical Society his bifilar vibration galvanometer. Further inventions included a high-frequency alternator, and an alternator for producing sine waves of 2,000 frequency.

Duddell's activities were not confined to research work. He made a reputation as a brilliant lecturer who was exceedingly happy in his choice of illustrative experiments. He was an active member of many scientific societies, and by his good judgment and tact was able to render valuable services to them. In 1907 he was President of the Röntgen Society. He passed through all the grades of membership of the Institution of Electrical Engineers, and was President from 1912 to 1914. He did invaluable work for the Institution as Chairman of the Library Committee, and the modern character of the Library is largely due to his efforts. He also keenly interested himself in the Institution Museum and in the Science Museum at South Kensington. He acted as Treasurer of the Physical Society of London from 1910-17. It was largely due to his efforts that the Physical Society was able to publish its Proceedings in their present form. He was elected a Fellow of the Royal Society in 1907, and was awarded the Hughes Medal in 1912.

During the war he devoted much time to the solution of scientific problems which the war brought into prominence, and it is probable that excessive work in this direction was responsible for his premature death. Duddell was always willing to help anyone who submitted an electrical problem to him. He brought to his work the skill of the trained mechanic, and himself constructed the most delicate parts of his first models. Over and above sound theoretical attainments, he possessed a wonderful knowledge of materials and the methods of handling them, together with an extensive command of modern technical literature. R.S.Wh.

A strong desire having been widely expressed that the work of Duddell should be fitly commemorated, the Council of the Physical Society in February, 1920, appointed a committee to consider the proposal. The committee consisted of Sir William Bragg, Sir Horace Darwin, Sir Richard Glazebrook, Prof. T. Mather, and Mr. R. S. Whipple, with Dr. R. Knox (representing the Röntgen Society), and Mr. Roger T. Smith (representing the Institution of Electrical Engineers). As a result the Council in October, 1920, resolved to institute a Duddell Memorial Medal. A gratifying response was made to the appeal for funds, the sum of £639 5s. 8d. being subscribed by 213 individuals. The Physical Society made a contribution of £50, and the Röntgen Society of £10 10s. Thus by April, 1923, including accumulated interest, a sum of £739 18s. 4d. was at the disposal of the Committee. The medal was designed by Mrs. Mary Gillick. Mr. Emery Walker prepared a diploma to accompany the medal. The first award will be made in 1924.

Illustrations of the Medal accompany the present Part.

Proceedings at the Meeting held on May 11, 1923, at the Imperial College of Science.

The President, Alexander Russell, M.A., D.Sc., in the Chair.

THE DUDDELL MEDAL.

Sir William Bragg, F.R.S., Chairman of the Duddell Memorial Committee, handed over to the President of the Physical Society a copy of the Memorial Medal, together with the dies and a certificate for the unexpended balance of the memorial fund. In the course of his remarks Sir William Bragg said that the committee had been formed in response to a widely felt wish that the work of this distinguished scientist should be commemorated in a suitable manner. A sum of no less than £700 had been subscribed, thanks in the main to the energy of Mr. R. S. Whipple, secretary to the committee. As regards the design of the medal, great pains had been taken to ensure that it should be worthy of the memory of the dead and of the art of the nation to which he belonged. The work had been entrusted to Mrs. Mary Gillick, and the committee felt that their choice had been fully justified by the result. Mrs. Gillick had devoted immense trouble and care to her task, and the medal she had produced was one of which the Society might be proud.

Photographic slides of the medal were exhibited. On the obverse is a miniature of Mr. Duddell; and on the reverse a figure symbolical of Science examining the world as she finds it, with legends "The Physical Society of London" and "Rerum Naturam Expandere."

Dr. Alexander Russell, President, in accepting the medal on behalf of the Society, said that he did so with gratitude to all who had combined to make this memorial possible, and particularly to Sir William Bragg and Mr. R. S. Whipple, whose initiative had set the undertaking on foot, while their perseverance had carried it to such a successful issue. He had been associated with Mr. Duddell at the City and Guilds Technical College and in other connections. He had always marvelled at the encyclopædic character of Mr. Duddell's technical knowledge, which, coupled with his mechanical skill and inventive ingenuity, had enabled him to make so many contributions of permanent importance to the progress of science and of industry. He had been Treasurer, Member of Council, and Vice-President of the Physical Society, and President of the Institution of Electrical Engineers.

Sir Richard Glazebrook said that it gave him great pleasure to express his warm and intense admiration for Mr. Duddell. The latter had been a cordial helper of the National Physical Laboratory, especially during the war, and had always been ready to give generously from his great store of knowledge and of scientific resource. His memory would be kept alive not only by his own work, but by the admirable medal which had been designed; it was to be hoped that the latter would encourage others to follow his example and to engage in the field of scientific enterprise in which Duddell had been such a prominent worker.

Terms of Award of the Duddell Medal.

The following resolution was adopted by the Council on May 11, 1923:—

DUDDELL MEMORIAL MEDAL.

Resolution.—That the £400 War Loan 5 per cent. 1929/47 Inscribed Stock be accepted in trust from the Duddell Memorial Fund Committee, and that the income therefrom be expended in accordance with the following regulation:—

A bronze medal shall be awarded by the Council not more frequently than once a year to persons who have contributed to the advancement of knowledge by the invention or design of scientific instruments or by the discovery of materials used in their construction. The award shall be made without restriction as to nationality or Fellowship of the Society. A parchment Certificate of Award and a sum of money may accompany the medal.

Should more than one person be connected with the invention or discovery for which the award is made, the Council may at its discretion present more than one medal and apportion the money accordingly.

The terms of award shall not be varied except by Special Resolution passed by the Society at a General Meeting and confirmed at a subsequent Meeting.

THE EIGHTH GUTHRIE LECTURE.

"*The Present Position of the Radiation Problem.*"

By J. H. Jeans, *D.Sc., LL.D., F.R.S.*

Summary.

Since about 1900 it has been obvious that classical dynamics conflicts with experience in certain respects, and particularly with reference to the radiation problem. According to the classical dynamics, for instance, (1) radiation in a steady state with matter should tend to run entirely into waves of very high frequency, and (2) the spectrum of any element, say hydrogen, should be continuous, whereas the observed spectrum is a line spectrum. These contradictions can indeed be shown to occur under any system of mechanical laws which is entirely continuous. The discrepancies with experience accordingly suggest that the laws of nature must in some way be discontinuous. To explain the observed nature of black-body radiation, Planck propounded the quantum-theory ; in the hands of Bohr it soon became apparent that the quantum-theory contained also the clue to the line spectrum.

Hence arose hopes of rapid development which should explain the whole of the radiation problem and reduce molecular physics to order. The more sanguine of these hopes have not been fulfilled. Progress has been slow and difficult. No royal road has been discovered and what progress there has been is concerned with the structure of matter and not with radiation.

One solitary attempt at progress with the radiation problem must be mentioned, namely, Einstein's hypothesis of light quanta, which assumed light to be of an almost corpuscular nature, or at any rate to move in complete quanta. The obvious advantage of this hypothesis was that it gave an immediate explanation of the photo-electric effect ; indeed, it enabled Einstein to predict the then unknown law connecting the electronic velocities with the frequency of the radiation. Incidentally it also provided a simple explanation of Planck's radiation formula. The hypothesis soon had to give way before the destructive criticism of Lorentz and others. In brief this criticism centred round the impossibility of reconciling the hypothesis with the known facts of the undulatory theory of light. A direct experimental test by G. I. Taylor also gave results which must be regarded as fatal to the hypothesis of light quanta.

Hence has become established a general belief that the classical theory as expressed by Maxwell's equations for free ether must remain true at distances sufficiently removed from matter. In the general Maxwell equations—

$$\frac{4\pi}{c}\rho u+\frac{1}{c}\frac{dX}{d}=\frac{\partial\gamma}{\partial y}-\frac{\partial\beta}{\partial z}$$

$$-\frac{1}{c}\frac{d\alpha}{dt}=\frac{\partial Z}{\partial y}-\frac{\partial Y}{\partial z}$$

it appears to be the first term ρu that requires modification. All the terms except this are concerned with the flow and spread of radiation in free space ; this term is concerned with the interchange of energy between radiation and matter.

The different methods of interchange of energy between matter and ether or radiation may be divided into three classes which we may call sub-atomic, atomic and mass transfers. These three mechanisms of interchange must, of course, all give the same formula for black-body radiation, and we can test any hypothetical mechanism by examining whether it would by itself lead to Planck's formula.

Typical of a sum-atomic interchange is the emission or absorption of radiation by a Bohr atom. Here the equation covering a transfer of energy ΔW is

$$\Delta W = h\nu.$$

Einstein, in a very remarkable Paper which appeared in the Phys. Zeits., in 1917, showed that this law, subject to certain assumptions to be considered later, leads to Planck's law.

Typical of an atomic transfer of energy is the emission and absorption of heat by a solid. Debye's theory of specific heats indicates that here again the law of transfer is given by the same equation—

$$\Delta W = h\nu.$$

Typical of a mass transfer is the transmission of momentum occurring when a beam of radiation falls upon the surface of a perfect reflector moving with a known velocity u. Here it is easily shown that the transfer of energy cannot be in accordance with the above equation, since the transfer of energy ΔW can be made to have any value we please, positive or negative, by varying the frame of reference according to which the velocity u is measured. In this case, since the transfer of energy clearly cannot be by whole quanta, it is natural to suppose that it must be in accordance with classical mechanics. This does not lead, it is true, to Planck's law, but as is well known, it leads to Wien's law, which includes Planck's law as a special case. The effect of a transfer of energy of this type is not to diminish or increase the number of quanta of radiation, but merely to change the frequency, and so also the energy, of the quanta already in being.

We are in this way led simply and almost inevitably to a consistent view of the transfer of radiation—namely, that physical and chemical transfers take place by quanta while mechanical transfers take place according to the classical laws.

The application of these general principles enables us at least to conjecture what may happen in special problems. Consider, for instance, the exchange of energy between a free electron and a field of radiation. This interchange cannot be covered by the classical laws, since it is easily seen that these would lead to the equipartition formula $8\pi RT\lambda^{-4}d\lambda$ for black-body radiation and not to Planck's law. Further, the interchange of energy cannot be by quanta, for, if an electron absorbed a quantum $h\nu$ of energy, its final velocity v would be given by an equation of the type

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mu^2 + h\nu.$$

The two terms on the right of this equation are necessarily positive, while the term on the left is zero for one observer—namely, for an observer moving with the final velocity of the electron. For this observer the equation makes zero equal to the sum of two positive quantities, which is an absurdity. Since a free electron can apparently absorb or emit neither according to the classical mechanics nor according to the quantum law, it seems probable that it can have no exchange of energy at all with a field of radiation. It is probably only in the case of bound electrons that interchanges of energy can take place.

In the limiting case in which the radiation is of zero frequency the electric forces approximate to those of an electrostatic field. In the extreme limit in which the field actually is electrostatic, a large mass of experimental evidence seems to suggest that electrons are acted on by the mechanical forces given by the classical formula. It seems probable that an electron e in a field of force X experiences a mechanical force eX when the field is steady, but not when it is a field of radiation. We may, perhaps, conjecture that the true force is given by a formula of some such general type as

$$\frac{e}{t}\int X dt,$$

but it has to be admitted that at present this cannot be anything more than conjecture. We may express the same idea somewhat differently by supposing the force X expanded in a Fourier series in the form

$$X = X_0 + \Sigma X_p \cos pt,$$

and supposing that the force X_0 acts on the electron in the manner supposed by the classical laws, while the forces represented by X_p do not influence the electron at all except through cataclysmic motions of the kind contemplated in Bohr's theory. We may perhaps regard the values of X_p, &c., as giving an indication of the probability of cataclysmic motion occurring. A conception of this kind is used by Einstein in his 1917 derivation of Planck's law, to which I have already referred. It seems difficult to interpret this conception of Einstein's except by supposing that the electric forces with which we are familiar are to be regarded as manifestations of a sub-universe more fine grained than anything we have so far imagined. Although some thought of this kind seems to be suggested by Einstein's Paper, most of us will feel that many possibilities must be explored before we find ourselves driven to believe in a sub-universe of this kind. On the other hand, the direct evidence of the nature of radioactive transformation gives considerable support to Einstein's hypothesis together with its accompanying suggestion of some sort of sub-universe.

The general conception that X_p, &c., instead of measuring mechanical forces on electrons, may give an indication as to the probability of the occurrence of cataclysmic motion, appears to give promise of removing many of the difficulties which now beset molecular physics and the theory of radiation. If $p/2\pi = \nu$, the probability of a cataclysmic exchange of energy of amount $h\nu$ may be supposed to depend only on X_p and ν; to secure agreement with the classical electrodynamics in the limiting case of $\nu = o$, we may assume the probability to be proportional to $X_p^2/h\nu$. If radiation, generated at a point P, spreads out in spherical waves in accordance with Maxwell's equations, the value of X_p^2 at Q will fall off as $1/PQ^2$, while ν will not vary with PQ at all, so that $X_p^2/h\nu$ will vary as $1/PQ^2$. In the special case in which monochromatic radiation of frequency ν is alone concerned, any transfer of energy to matter must be by quanta of amount $h\nu$ and the probability of such a transfer falls off as $1/r^2$. There is no longer any conservation of energy as between the radiation and the matter in any small region of space, but the conservation of energy now reappears as a statistical law, of validity somewhat similar to that of the second law of thermodynamics. There is no longer any contradiction between the theory of quanta and the undulatory theory of light, and such phenomena as the photoelectric effect appear as the inevitable consequence jointly of the quantum equations and of Maxwell's equations for free ether.

XXII *The Effect of Torsion on the Thermal and Electrical Conductivities of Metals.* *By* CHARLES H. LEES, *D.Sc., F.R.S., and* J. E. CALTHROP, *B.A., B.Sc.*

RECEIVED MAY 7, 1923.

ABSTRACT.

A method is described which enables the effect on the thermal conductivity of a wire of twisting the wire to be measured. In each of the steel, aluminium, copper and lead wires tested the twist decreases the conductivity along the wire by a small amount which is approximately proportional to the square of the twist per unit length.

The change of electrical conductivity is found to be in general less than the change of thermal conductivity, but it is also approximately proportional to the square of the twist per unit length.

I.

THE close connection which all forms of the electron theory of flow of heat and electricity in metals disclose between the conductivities k for heat and K for electricity makes it desirable that these quantities should be determined for as many metals and under as many different conditions as possible in order to see the extent to which the theories agree with the facts.

Both conductivities have been measured for a number of metals and alloys at temperatures between 18° C. and 100° C. by Jäger and Diesselhorst* and between −180° and 15° C. by Lees.† The latter observations show that the constancy of the quantity $\frac{k}{KT}$ where T is the absolute temperature which is maintained above 0° C. for the pure metals, ceases for them at lower temperatures, and both sets of observations show that the quantity has not the same value for alloys as for pure metals, nor does it remain constant as the temperature is varied.

Johnstone ‡ has shown that when a wire of copper, zinc, aluminium, nickel, iron, steel or brass is stretched, the conductivity for heat along it is increased slightly (0·5 per cent.) by a tensile stress about 0·7 of the elastic limit for the material, while the measurements of Tomlinson, Gray, Smith, Credner and others§ show that the electrical conductivities of metals are under the same circumstances decreased by approximately 1/10 this amount.

Lussana‖ and Bridgman¶ have measured the effect of pressure on the heat conductivities of a number of metals, but unfortunately their results differ as to both sign and magnitude of the effect in many cases. Bridgman finds the conductivity increases with pressure for lead, tin and zinc, and decreases for iron, copper, silver, nickel, platinum, bismuth and antimony, at rates which vary greatly from one material to another.

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* Jäger and Diesselhorst, Abh. Phys.-Tech. Reichsanstalt 3, p. 269 (1900).

† Lees, Trans. Roy. Soc., A. 208, p. 381 (1908).

‡ Johnstone, Proc. Phys. Soc. 29, p. 195 (1917).

§ *See* Lees, Proc. Phys. Soc. 29, p. 203 (1917), for a discussion of the question and references to further determinations.

‖ Lussana, Nuovo Cim. 15, p. 130 (1918).

¶ Bridgman, Proc. Amer. Acad. 57, p. 77 (1922).

By combining these results with his observations of the effect of pressure on the electrical conductivities* Bridgman shows in his Paper of 1922, referred to above, that the quantity $\frac{k}{KT}$, which should on the usual electron theories remain constant, changes very considerably under pressure, and he has put forward a modified electron theory of conduction† in better accord with the observed facts.

Since the appearance of Johnstone's Paper, Bridgman has given the results of his measurements of the effect of tension on the electrical conductivities of eight metals and alloys.‡ For strontium, bismuth, manganin and therlo the conductivity increases with increase of tension at rates of the order 10^{-6} of itself per kilogram per square cm. of tension, and for lithium, calcium, antimony and cobalt it decreases at rates of the same order.

The normal metal, according to Bridgman's measurements, decreases in electrical conductivity under tension, as was found by the earlier observers. In the case of the normal metals tension thus increases the thermal and decreases the electrical conductivity.

II.

So far as we are aware, the only determinations of the effects of torsion on the thermal conductivities of metals are those of Smith§ on rods of iron, steel, copper and brass $\frac{5}{16}$ in. diameter and 4·5 ft. long, which showed decreases proportional to the twist. Both Smith and von Szily‖ have determined its effects in the electrical conductivities, and find decreases approximately proportional to the twist.

If the metal is obtainable in the form of wire the effect on the electrical conductivity can readily be determined by either the bridge or the potentiometer method. The measurement of the effect on the thermal conductivity is not so easily made to the accuracy necessary, but a method similar to that used by Johnstone for the determination of the effect of tension seemed suitable, and it has proved successful.

III. OUTLINE OF METHOD.

The material to be tested is in the form of a wire the ends of which are held in water-cooled clamps, one of which can be rotated through a measured angle with respect to the other to apply the twist. The heat is supplied to the centre of the length of wire by an insulated coil of manganin wire wound round the test wire, through which a measured electric current is sent. The temperatures at two points on the same side of the centre of the wire are measured by two insulated platinum wires wound on the wire under test. The platinum thermometers can be connected to a resistance bridge in such ways that either their resistances or the difference of their resistances can be measured.

The wire under test is entirely surrounded by a water jacket kept at the same temperature as the clamps. Measurements of resistances and differences are made with and without torsion, sufficient time being allowed between two sets of observations to ensure that all temperatures have become steady. A platinum coil in contact with the clamp or wall of the enclosure gives the temperature of the air surrounding the wire.

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* Bridgman, Proc. Amer. Acad. 52, p. 573 (1917), 56, p. 61 (1921), and 58, p. 151 (1923).

† Bridgman, Phys. Rev. 19, p. 114 (1921).

‡ Bridgman, Proc. Amer. Acad. 57, p. 41 (1922).

§ N. F. Smith, Phys. Rev. 6, p. 429 (1909).

‖ v. Szily, C.R. 128, p. 927 (1899). Math. u. Nat. Ber. Ungarn 16, p. 298 (1898).

IV. Details of the Apparatus.

The wire under test, *WW′*, Fig. 1, No. 14, diameter 0·2 cm., was held at each end by a clamp *CC′*, which formed part of a cylindrical brass box through which water was kept circulating. The box *C′* could be rotated about its axis with respect to the box *C* through an angle measured by the dial and pointer, and could be fixed by the clamp. A hollow cylindrical water jacket surrounded both wire and boxes.

The heating coil *H* consisted of 14 cm. of No. 40 manganin wire, silk-covered, wound directly on the wire tested and covering 0·45 cm. of that wire. The ends of the manganin wire projected 0·5 cm. from the coil, and to them were soldered lengths of No. 30 cotton-covered copper wire leads. The resistance of the coil was 4 ohms. The current through the coil was supplied by a small storage cell, and was read and kept constant to within 1 part in 2,000 by means of a shunted Weston milliammeter and adjustable resistance. The platinum thermometers consisted each of 35 cm. of No. 40 single silk-covered platinum wire wound directly on the wire under test and covering 1·05 cm. of that wire. Each was provided with copper leads, as in the case of the heating coil. Their resistances were 2·8 ohms at 15° C. One *B* was placed near the heating coil and the other *A* near the clamp *C*. Their distances from each other and from the heating coil were maintained constant by means of a mica strip *M*, through which the leads were threaded. The diameter outside the windings of heating and thermometer coils was 0·25 cm. The thermometer giving the temperature of the clamps and surrounding jacket was similar in construction. The leads from the various coils emerged from the surrounding jacket through a tube *T* in the end cylinder *C*.

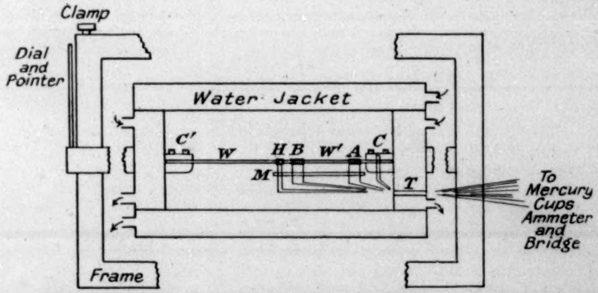

Fig. 1.—*WW′* Wire, *H* Heating Coil, *A*, *B* Thermometer Coils, *M* Mica Strip.

The resistances were measured by means of a Griffiths-Callendar bridge and Gambrell galvanometer, a deflection of 1 mm. on the galvanometer scale corresponding to a difference of resistance of 0·0002 ohms.

V. THEORY OF THE METHOD.

If a wire within an enclosure at constant temperature has its ends secured to the enclosure so that they have the same temperature as the enclosure, while heat is supplied at a constant rate to the middle portion of the wire, the temperature of the middle portion will be raised above that of the enclosure and heat will flow along the wire in both directions from the middle to the ends. If Q is the quantity of heat which leaves the short heated portion of the wire in each direction per second, and v is the excess of the temperature at a distance x from the end of the wire over the temperature of the enclosure, we have*

$$v=\frac{Q}{qka}\cdot\frac{\sinh ax}{\cosh ax_0} \qquad (5.1)$$

where q is the area, p the perimeter of the section of the wire, k the thermal conductivity of the material of the wire, h the emissivity of its surface, $a^2=ph/qk$, and x_0 is the distance of the near end of the heated portion from the clamp at the end of the wire.

If heat is supplied at a uniform rate to the surface of the wire between x_0 and the centre x_C, the mean temperature excess of the surface of the heated portion may be taken at that given by equation (5.1) for a point x_D one third of the distance between the edge x_0 and the centre x_C of the heated portion of the surface—that is for $x_D=x_0+\frac{1}{3}(x_C-x_0)$. If the total supply of heat per second to each half of the wire is H, then

$$H=Q+hsv_D=Q\left(1+\frac{hs}{qka}\cdot\frac{\sinh ax_D}{\cosh ax_0}\right) \qquad (5.2)$$

where s is the surface and v_D the mean temperature as thus determined.

If the mean temperature excesses v_A and v_B of two short lengths $2b$ of the wire whose centres are at distances x_A and x_B from the clamp at one end of the wire are observed, we have

$$Q=qka\cosh ax_0\left(\frac{v_B-v_A}{\sinh ax_B-\sinh ax_A}\right)\cdot\frac{ab}{\sinh ab} \qquad (5.3)$$

Hence

$$H=(qka\cosh ax_0+sh\sinh ax_D)\,\frac{v_B-v_A}{\sinh ax_B-\sinh ax_A}\cdot\frac{ab}{\sinh ab} \qquad (5.4)$$

If S is the resistance of that portion of the heating coil encircling the wire† and A is the current through it in amperes,

$$H=SA^2/4.18 \qquad (5.5)$$

If the resistances of the platinum thermometers at the points B and A when both are at the temperature of the water jacket (about 15°C.) are R_0 and r_0 respectively, and when heat is flowing along the test wire are R_1 and r_1 respectively then if γ is the temperature coefficient of the increase of resistance of the platinum wire, i.e., $R_v = R_0(1+\gamma v)$*, and we take into account the fact that only 34 cm. of the 35 cm. of platinum wire is wrapped round the test wire, we find that

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* Lees, Phil. Trans. Roy. Soc., A 208, p. 381 (1908).

† Taken throughout as 13/14 of the whole heating coil wire.

$$v_B - v_A = \frac{35}{34} \cdot \frac{1}{\gamma}\left(\frac{R_1}{R_0} - \frac{r_1}{r_0}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.6)$$

On substituting these values in equation (5.4) we get

$$\left(qk + sh \,.\, x_D \frac{\sinh ax_D}{ax_D} \cdot \frac{1}{\cosh ax_0}\right) \frac{\cosh ax_0}{x_B \dfrac{\sinh ax_B}{ax_B} - x_A \dfrac{\sinh ax_A}{ax_A}} \cdot \frac{1}{\dfrac{\sinh ab}{ab}} = \frac{SA^2}{\dfrac{4 \cdot 34}{\gamma}\left(\dfrac{R_1}{R_0} - \dfrac{r_1}{r_0}\right)}\dagger \qquad (5.7)$$

When k is fairly large as in the case of copper, a is small, the terms cosh x and sinh x/x approximate to unity, and may be expanded in ascending series. If terms above x^3 are neglected $\sqrt{k}$ disappears from the equation which becomes linear in k, and may be solved directly. When k is not large enough to make terms beyond x^3 negligible, the equation is best solved by successive approximation. With an assumed value of k taken from physical tables and with h taken 0·00030, a is found, and the value of the left-hand side of the equation calculated. On comparison with the value of the right-hand side it is seen whether the assumed value of k is too small or too large, and a new value is assumed accordingly. The calculation of the left-hand side is repeated, and the two values of it found when compared with the right-hand side allow a fairly close approximation to the true value of k to be found. The approximation is carried out to 1 part in 1,000. When the small change of value of k due to twist is to be calculated the process is carried out easily and quickly.

VI. METHODS OF OBSERVATION.

Water from the mains was circulated through the space between the double walls of the enclosure and electric current through the heating coil for about an hour in order to allow the steady state to be attained. The resistance of the enclosure coil was then read on the Callendar-Griffith bridge, then that of the hotter coil, and finally the difference between the hotter and the cooler coil. The heating current was kept constant. The current was then stopped. About 10 minutes was necessary for the wire to take up throughout its length the temperature of the enclosure. Resistance observations were now made in the order difference, hotter, enclosure. Unless the final temperature of the enclosure agreed closely with the initial temperature the experiment was regarded as unsatisfactory and was repeated.

A twist of 5—10° was usually given to one end of the wire and the observations repeated. The twist was then increased and further observations made. In some cases a still greater twist was applied. Finally the twist was removed and the observations repeated.

The positions of the heating and thermometer coils were determined before and after the observations to ensure that no change had taken place.

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* It may be noted that γ is not the coefficient of increase of resistance of platinum at 0°, that is 0·00392, but 0·00392/(1+15×0·00392), that is 0·00368.

† In this term a slight correction has been made as the thermometer coils are approximately 1 per cent. cooler than the wire with which they are in contact, owing to the electrical insulation of the platinum wire.

The following observations taken with a copper wire show the accuracy and consistency of the readings :—

No. 14 copper wire for electrical use, diameter 0·20 cm., length between clamps 13·60 cm. $x_A=1{\cdot}37$ cm., $x_B=5{\cdot}87$ cm. Length of each thermometer coil 1·05 cm., of heating 0·40 cm. along wire. Date, 1923/11/2. Water bath at start 14·63°C., at end 14·67°C.

Twist per cm.	Heating.	Thermometer readings, ohms.			
		Case before.	A.	B.	Case after.
0	On.	2·8966	3·0066	3·0042	
0	Off.		2·8657	2·8648	2·9015
0·51 *L*	On.	2·8978	2·9050	3·0044	
0·51 *L*	Off.		2·8624	2·8619	2·8990
0	On.	2·8965	2·9041	3·0088	
0	Off.		2·8621	2·8612	2·8970

The fourth figure after the decimal point represents hundredths of a degree C. very nearly.

The electrical conductivity was measured by the potentiometer method, a constant current being sent through the wire and the fall of potential down a given length determined with the wire twisted and without twist. In some cases the resistance of a considerable length of the same wire was measured directly by the bridge method and the change on twisting found.

VII. TABLES OF RESULTS.

(1) Steel wire 0·203 cm. diameter, 13·6 cm. long of mild steel, from the London Electric Wire Co.

Heat Conductivity.

Distance between temperature coils 4·5 cm. Heating current 0·200 amps. *k* by equation (5·7), 0·19.

Date.	Twist. (Deg. per cm.)	(Twist)^2.	Conductivity.	Per cent. decrease.
1922/7/18	0	0	1·0	0
	0·22 *Left*	0·048 *L*	0·9975	0·25
	0·51 *L*	0·26 *L*	0·9895	1·05
	0·74 *L*	0·55 *L*	0·9850	1·5
	0·88 *L*	0·77 *L*	0·9785	2·15
/7/19	0	0	0·9925	0·75
/7/18	0	0	1·0	0
	0·88 *Right*	0·77 *R*	0·9980	0·2
	1·48 *R*	2·19 *R*	0·9870	1·3
	0	0	0·9955	0·45
/7/12	0	0	0·9980	0·2

Electrical Conductivity.

1922/10/10	0		1·0	
	0·74 *R*	0·55 *R*	0·999	0·1
	1·84 *R*	3·4 *R*	0·993	0·7
	0	0	0·996	0·4
/10/11	0	0	1·0	
	1·10 *L*	1·2 *L*	0·998	0·2
	1·47 *L*	2·2 *L*	0·996	0·4
	1·84 *L*	3·4 *L*	0·993	0·7
	0	0	0·998	0·2

(2) Aluminium wire 0·203 cm. diameter, 13·6 cm. long, of commercially pure metal, from the London Electric Wire Co.

Heat Conductivity.

Distance between temperature coils 4·6 cm. Heating current 0·300 amperes. *k* by equation (5·7), 0·39.

Date.	Twist.	(Twist)^2.	Conductivity.	Per cent. decrease.
1922/12/18	0	0	1·0	0
/19	1·47 *L*	2·2 *L*	0·9980	0·2
/20	0	0	1·000	
/21	1·47 *R*	2·2 *R*	0·9980	0·2
	1·84 *R*	3·4 *R*	0·9910	0·9
1923/1/15	0		1·0	
1923/1/15	0		1·0	
	1·47 *L*	2·2 *L*	0·9975	0·25
	1·84 *L*	3·4 *L*	0·9930	0·7
1/17	0	0	0·9985	0·15

Electrical Conductivity.

1923/2/14	0	0	1·0	
	1·10 *R*	1·2 *R*	0·9997	0·03
	1·36 *R*	1·85 *R*	0·9994	0·06
	1·84 *R*	3·4 *R*	0·9984	0·16
	0	0	1·0	0
	1·36 *L*	1·85 *RL*	0·9976	0·24
	1·84 *L*	3·4 *L*	0·9965	0·35
	0		1·0	

(3) Copper wire of 0·203 cm. diameter, 13·6 cm. long, of high conductivity copper, from the London Electric Wire Co.

Heat Conductivity.

Distance between temperature coils 4·5 cm. Heating current 0·410 ampere. *k* by equation (5·7), 0·99.

Date.	Twist.	(Twist)^2.	Conductivity.	Per cent. decrease.
1922/10/19	0	0	1·0	
	0·59 *R*	0·35 *R*	0·9990	0·1
	0	0	0·9990	0·1
/10/20	0	0	1·0	
	0·74 *R*	0·55 *R*	0·9965	0·35
	0	0	0·9965	0·35
/11/1	0	0	1·0	
	0·65 *L*	0·42 *L*	0·9955	0·45
	0	0	1·0	
/11/2	0	0	1·0	
	0·51 *L*	0·26 *L*	0·9975	0·3
	0	0	0·9999	0·01

Electrical Conductivity.

	0	0	1·0	
	0·74 *L*	0·55 *L*	0·9985	0·15
	0	0	1·0	0

(4) Lead wire 0·195 cm. diameter, 13·6 cm. long, of commercially pure lead, from Messrs. Johnson & Matthey.

Heat Conductivity.

Distance between temperature coils 4·50 cm. Heating current 0·24 ampere. *k* by equation (5·7), 0·092.

Date.	Twist.	(Twist)^2.	Conductivity.	Per cent. decrease.
1923/1/30	0	0	1·0	
	0·29 *R*	0·08 *R*	1·0	0
	0·59 *R*	0·35 *R*	0·9965	0·35
	0	0	1·0	0
/2/5	0	0	1·0	
	0·81 *L*	0·66 *L*	1·0	0·0
	1·10 *L*	1·2 *L*	0·9975	0·25
	0	0	1·0	0
/2/6	0	0	1·0	
	0·59 *L*	0·35 *L*	0·9980	0·2
	0	0	1·0	0
/2/8	0	0	1·0	
	0·90 *L*	0·8 *L*	0·9965	0·35
	0	0	1·0	0

Electrical Conductivity.

	0	0	1·0	
	0·90 *R*	0·8 *R*	0·9985	0·15
	0	0	1·0	0

Fig. 2 shows graphically the results obtained.

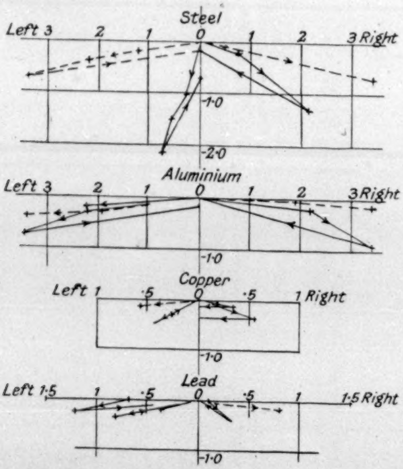

FIG. 2.—ABSCISSAE REPRESENT TWIST IN DEGREES PER CM.^{2}, ORDINATES PER CENT. DECREASE OF CONDUCTIVITY, FULL LINE THERMAL, DOTTED ELECTRICAL.

VIII. DISCUSSION OF THE RESULTS.

It will be noticed that in several cases the heat conductivity of a wire after the wire has been twisted does not return immediately to the value it had before the twist. This is so especially in the case of steel. The cause of this we have not yet been able to trace, but the same difficulty was encountered by Smith apparently to a greater degree. Apart from this irregularity the effect of twist in most of the wires tested is to decrease the heat conductivity by a few parts in in a thousand. In steel only does the decrease exceed 1 per cent. The decrease appears to be independent of the direction of twist and for small twists proportional to the square of the twist. Its magnitude agrees as well as could be expected with that found by Smith, who took it as proportional to the twist.

The effect of twist on the electrical conductivity is in all cases less than that on the heat conductivity, but follows roughly the same law—that is, it is proportional to the square of the twist. Its magnitude is of the same order as those found previously by Kelvin, Witkowski, Ascoli and von Szily,* who took it as proportional to the twist.

DISCUSSION.

Mr. C. R. **DARLING** said that the method described seemed an admirable one, but wished to know whether the wires were annealed before the experiment ? The initial condition of the specimens would doubtless affect the results obtained, and might account for the discrepancies between the conclusions arrived at by different observers.

Mr. **ROLLO APPLEYARD** also referred to the effect of annealing the wire under test, and added that the effect of stretching on resistance is at least as important as that of twisting. In stranding copper wires for a cable the outer wires are stretched as well as twisted, and any permanent increase of resistance due to such causes is of some consequence in practice. It would be interesting if the experiments could be carried out on a large scale, instead of on a laboratory scale.

Prof. A. O. **RANKINE** inquired whether the effect differs according as the direction of twist is clockwise or counter-clockwise as viewed in the direction of flow of the heat or electric current.

Mr. F. E. **SMITH** suggested that the experiments might be simplified by means of an arrangement for producing a potential difference between the two clamps C, C' (Fig. 1 of the Paper) at the ends of the wire. The necessary heating would be effected by the current thereby set up along the wire, which would have a maximum temperature at its middle point owing to the cooling effect of the masses of metal at its ends. The required thermal and electrical measurements could be made by means of wires twisted on to the principal wire as described in the Paper.

Dr. **ALEXANDER RUSSELL** expressed his admiration for the methods of measurement devised by the authors. He pointed out that the twisting of the strands in a stranded cable produces a somewhat similar effect. For example in a seven-strand cable the centre strand is straight and the other six strands form helices enclosing it. If r be the resistance of the central strand the resistance of the cable is greater than $r/7$, for the bulk of the current continues to flow along the helical strands and little of it flows from strand to strand. Hence, if the " lay " of the helical strands be 1 in n (*i.e.*, if the pitch of a helical strand be n times its diameter) then the resistance R of the cable is given by

$$1/R = 1/r + 6\Big/\left\{r\left(1+\frac{\pi^2}{n^2}\right)^{\frac{1}{2}}\right\},$$

and thus

$$1/R = (7/r)(1 - 3\pi^2/7n^2)$$

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* Kelvin, Nature, 18, p. 180 (1878). Witkowski, Trans. R. Soc. Edin., 30, p. 413 (1881), and Ann. der Phy., 16, p. 161 (1882). Ascoli, Mimori. Acc. Lincei, 4, p. 406 (1887), Rend. Acad. Lincei, 6, p. 502 (1890), and 1, p. 10 (1892). Von Szily, Journ. de Physique, 8, p. 329 (1899), found the electrical resistance of constantan increased proportionally to the twist up to the elastic limit. For a constant twist the resistance decreased slowly with time. Less accurate observations on copper, nickeline and maillechort showed the same general results.

approximately when $(\pi/n)^2$ is small. Hence, the diminution of the conductance of the cable due to the lay of the wires is proportional to the square of the twist.

Prof. LEES, in reply to the discussion, said that the wires were soft and not annealed. The object of the experiments was to test the electron theory of conduction, and it was sufficient for this purpose that the state of the wire should merely be the same for the electrical as for the thermal measurements. The large scale tests which had been suggested presented attractions for the engineer, but any accuracy lost by performing the experiments on a small scale was fully compensated by the more precise methods available in a laboratory, while the relatively small cost of the latter methods is a serious consideration to the physicist. The effect of stretching has been investigated by Johnstone, and a reference to his work is given in the Paper. The thermal conductivity is increased by stretching while the electrical conductivity is diminished—another instance of failure in the relation predicted by the electron theory. Prof. Rankine's suggestion was an interesting one. It had been considered, but rejected as improbable; it might, however, be worth investigation. The method suggested by Mr. Smith was somewhat similar to that of Jäger and Diesselhorst, but the latter requires the thermal flow to pass through joints, and inaccuracies arise from this circumstance. As regards stranded cables, the effects of twisting and stretching are in opposite directions for thermal conductivity, but aid one another in the case of electrical conductivity, so that the matter would probably repay investigation.

XXIII. *The Use of the Wien Bridge for the Measurement of the Losses in Dielectrics at High Voltages, with Special Reference to Electric Cables.* By A. ROSEN, *A.C.G.I., B.Sc. (Engineering), A.M.I.E.E.*

RECEIVED MARCH 25, 1923.

ABSTRACT.

In the preliminary section, the loss angle of an imperfect condenser is defined, the equations for the Wien bridge are derived, and the effects of variations of frequency and voltage on the balance are discussed.

One of the difficulties in the application of large potential differences to a bridge is the effect on the arm which has to withstand the high voltage. In the arrangements due to Monasch and Schering, this arm is the known condenser; in the bridge as used by the author, the voltage is applied to the ratio coils. The errors introduced by earth impedance are eliminated by using the Wagner auxiliary bridge.

The applications to measurements on cables are considered, and the use of the double bridge in determining the "wire-to-wire" and "wire-to-sheath" losses in a multi-core cable is described.

In Appendix I. the corrections due to imperfections of the bridge arms are discussed, and in Appendix II. a simple quantitative theory of the double bridge is given.

PRELIMINARY.

Loss Angle of a Condenser.

CONSIDER a periodic E.M.F. of pure sine form and frequency $\frac{\omega}{2\pi}$, represented by $v=\hat{V}\sin\omega t$, applied to the terminals of a condenser; the resulting current, which may possibly be complex, can be represented in the general case by

$$i=\hat{I}\sin(\omega t-\varphi)+\hat{I}_3\sin(3\omega t-\varphi_3)+\hat{I}_5\sin(5\omega t-\varphi_5)+\ldots.$$

The mean power dissipated will be $W=\frac{1}{T}\int_0^T vi\,dt$, T being taken over a whole number of periods, $=\frac{1}{2}\hat{V}\hat{I}\cos\varphi$, so that, in measuring the power, we may disregard the harmonics in the current wave, and consider only the fundamental, $\hat{I}\sin(\omega t-\varphi)$. For any particular values of $\hat{V}$ and ω we can imagine the condenser replaced by a combination of a perfect, unvarying capacity C, shunted by a resistance which obeys Ohm's law, of conductance G, such that the total current is $i=\hat{I}\sin(\omega t-\varphi)$; C will be the effective capacity of our condenser, and the loss at a root-mean-square voltage V will be V^2G. The current leads on the P.D. by an angle $\varphi=\frac{\pi}{2}-\theta$, such that $\tan\theta=\frac{G}{\omega C}$, and the power may be expressed by $W=V^2\omega C\tan\theta$; θ is therefore a measure of the loss in the condenser, and is referred to as the "loss angle." The "power factor" as defined by $\int_0^T vi\,dt\Big/\left(\int_0^T i^2dt\,.\int_0^T v^2dt\right)^{\frac{1}{2}}$ equals $\cos\varphi\,\frac{\hat{I}}{(\hat{I}^2+\hat{I}_3^{\,2}+\hat{I}_5^{\,2}+\ldots)^{\frac{1}{2}}}$, so that only when the current wave is free from harmonics can $\cos\varphi$ be strictly termed the power factor of the condenser; the difference can, however, usually be neglected.

The Wien Bridge.

The Wien Bridge,* a particular application of which is described in this Paper, measures the values of C and G, and thus θ directly. Diagrammatically it is represented in Fig. 1. The arms AB, AD consist of pure resistances R_1, R_2 respectively.

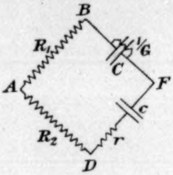

FIG. 1.—WIEN BRIDGE.

The unknown condenser to be measured is placed in the arm BF, and is represented by the capacity C shunted by the resistance $\frac{1}{G}$. The fourth arm is occupied by a perfect condenser c in series with a pure resistance r. An alternating P.D. of frequency $\frac{\omega}{2\pi}$ periods per second, and a suitable detector are placed across the conjugate points BD, AF. The impedance of the arm BF is $Z_3=\frac{1}{G+j\omega C}$, and of DF is $Z_4=r-\frac{j}{\omega c}$. The condition for no current in the detector is

$$\frac{R_1}{R_2}=\frac{Z_3}{Z_4}=\frac{1}{(G+j\omega C)\left(r-\frac{j}{\omega c}\right)}.$$

Equating the real and imaginary terms, we obtain, after transformation,

$$G=\frac{R_2}{R_1}\cdot\frac{\omega^2c^2r}{1+\omega^2c^2r^2}$$

$$C=\frac{R_2}{R_1}\cdot\frac{c}{1+\omega^2c^2r^2}$$

$$\tan\ \theta=\frac{G}{\omega C}=\omega cr.$$

In the majority of cases, $\tan\theta$ is a small quantity, and its square can be neglected in comparison with unity. Writing $\frac{R_1}{R_2}=m$, the expressions reduce to

$$G=\frac{\omega^2c^2r}{m}\ \text{mhos}$$

$$C=\frac{c}{m}\ \text{farads}$$

$$\theta=\omega cr\ \text{radians},$$

c and r being in farads and ohms respectively.

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* M. Wien, Wiedemann's Annalen, Vol. 44, p. 689 (1891).

For a given set of conditions, G and C are fixed, and to satisfy these equations we must vary any two of c, r and m. In the arrangement described in detail below m is kept fixed, and c and r are varied for a balance.

Variation of Balance with Frequency.

This will depend on the properties of the dielectric being tested. It has been found experimentally* that for practically all the materials used as dielectrics for condensers the variation of the conductance G with frequency may be represented approximately by an equation of the form $G=a+b\omega$, where a and b are constants which are determined by the temperature ; also the capacity C is nearly independent of frequency. Substituting in the above equations, we obtain

$$r \propto \frac{a}{\omega^2}+\frac{b}{\omega}$$

$$c \doteq \text{constant.}$$

We may say that r varies inversely as some power of the frequency between the first and second, the precise number depending on the relative values of a and b. (For electric cables with impregnated paper insulation, at normal temperatures, a is relatively small, and r varies very nearly inversely as the frequency.)

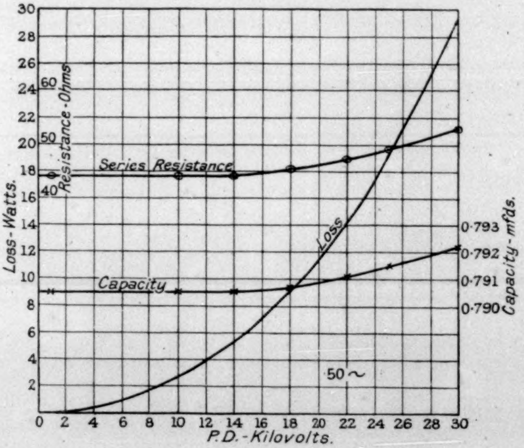

Fig. 2.—Variation of Balance with Voltage.

It follows that if the current in the condenser tested is complex—i.e., contains terms of more than one frequency, the current in the detector can never be zero. However, by using an instrument that will respond to only one periodicity—e.g., a tuned vibration galvanometer—a balance can be obtained for the fundamental, and the effect of the harmonics is eliminated. As no loss is caused by the harmonics, the result still gives the power spent in the dielectric. Further, if the P.D. applied to the bridge is not of pure sine form, the balance obtained will be in terms of the fundamental components of the voltage and current—i.e., the bridge will measure only the loss caused by the fundamental; in other words, the readings are independent of the wave-form of the applied voltage; this has been experimentally verified by Monasch.*

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* Fleming and Dyke, Journal I.E.E., Vol. 49, p. 323 (1912).

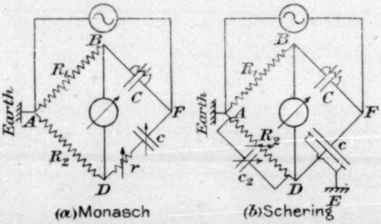
Variation of Balance with Voltage.

The change of capacity and loss angle of a condenser with voltage, if any, is generally not rapid, and, in any case, if the loss varies as V^n, to balance the bridge, r will vary approximately as V^{n-2}. This is a considerable advantage compared with the wattmeter method of measuring power, in which the reading varies as V^n, as firstly, the balance is not much disturbed by fluctuations of voltage, and, secondly, errors in the voltmeter which measures the P.D. are of less importance. In Fig. 2 are plotted some figures relating to a paper-insulated cable, and show the slow variation of r and c with voltage and the comparatively rapid change in the power loss.

PREVIOUS WORK.

Monasch's Method.†

The application of high voltages to a bridge presents certain difficulties which were only partially overcome by Monasch's arrangement. Referring to Fig. 3(*a*), the P.D. was applied to the points AF, A being earthed, and a tuned optical telephone was placed as detector across BD. It will be seen that the full voltage on the condenser tested was applied to the adjustable capacity c, and this introduced serious limitations. After many efforts, a variable air condenser was constructed capable of withstanding 11 kilovolts without appreciable loss due to ionisation of the air, its maximum capacity being 230×10^{-6} mfd.; this set a definite upper limit to the testing pressure. The large surface area of this condenser introduced considerable earth capacity effect, so that the readings obtained were only



(*a*)Monasch (*b*)Schering

FIG. 3.—BRIDGES USED BY MONASCH AND SCHERING.

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* B. Monasch, Electrician, Vol. 59, p. 416 (1907).

† Electrician, loc. cit.

comparative and not suitable for calculating the true loss angle. For his quantitative measurements, Monasch used a much lower P.D. and larger capacities, as he had concluded that the balance was independent of the voltage. As regards cable, this is only true for the comparatively low dielectric stresses used by Monasch and for the higher values it is necessary to test at the full voltage.

Schering's Method.

A more successful arrangement due to Prof. Schering, and described by Alfred Semm,* is shown in Fig. 3(*b*). It differs from Monasch's scheme in that the leakance in condenser C is compensated by a capacity c_2 shunted across R_2. The equations for balance are of the same form as previously, viz.,

$$C=\frac{R_2}{R_1}\cdot\frac{c}{1+\omega^2c_2{}^2R_2{}^2}$$

$$\tan\theta=\omega c_2R_2$$

and balance was obtained by varying R_2 and c_2. The air condenser c, which was of fixed capacity, had a " guard plate " connected to earth ; this placed the " edge-effect " with the possibility of ionisation outside the bridge arm. The effect of the earth capacity of the L.T. plate was negligible, as it was in parallel with c_2, which was of the order of 1 mfd. By making the distance between the plates large, c was made capable of withstanding 100 KV., but the capacity was small, being only 50 cm., i.e., $55{\cdot}6\times10^{-6}$ mfd. A defect is that the impedances of the bridge arms were very unequal, causing loss of sensitivity ; e.g., R_1 is given as usually 200 ohms, which is $\frac{1}{300,000}$ of the impedance of c at 50 cycles per second. Further, a serious practical disadvantage when testing cables is that the low-tension side of C has to be well insulated from earth, as any earth impedance shunts the arm R_1.

AUTHOR'S ARRANGEMENT.

The method used is shown in Fig. 4 ; it consists essentially of the usual Wien bridge with the Wagner earthing arrangement.† The arms of the bridge *ABFD* are as in Fig. 1, and in addition an auxiliary bridge, consisting of the condensers C', c' and a resistance r', is placed across the points *BD*, the junction *O* of C' and

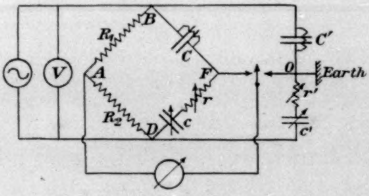

FIG. 4.—WAGNER DOUBLE BRIDGE.

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* Archiv für Elektrotechnik, Band 9, p. 29 (1920).

† K. W. Wagner, E.T.Z. Vol. 40, p. 1001 (1911).

r' being put to earth. The detector is switched alternately between the points AF, AO, c, r and c', r' being varied respectively to obtain a balance. When finally there is no deflection in either position, A, F and O are at the same, i.e., earth potential, and c, r give the true readings required. Considering the currents to earth from the points $ABFD$, those from B and D will merely modify the impedance of the arms BO, DO, and since A, F and O are at the same potential, no earth currents can flow from A and F. The effect of the impedance to earth of the apparatus, including the source and the detector, is thus eliminated.* (*See* Appendix II.)

The following points are to be noticed :—

(1) By connecting the source across the ratio coils, the arm of the bridge which has to withstand the high voltage is a resistance and not a capacity ; it is easier to construct a satisfactory resistance for this purpose than a condenser.

(2) By using a large value for the ratio $\frac{R_1}{R_2}$, e.g., 100 : 1, the P.D. across the measuring condenser c can be brought to within everyday values, so that ordinary condensers can be used for this purpose, and there is no limit as to their size.

(3) The galvanometer is brought to earth potential, and the P.D. between the adjustable arm FD and earth is small, so that the apparatus can be manipulated conveniently and in safety, which contributes to speed and accuracy in working.

(4) The voltage on the condenser tested is a fixed proportion of the P.D. of the source, which can be read directly on a voltmeter.

PRACTICAL DETAILS.

Ratio Arms.

In obtaining the equations for the Wien bridge, it was assumed, for simplicity, that the phase angle of the resistances R_1, R_2 was zero. The equations still hold good if the resistances are not perfectly non-reactive, providing they have the same time-constant, in which case their ratio, m, has zero angle. This result can be largely attained by constructing both arms of identical units. R_1, R_2 consisted of 100 and one 5,000 ohm coils respectively, the total resistance being 505,000 ohms. They were wound on flat ebonite cards 0·036 in. thick with 0·002 in. d.s.c. Eureka wire in one layer, the direction of winding being reversed three times ; this form of winding gives a coil with small inductance and self-capacity suitable for withstanding high voltages. The cards were mounted in supports in such a way that the voltage and capacity between adjacent ones was small, and the whole was immersed in oil in a wooden tank supported on insulators. This resistance was capable of withstanding 30 KV. without undue temperature rise, and up to 50 KV. for a short time. A smaller coil of 2,500 ohms resistance was provided for use above 30 KV., giving a ratio of 200 : 1, and thus a lower voltage on the adjustable condensers.

For higher potential differences resistances become too elaborate, besides absorbing considerable power. In this case it is better to use inductances of the same phase angle for the ratio arms. A tapping from the H.T. winding of the transformer would serve, but, apart from the difficulty of adapting an existing transformer, this has the disadvantage that the time-constant ratio would probably alter with the voltage and load. A more suitable arrangement would be a specially designed iron-cored choke coil, the winding being built up of a number of similar units in series. Each one of these would then have the same phase angle under all conditions, and wherever the tapping is made the two parts would have equal time constants.

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* S. Butterworth, Proc. Phys. Soc., Vol. 34, p. 8 (1921).

Adjustable Capacity Arms.

For general work, three condensers were used in parallel as the variable capacity c, (1) mica dielectric, adjustable by means of plugs up to 20 mfds. in steps of 1 mfd., (2) mica dielectric of 3-decade type with dial switches giving continuous variation up to 1 mfd. in steps of 0·001 mfd., (3) air dielectric, continuously variable up to 0·0018 mfd. For the " extra " bridge capacity, accuracy of calibration and small loss angle are not needed, and Mansbridge condensers with paper dielectric were used for the values from 1 to 10 mfds. ; a 3-decade condenser in parallel gave continuous adjustment from 0·001 to 1 mfd. The balancing was much facilitated by using the condensers with rotating switches.

Source.

The source of current used was an alternator of 60 KW. capacity directly driven by a D.C. shunt motor. The frequency could be varied by altering the excitation of the motor field, and was measured by a frequency meter of the vibrating-reed type, actuated by a device mounted on the main shaft. The meter could be read to within $\frac{1}{2}$ cycle per second, and a variation of about $\frac{1}{10}$ cycle per second could be detected.

Leads.

The lay-out of the testing plant was such that all parts at a dangerous potential were completely protected, e.g., the transformer was housed in an enclosed chamber, and the voltmeter guarded by a large glass screen. It was desirable that the addition of the apparatus described should not alter this general rule. The ratio arms, which have to withstand the full voltage, were therefore placed in the chamber alongside the transformer, and the galvanometer, variable condensers and resistances, which are all at low working potential, were placed conveniently near the controls for the frequency and voltage. This necessitated somewhat long connecting leads ; these were brass-taped over the insulation and then laid up in pairs, the screens being joined to earth. Thus, the possibility of inductive interference from the primary circuit was avoided ; further, the impedance between the leads was eliminated, as it was taken account of by the Wagner auxiliary bridge. This impedance consists, not only of the capacity and leakance of the leads, but also leakage over the ends, and this last may, in certain cases, cause serious errors unless avoided by having the leads screened.

When tests were taken on drums of cable, the P.D. was conveyed through highly-insulated leads, about 20 yards in length, consisting of two single core cables with impregnated paper insulation, the lead sheathing being earthed. Here, again, the double bridge arrangement eliminated the impedance to earth of these leads, and no correction was needed to take account of them. The resistance of these H.T. leads was negligible, as the equivalent series resistance of the cable never amounted to less than several hundreds of ohms. However, the resistance of the leads to the adjustable arm *FD* could not in certain cases be neglected, and its value, 0·20 ohm., was added to the readings of the series resistance *r*.

Detector.

A vibration galvanometer made by H. Tinsley & Co. was used as detector A step-down transformer was found greatly to increase the sensitivity. (Actually

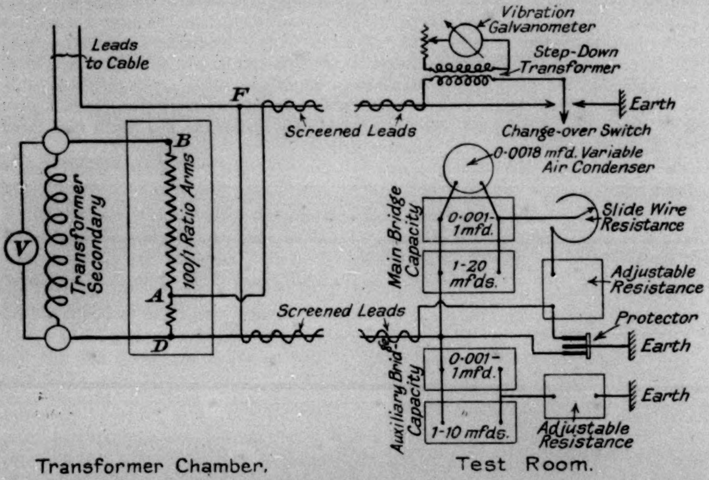

Leads
to Cable
Vibration
Galvanometer
Step-Down
Transformer
F
Earth
Screened Leads
Change-over Switch
B
0·0018 mfd. Variable
Air Condenser
Transformer
Secondary
100/1 Ratio Arms
V
Main Bridge
Capacity
0·001-
1 mfd.
Slide Wire
Resistance
1-20
mfds.
Adjustable
Resistance
A
Screened Leads
Protector
D
Earth
0·001-
1 mfd.
Auxiliary Bridge
Capacity
Earth
1-10 mfds.
Adjustable
Resistance
Transformer Chamber.
Test Room.

FIG. 5.—ARRANGEMENT FOR ROUTINE TESTING OF CABLES.

a voltmeter transformer 6,000 V./110 V. was used, because it happened to be available, and this gave an increase of about ten times in the deflection.) An adjustable resistance was put in the circuit to reduce the deflection to a suitable value when the bridge was out of balance. Theoretically best results are obtained when the galvanometer is exactly tuned to the supply frequency. It was found that this made the balance very sensitive to speed variation, and further, the response to adjustments of the bridge was sluggish. As there was generally no lack of sensitivity, it was found better to work with the galvanometer adjusted to about 3 cycles from the working value; even so, for the higher voltages the deflections were very large, and it was usually advisable for rapid working to cut them down by means of the series resistance.

Protective Devices.

Under normal working conditions no part of the apparatus to be adjusted is at a higher potential than 300 volts (A.C.) from earth; if, however, the dielectric of the test condenser breaks down, then the full voltage appears on the measuring condenser. This was guarded against by connecting the points *F* and *D* through spark gaps to earth. A protector designed by the Post Office for guarding their aerial lines against lightning discharges was used very successfully. It consisted of carbon blocks mounted on a porcelain base and separated by thin sheets of mica pierced with holes. This acted as a perfect insulator until the P.D. rose to about 450 volts, at which value the air broke down and a low resistance arc was formed; this was capable of passing enough current to operate an overload release in the primary circuit of the transformer.

DETERMINATION OF CORRECTIONS.

In Appendix I. the effect of the imperfections of the bridge arms is discussed, and it is shown that it is only necessary to take into account the loss angle of the variable condenser *c*, measured with the bridge ratio arms. This correction was determined for the various mica condensers by comparison with a known air condenser and a non-reactive series resistance placed in the arm *BF*, the rest of the circuit being as usual; the capacity to earth of the leads served as the corresponding auxiliary bridge condenser. The P.D. of the transformer secondary was cut down to 200 V. (this being a safe figure for the air condenser) by using a 10 V. accumulator to excite the field of the alternator instead of the usual 110 V. mains. By employing exact tuning of the galvanometer, the power factor could be measured to within a few per cent. On the large 1-20 mfd. condenser the values were of the order 0·001, and on the 3 decade condenser about 0·0008. The correction is thus roughly 10% when measuring power factors of 0·01, and an error of as much as 10% in the determination of the loss angle of the mica condensers would produce an error of only 1% in the power measurement.

It will be seen that the assumption is made that the constants of the bridge arms do not alter as the voltage is raised. This was checked by direct comparison with a simple air condenser constructed as follows:—

The two plates were formed of thin sheets of tinned iron 6 ft. × 2 ft. 6 in. fixed on to flat wooden frames. The L.T. plate was cut into two portions by a continuous slot parallel to and about 4 in. from the sides, the sharp edges being turned down. The upper (H.T.) plate was supported by porcelain insulators standing on the outer ring, which served as a guard plate and was earthed; thus there was only air as dielectric between the plates of the condenser proper.

To assist in obtaining a balance, a coil having an inductance of 16 millihenries and resistance of 20 ohms was placed in series with the 5,000 ohm ratio arm. With the plates about 2 in. apart, the capacity was 0·000197 mfd., and the bridge balance was constant up to 12 KV., beyond which an increase in the series resistance *r* was necessary. On altering the distance to 4½ in. by using bigger insulators, the capacity was reduced to 0·0000828 mfd., and the balance was undisturbed up to 25 KV. It is evident that no change was occurring in the bridge arms, the alteration being due to ionisation of the air in the H.T. condenser. Similar results were obtained using both air and mica condensers as the adjustable capacity *c*, and it was concluded that the bridge constants do not alter with voltage.

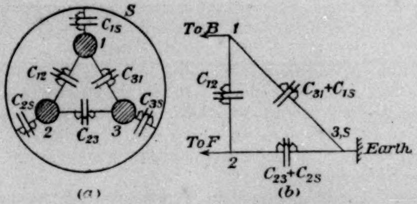
From the results of these tests, the phase angle of the ratio arms (a) was calculated as $a=\frac{\omega L}{R_2}-\omega cr$, L being the inductance (16 mh.) in series with R_2. When the capacity of the known air condenser c was 0·00828 mfd., the series resistance at 50 $\sim$ was 400 ± 10 ohms; thus $a = -0{\cdot}00004$, which is less than the experimental error. This result shows that the unit method of constructing the ratio arms is very satisfactory.

MEASUREMENTS ON CABLES.

In a single-core cable, the conductor forms one plate of a condenser and the outer lead sheath the other. The core is connected to the high potential point B on the bridge, and the sheath to the low potential point F. A second length of cable capable of withstanding the voltage, forms a suitable condenser C' for the auxiliary bridge, the sheath being put to earth. The sheath of the cable being tested must not be earthed, but the insulation need not be of a high order, e.g., satisfactory measurements were obtained on a coil wound on a dry wooden drum.

Multicore Cables.

Consider a cable containing n cores insulated from each other and from the metal sheath surrounding them. The "wire to wire" capacities between them may be represented by a series of $\frac{1}{2}n(n-1)$ leaky condensers connecting every pair of wires, and the "wire-to-sheath" capacities by a further n condensers connecting each core to the sheath.* A three-core cable is shown diagrammatically in Fig. 6(a). The Wagner double bridge enables us to determine the loss in, and capacity of any of these condensers directly.† To measure C_{12}, core 1 is connected to B in the bridge, and core 2 to F, the remaining core being joined to the sheath, which is earthed; the resulting system of condensers is shown in Fig. 6(b). The capacity of 1 to earth is $C_{31}+C_{1S}$, and this forms the condenser C' in the auxiliary bridge circuit. When final balance is obtained core 2 is brought to the same potential as S, and thus the capacity of 2 to earth, viz. $C_{23}+C_{2S}$ does not affect the main bridge; condenser C_{3S} is short-circuited, since 3 is joined directly to S. We have therefore eliminated all the capacities but the one to be measured, viz., C_{12}. Similarly to determine a



(a) (b)

FIG. 6.—CAPACITIES IN 3-CORE CABLE.

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* A. Russell, Alternating Current Theory, Vol. 1, Chap. 4.

† K. W. Wagner, E.T.Z., Vol. 25, p. 635 (1912).

" wire-to-sheath " capacity, say C_{1S}, we connect 1 to the high potential point *B*, the sheath to *F*, and the other cores to earth. It is necessary for this test, as when taking a single core cable, that the sheath should be insulated from earth.

Cable Sheath Earthed.

If the cable sheath is already earthed, as when the coil is immersed in water, it is not possible to obtain any " wire-to-sheath " measurements in this way, and we have to fall back on the ordinary single bridge. We can allow for the earth impedance of the bridge arms and of the leads to the cable, if we first take a reading with the cable disconnected ; this gives the value of the resultant earth impedance imagined all located in the arm *BF*. In what follows, the earth impedance of the bridge arms is understood to be included in the term " leads."

Let C_l, θ_l be the capacity and loss angle respectively of the leads.

C_c, θ_c be the capacity and loss angle respectively of the cable.

C_{c+l}, θ_{c+l} be the capacity and loss angle respectively of the leads and cable.

Then $C_c=C_{c+l}-C_l$,

and $\tan\theta_c=(C_{c+l}\tan\theta_{c+l}-C_l\tan\theta_l)/(C_{c+l}-C_l)$.

With the existing double bridge arrangement we can compensate for the leads and measure the cable directly as follows :—

With the cable disconnected and the arm *FD* (Fig. 7) open, the auxiliary bridge arm *OD* is adjusted to balance the leads. Then when the cable is on, the main bridge condenser and resistance are adjusted for balance, and the readings give the capacity and loss angle of the cable in the usual way.

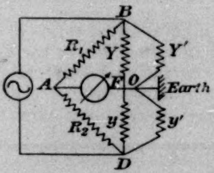


FIG. 7.—CABLE SHEATH EARTHED.

For, let Y' represent the admittance of the leads, which is balanced by y'.

$$\therefore \frac{R_1}{R_2}\cdot Y'=y'.$$

Again Y and Y' in parallel are balanced by y and y' in parallel.

$$\therefore \frac{R_1}{R_2}(Y+Y')=y+y'.$$

Hence $\frac{R_1}{R_2}\cdot Y=y$, or y is the required balance for Y.

Loss in a 3-core Cable.

The dielectric loss in a 3-core cable is given by

$$W=3\omega(V_w^2 \tan \theta_w C_w + V_e^2 \tan \theta_e C_e)$$

the subscripts w and e referring to " wire-to-wire " and " wire-to-sheath " quantities respectively; when the voltages are balanced, $V_e=\frac{V_w}{\sqrt{3}}$, V_w being the line pressure.

Two tests were employed to determine W: (1) the three cores against sheath, at a voltage $\frac{V_w}{\sqrt{3}}$, giving a loss W_e; it the sheath were earthed, the single bridge method as described in the previous section, was used; (2) a " wire-to-wire " test with the double bridge, at the full voltage V_w, giving a loss W_w; then $W=W_e+3W_w$.

Routine Work.

From the apparently complicated series of adjustments required to obtain a balance using the double bridge, it might appear that this method is not so suitable for routine testing when time is a consideration, as the simpler wattmeter. This is not the case. It is shown in Appendix II. that, because of the high ratio of R_1 to R_2, a balance is quickly obtained, and only one cycle of adjustments, i.e., one of the auxiliary and then one of the main bridge, is needed for each subsequent step in voltage. Further, as was pointed out in the preliminary discussion of the Wien bridge, the balance is much less affected by fluctuations of speed and voltage than a wattmeter.

When testing cables, a reading was first taken at a low voltage (1 or 2 *KV*.). This indicated if the circuits were correct before the high tension was applied, and provided a datum for determining the change in the capacity and loss angle with voltage. As a rule, a series of readings at 4 or 5 different pressures was taken in addition to the working voltage, as not much extra time was occupied in obtaining the subsequent balances.

The following is an example of a " wire-to-wire " test on a 22,000 volt 3-core cable at 50 ∿. Ratio=100 : 1.

(1) Voltage V kilovolts.	(2) Capacity, c mfds.	(3) Resistance reading ohms.	(4) Resistance, r ohms.	(5) ωcr.	(6) $\tan \theta$	(7) Loss $=V^2\omega C \tan \theta$ watts.
1	0·7905	40·5	40·7	0·0101	0·0109	0·027
10	0·7905	40·5	40·7	0·0101	0·0109	2·70
14	0·7905	40·5	40·7	0·0101	0·0109	5·30
18	0·7907	42·1	42·3	0·0105	0·0113	9·10
22	0·7911	44·1	44·3	0·0110	0·0118	14·2
25	0·7915	46·0	46·2	0·0115	0·0123	19·1
30	0·7922	49·7	49·9	0·0124	0·0132	29·6

Column (4) is obtained from (3) by adding 0·2 ohm, the resistance of the connecting leads to the variable arm. Column (6) is obtained from (5) by adding 0·0008, the loss angle of the measuring condenser. These results are plotted in Fig. 2, where, however, the resistance figures are adjusted to include the effective series resistance of the condenser.

COMPARISON WITH WATTMETER.

Comparative tests were carried out using a Duddell-Mather dynamometer wattmeter with the scheme of connections shown in Fig. 8. The effect of the capacity current flowing between the series resistance and its metal containing tank was eliminated by joining the case to the junction of the wattmeter coils. Correcting only for the loss in the current coils, the results, when measuring power factors of the order 0·01 agreed with the bridge figures to within 3 per cent., which was about the estimated experimental error due to unavoidable fluctuations in speed and voltage. An example is appended :—

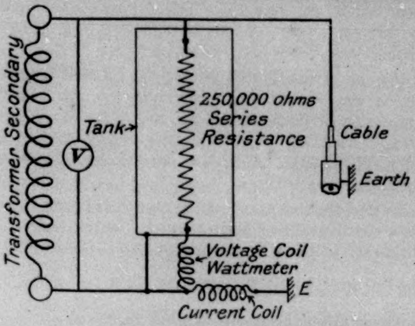


FIG. 8.—WATTMETER CONNECTIONS.

Bridge. 50~. Ratio = 100 : 1.

Voltage *KV.*	Capacity, c mfds.	Total Resistance, r ohms.	ωcr	$\tan\theta$	Loss watts.
12·7	8·103	3·40	0·0086	0·0096	39·5
22·0	8·117	4·17	0·0106	0·0116	143

Wattmeter 50~.

Voltage *KV.*	Constant, watts per division.	Reading of torsion head divisions.	Reading less lead correction divisions.	Loss watts.	Loss in current coils watts.	Loss in cable watts.
12·7	12·5	3·65	3·27	40·9	2·5	38·4
22·0	12·5	13·8	12·3	154	7·5	146·5

GENERAL REMARKS.

The bridge arrangement described in this Paper is the result of an attempt to devise an alternative to the wattmeter for measuring the dielectric losses in cables using only such apparatus as was already available, or could be easily made. It has proved more successful than was originally anticipated, in particular regarding the accuracy and ease of balancing. Whilst it is most suitable for the purpose for which it was designed, it has shown a surprising degree of flexibility and has proved capable of measuring a large range of capacities of varying power factors, at low and high voltages, without alteration of the connections.

In the measurement of dielectric loss, the accuracy obtainable is limited by the closeness with which the frequency can be controlled and measured. For ordinary work, 1 per cent. was considered sufficient, and with special care, an accuracy of one-fifth per cent. has been obtained.

Arising from the use of the double bridge, the method is particularly adapted to the application of the "guard-wire" principle, e.g., the "wire-to-wire" tests of cables, the elimination of lead corrections, and the measurement of condensers with "guard-plates." This, combined with the ability to deal with small capacities, of the order 0·0001-0·001 mfd., has proved very useful in the testing of dielectric materials in small quantities.

It should be noticed that one pole of the condenser tested is brought to earth potential; this is a disadvantage when taking a "wire-to-wire" test on a cable, as the voltage is limited to the safe "wire-to-earth" pressure; on a three-phase system with earthed neutral, the W/E voltage is about $\frac{1}{\sqrt{3}}$ of the W/W voltage.

In conclusion, the author desires to express his thanks to the management of Messrs. Siemens Bros. & Co., Ltd., Woolwich, for permission to present this Paper; to Mr. B. R. Chaplin, for his help in the construction of apparatus, and in particular, to Mr. E. A. Beavis, A.C.G.I., B.Sc.(Eng.), A.M.I.E.E., for his co-operation both in that part and in the experimental portion of the work.

APPENDIX I.

Corrections for Imperfections of Bridge Arms.

The possible departures from the assumptions made in obtaining the equations for balance of the bridge are :—

(1) The time constants of the ratio arms may not be the same, in which case $\frac{R_1}{R_2} = m\angle\alpha$, where α is a small angle.

(2) The condenser used as the balancing capacity c may have a small loss angle β.

(3) The series resistance r may have a small phase angle γ.

Then
$$Z_4 = r\angle\gamma - \frac{j}{\omega c\angle\beta} = r + \frac{\beta}{\omega c} - j\left(\frac{1}{\omega c} - \gamma r\right).$$

Since r is small compared to $\frac{1}{\omega c}$ and γ is small compared to r, γr may be neglected in comparison with $\frac{1}{\omega c}$, i.e., the phase angle of the series resistance does not introduce any appreciable error.

Further, $Z_3=m\angle a\,.\,Z_4=m(1+ja)\left(r+\frac{\beta}{\omega c}-\frac{j}{\omega c}\right)=m\left(r+\frac{a+\beta}{\omega c}\right)-j\,.\,\frac{m}{\omega c},$

neglecting ar and $\frac{a\beta}{\omega c}$ in comparison with $\frac{1}{\omega c}$.

Thus the corrections amount to adding a small quantity $\frac{a+\beta}{\omega c}$ to the value of the series resistance r. In the expression for the capacity C, r itself enters only as a small correcting factor; so that the effect of the angles a and β can be here safely ignored. For the loss angle, we have

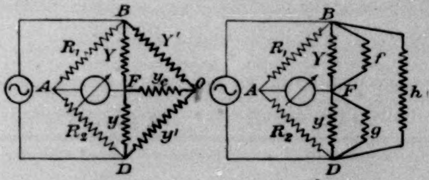
$$\tan\theta=\omega c\left(r+\frac{a+\beta}{\omega c}\right)=\omega cr+a+\beta.$$

The correction consists simply in adding the constant $(a+\beta)$ to the loss angle as determined in the usual way; $a+\beta$ may be regarded as the loss angle of the measuring condenser in terms of the particular ratio arms used.

APPENDIX II.

Theory of Double Bridge.

The general network of admittances resulting from the double bridge is shown in heavy lines in Fig. 9(*a*). To obtain the effect of the parallel system *BOD* with the "link" admittance *OF* when balancing *BFD*, we can transform the "star" *OB*, *OF*, *OD*, into its equivalent "mesh" as shown in Fig. 9(*b*).* The arrangement reduces to a simple single bridge; the admittance f is in parallel with the unknown arm Y, g is in parallel with the known arm y, while h is a shunt to the source and does not affect the balance.



(*a*) (*b*)

FIG. 9.—TRANSFORMATION OF DOUBLE TO SINGLE BRIDGE.

$$f=\frac{y_eY'}{Y'+y'+y_e}=pY' \text{ where } p=\frac{y_e}{Y'+y'+y_e}$$

$$g=\frac{y_e y'}{Y'+y'+y_e}=py'.$$

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* S. Butterworth, Proc. Phys. Soc., Vol. 33, p. 315 (1921).

If the bridge ratio is m, the net effect is to add $g-mf$ to the known arm, i.e., $p(y'-mY')$. The conditions for no effect are (1) $y_e=0$, in which case the auxiliary circuit is not necessary, (2) $y'=mY'$, which is obtained when the bridge is finally balanced.

If all the admittances consist of capacities of fairly small phase angle, p becomes approximately $\frac{c_e}{C'+c'+c_e}$, where c_e is the "link" capacity; (when the bridge is balanced, $c'=mC'$ and $p=\frac{c_e}{(m+1)C'+c_e}$).

Now $y+py'=m(Y+pY')$.

As an approximation p is assumed unaffected by small changes in y'.

∴ $$y+py'=\text{constant},$$

i.e., $$\omega^2c^2r+j\omega c+p(\omega^2c'^2r'+j\omega c')=\text{constant},$$

whence $$c+pc'=\text{constant},$$

$$r+p\left(\frac{c'}{c}\right)^2r'=\text{constant}.$$

Thus small changes $\delta c'$, $\delta r'$ in c', r' respectively, produce the same effect on the balance of the main bridge as variations $p\delta c'$, $p\left(\frac{c'}{c}\right)^2\delta r'$ in c, r respectively. Conversely, when balancing the auxiliary bridge, to obtain the effect of small changes in c and r, multiply by p', $p'\left(\frac{c}{c'}\right)^2$ respectively, where $p'=\frac{c_e}{C+c+c_e}$.

A given "out-of-balance" in the main bridge capacity will give rise to $\frac{1}{p}$ of the error when adjusting the auxiliary arm, and this again, when we return to the main bridge, will cause a new "out-of-balance" $\frac{1}{pp'}$ of the original value. Thus, in one cycle, we have reduced the error by the over-all factor pp'; in the same way, the over-all reduction factor per cycle for the resistance is also pp'. This shows that, to facilitate working, the "link" capacity c_e should be kept as small as possible, and also the advantage of using a large value for the ratio m.

In a particular 3-core cable, the "wire-to-wire" capacity was 0·007 mfd., and the "wire-to-earth" capacity 0·0275 mfd. per core. When taking a "wire-to-wire" test, the calculated figures allowing for the capacity of the leads, were as follows:—

$C=0{\cdot}00700$ mfd., $c=0{\cdot}700$ mfd., $C'=0{\cdot}0368$ mfd., $c'=3{\cdot}68$ mfd., $c_e=0{\cdot}0463$ mfd.

Main bridge balance,

capacity factor, $$p=\frac{0{\cdot}0463}{0{\cdot}037+3{\cdot}68+0{\cdot}046}=0{\cdot}0123(0{\cdot}016)$$

resistance factor, $$p\left(\frac{c'}{c}\right)^2=0{\cdot}0123\left(\frac{3{\cdot}68}{0{\cdot}70}\right)^2=0{\cdot}34\ (0{\cdot}33).$$

Auxiliary bridge balance,

capacity factor, $$p'=\frac{0{\cdot}0463}{0{\cdot}037+3{\cdot}68+0{\cdot}046}=0{\cdot}062(0{\cdot}05)$$

resistance factor, $$p'\left(\frac{c}{c'}\right)^2=0{\cdot}062\left(\frac{0{\cdot}70}{3{\cdot}68}\right)^2=0{\cdot}0022(0{\cdot}0033).$$

Over-all reduction factor, $pp'=0{\cdot}00076(0{\cdot}00080)$.

The figures in brackets are the measured results, and these agree closely enough with the calculated figures to prove the approximate theory given above. In this case, the capacity was read to the nearest 0·0001 mfd., and the resistance to the nearest 0·1 ohm ; hence the permissible error in balancing the auxiliary circuit was $\frac{0{\cdot}0001}{0{\cdot}016}=0{\cdot}006$ mfd., and $\frac{0{\cdot}1}{0{\cdot}33}=0{\cdot}3$ ohm. The penultimate adjustments on the main bridge could be to within $\frac{0{\cdot}0001}{0{\cdot}0008}=0{\cdot}125$ mfd. and $\frac{0{\cdot}1}{0{\cdot}0008}=125$ ohms without causing error, i.e., 0·062 mfd. and 62 ohms from the final values. Actually, the maximum variation with the usual voltage steps never approached these figures, so that once the first balance was obtained, only one cycle (i.e., one adjustment of the auxiliary and then one on the main bridge) was needed for each step in voltage.

DISCUSSION.

Dr. E. H. RAYNER : This description of some of the methods available for what is an increasingly important branch of electrical measurements is very useful to have on record. I should like to point out one or two practical details more as a warning than a criticism. The author suggests a series of iron-cored inductances for a potential divider. It is to be remembered that the important region of phase angle in this kind of work is from about 2 deg. to zero, and any want of accuracy or similarity in subsidiary apparatus may become of first importance. I should not trust to any iron-cored inductances with joints in the iron. Our experience with nominally exactly similar potential transformers, which might make excellent potential dividers, is to the effect that their phase-angles may differ in the ratio of 3 to 2, and I am informed that the value can be made to vary by a hammer blow which may open or close the iron joints.

Another point to beware of is distributed capacity in the 500,500 ohm potential divider, either from the sections to earth or between sections. The same applies to the series resistance of the Duddell wattmeter. At 50 ∼ it may be negligible ; but it is one of the most difficult technical points to deal with, and the use of condensers, which are much more " pure " than resistances or inductances, is always a great point in their favour when phase angles of minutes or seconds are of importance.

Mr. L. HARTSHORN : Having made many measurements of dielectric losses with the Wien Bridge and the Schering Bridge, I was surprised to find that Mr. Rosen had used the Wien Bridge for high voltage tests, instead of the Schering one. Grebe and Zickner have shown that the Schering Bridge is capable of giving great precision under suitable conditions, and the saving of time in doing away with the necessity of an auxiliary balance is very considerable. It seems unlikely that it is more difficult to make a condenser to stand the high voltages than a resistance. Could not the condenser constructed out of sheet iron by the author be used for this purpose ? As the resistance arms can be kept very small and yet the sensitivity remains quite adequate at such high voltages, the effect of the earth impedances on the low-tension side should not prove to be serious, provided it is made as small as possible by a suitable arrangement of the apparatus. An important advantage of the Schering Bridge is that it requires very little power. The resistance arms carry but small currents, so that they can be small and shielded, whereas the large resistances used by the author will, as Dr. Rayner pointed out, have considerable distributed capacity to earth, and as the Wagner earthing device only compensates for capacities to earth concentrated at the corners of the bridge, this bridge is not likely to be entirely free from error due to earth impedances. If comparative tests could be made, workers on dielectric losses would be glad to know how the two bridges compare as regards accuracy. In the matter of making the adjustments the Schering Bridge is unquestionably simpler.

AUTHOR'S reply (communicated): In reply in Dr. Rayner, I appreciate that it would be difficult to obtain two separate iron-cored inductances with the same phase-angle. However, what is suggested in the Paper is that the two ratio coils should have a common iron circuit; the effect of joints in the iron should then be the same for both. Further, the phase-angle of the ratio coils is easily measured and allowed for. This applies also to the resistances used as ratio arms. It is realised that the Wagner method of eliminating earth capacity is not perfect, but the resistances are connected in exactly the same way when their phase-angle ratio is measured, so that the correction takes account of any residual earth capacity.

In reply to Mr. Hartshorn, the objection to the Schering bridge is that the impedance between the L.T. pole of the condenser tested and earth must be of a high order. This is not always practicable in cable testing, whereas successful tests have been obtained with the arrangement described in which the resistance between the lead sheath and earth was only a few hundred ohms.

XXIV. *On a Coil-Galvanometer of Rapid Indication.* By Dr. W. J. H. Moll,

University of Utrecht.

Received March 6, 1923.

ABSTRACT.

The galvanometer is designed to secure rapid indication and steadiness of reading without unduly sacrificing the sensibility. The coil is long and narrow, and therefore of small moment of inertia: the mirror is supported by the wires forming the coil, between which it is slipped, and the coil is supported between an upper and a lower vertical wire, as distinct from strips, made of silicium bronze and put in tension.

In estimating the efficiency of a galvanometer it is customary to mention only its sensitivity—e.g., expressed as the deflection given by 1 microvolt at 1 metre.

In practical work, however, the interesting point to the physicist is chiefly what is the least measurable P.D.—a quantity which enters into the precision of every reading ? In the light of this criterion, the stability—i.e., the steadiness of the spot—is as important as the sensitivity.

A property of no less significance is the degree of rapidity of indication. As a matter of fact, the influence of this quantity on the precision of measurements is usually undervalued. In all galvanometer work disturbances of electrical origin are present, and their effect on the reading will in any case increase with the time during which they are active. Rapid indication, therefore, reduces the influence of such disturbances, and thus implies enhanced precision. The attendant economy of time is only a secondary advantage.

In the following will be pointed out the means by which it has been found possible to diverge from the usual construction of the coil-galvanometer, so as to obtain

(*a*) More rapid indication ;
(*b*) Greater stability.

(*a*) The rapidity of indication depends on the moment of inertia of the coil, the directive force of the suspension, and the magnetic field (as it affects the damping). These quantities, however, also determine the sensitivity. We will inquire whether it is possible to increase the rapidity without reducing the sensitivity.

When we confine ourselves to the case of aperiodicity, and neglect air-damping, the following three relations apply—

$$P=\frac{HF}{DR} \qquad T^2=\pi^2\frac{Q}{D} \qquad 4DQ=\frac{(HF)^4}{R^2}$$

where P is the sensitivity, T the semi-period on open circuit, D the restoring couple, Q the moment of inertia of the coil, F the total area of its windings, H the intensity of the magnetic field, and R the resistance of the circuit.

Between these seven quantities there exist still other relations, varying with the construction of the instrument. The equations will, therefore, only serve to indicate the direction in which we have to seek.

We may simplify the problem as follows : Suppose, for simplicity, the coil to be circular, and the number of its windings as well as the material from which it is made to be fixed. Assume, furthermore, a fixed relation between R and the resistance of the coil. In this supposition, R, F and Q are simple functions of the coil's diameter A, $R \propto A$, $F \propto A^2$, $Q \propto A^3$.

Or, dropping various constant factors and eliminating A

$$R = Q^{1/3}; \qquad F = Q^{2/3}.$$

By means of these relations we derive from the original expressions

$$P^4 = \frac{Q^{1/3}}{D^3}; \quad \text{and} \quad T^2 = \pi^2 \frac{Q}{D}$$

These equations show very clearly that what we have to do is to decrease Q. This will reduce T, but hardly affect P. By decreasing both Q and D, but D to a lesser degree, it will be even possible *to reduce T, while at the same time P increases.*

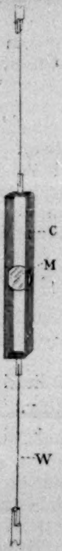

FIG. 1.

It may seem strange that this rather obvious procedure for reducing the moment of inertia of the coil, though repeatedly suggested, has never been applied in full consequence. Probably the reason for this is a difficulty which arises when the principle is applied. In the usual construction the coil is suspended by a strip, while the current is led off by means of a metal band so slack as to have no appreciable share in the directive couple. Now when the moment of inertia of the coil is made very small, and its weight consequently very slight, it will suffer from inevitable vibrations to a much higher degree than would be the case with a heavy coil.

(*b*) The desired stability has been obtained in a rather obvious way—namely, by simply stretching the coil between two metal wires, instead of suspending it.

One might suppose that this tension would increase the directive couple; but this is not the case. The directive couple is independent of the tension, and by using thin wires it may indeed be made as small as desired.

For this purpose wires are preferable to strips. As a matter of fact, strips, though generally used in galvanometers, are unsuitable. They are never absolutely straight; the result is that a strip when stretched (in the usual construction, by the weight of the coil) will undergo asymmetric deformation. Any external impulse will cause a temporary change in the tension, and may cause a slight deformation of the strip, resulting in oscillation of the system. In point of fact, the instrument proved to be much more indifferent to vibrations after the strips had been replaced by wires.

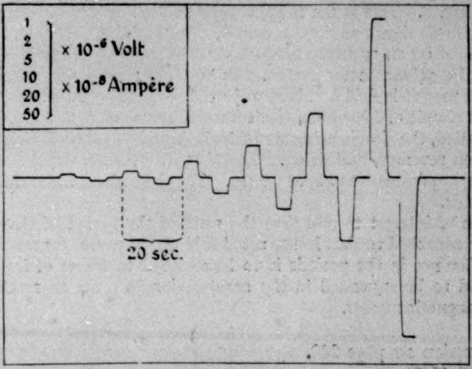

FIG. 2.

These considerations have led me to the construction of a new type of galvanometer. The principle of the moving part is shown in the accompanying sketch (Fig. 1). The coil, the mirror and the stretched wires are marked *C*, *M* and *W* respectively. Great care has been devoted to the attainment of symmetrical distribution of mass with respect to the axis of rotation. It proved possible to make the instrument practically indifferent to the vibrations of a solid wall-bracket.

The coil is made of copper wires about 0·07 mm. thick, free from iron and enamelled; it measures 5 × 40 mm., consists of 50 turns, and has a resistance of about 30 ohms. It has no former, but by means of a little shellac the windings have been made fairly rigid.

For the suspension-wires I use silicium-bronze, which has the same elastic qualities as the usual phosphor-bronze, but a lower specific resistance. The wires were 0·015 mm. thick.

The arrestment of the stretched coil presented some difficulty. A special mechanism has been designed in which, by turning a button, the coil is caught at front and back simultaneously and clamped.

A single permanent magnet yields a field of about 700 gausses, strong enough to reach aperiodicity when the outer circuit has a resistance of 50 ohms. By means of a magnetic shunt this field may be weakened, so as to make the indications dead-beat for smaller values of the external resistance. I have also designed for this galvanometer an electro-magnet; for some purposes this is preferable to the permanent magnet type, as it allows the exciting current (and thus the damping) to be regulated from a distance, and also since a stronger field is obtainable. The current is intended to be supplied by two or at most three small accumulator cells.

The reliability and the rapidity of this galvanometer may be judged by an inspection of Fig. 2. This is the full size reproduction of a photographically registered series of deflections and zero-positions. In obtaining this record the galvanometer was placed on an ordinary bracket fixed to the wall of my laboratory. The resistance of the galvanometer system was 45 ohms; with an external resistance of 55 ohms the magnetic field had been adjusted so that the galvanometer was dead-beat. In this circuit of 100 ohms, electromotive forces of 1, 2, 5, 10, 20, 50 micro-volts were applied, the circuit being closed for 5 seconds, opened during the following 5 seconds, then reversed, and so on. The distance between the galvanometer and the registering apparatus was about 70 cm. The deflections reach their final value within 2 seconds.

It may be of interest to add that the ratio of the period of the coil when on open circuit in zero field to that in the usual field of 700 to 800 gausses is as 100 : 97. The slight difference in the periods is to be ascribed to traces of iron in the coil. Much care had to be expended in the construction in order to reach such a low residuum of magnetic matter.

For Discussion *see* page 260.

XXV. *A Thermopile for Measuring Radiation.* *By* DR. W. J. H. MOLL, *University of Utrecht.*

RECEIVED MARCH 6, 1923.

ABSTRACT.

The thermopile is designed to be quick-reading and free from zero-errors, as well as sensitive. The cold junctions are in contact with metal masses which keep down their temperature, and in order that the hot junctions may have small heat capacity the bi-metallic strips composing the thermopile are made of plates of constantan and manganine silver-soldered along an edge, rolled in a direction parallel to the edge into thin foil, and then cut into strips perpendicular to the edge.

THE problem of how most efficiently to construct a thermopile has been repeatedly considered in the literature of the last 25 years. Without exception Papers dealing with this subject emphasise the necessity of high sensitivity.

When comparing, however, in practice, various types of thermopiles, it soon becomes obvious that there are still other qualities which are of paramount influence on the effectiveness of the instrument. The most important of these is the degree of stability of the zero. A second quality is the quickness, namely, the time elapsing between the admission of radiation and the moment at which the full electromotive force is reached.

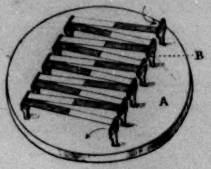

FIG. 1.

We shall describe a new method of construction which affords high stability and great quickness, together with high sensitivity.

In this method we have started from this principle: that it will be useful to minimise the heat-capacity of the active junctions, and to give a maximum value to that of the passive ones.

Fig. 1 is a diagrammatic sketch of the inner part of the new thermopile. *A* is a thick brass plate with two parallel rows of holes, in which flattened copper pins *B* have been clamped. A very thin coating of lacquer forms an insulating layer between these pins and the plate. On top of the pins the thermo-elements have been soldered. These consist of very thin blackened metal strips, made of the combination manganine-constantan.* With the exception of the first and last (indicated in the sketch by an arrow), all the pins bear two thermo-elements, and so form part of the passive junctions. The latter therefore possess a relatively rather high heat-capacity.

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* The combination manganine-constantan, though not showing a very high thermo-electric power (about 41 microvolts per 1°C.), has been chosen on account of the perfect elastic qualities of the two metals; these made it possible to give the thermo-elements their required thinness. An additional advantage lies in the durability of the metals, which neither rust nor corrode. For some applications it is, moreover, important that both manganine and constantan have a resistance which is independent of changes of temperature.

In order to minimise the capacity of the active junctions, the following method* has been introduced. Two rather thick plates of manganine (X) and constantan (Y) (*see* Fig. 2) are soldered together with silver. The superfluous silver projecting at both sides is polished away, after which a narrow seam of solder PQ remains. The bimetallic plate is then rolled out in the direction of the arrows. In this process the seam of solder is not broadened; with care it has been found possible to roll out this plate to a foil of only 0·005 mm. in thickness. By simply cutting this foil into narrow strips a large number of elements is obtained.

The plate A bearing the thermo-elements is tightly enclosed by a metal cover, provided with a transparent plate in front. All the necessary conditions have been thus fulfilled for obtaining both quickness and stability. The cover, together with the plate A and the pins B, form the "body," possessing a great capacity. In this body the thermo-elements and a little air are embedded. If no radiation is admitted, and no heat is conveyed to or carried from the body, the temperature of the active and the passive junctions will be the same. As soon as radiation begins, the temperature of the strips and the surrounding air will rise above that of the body. Thermal equilibrium will result, the energy received being transferred to the body. Continuous radiation will tend to raise the temperature of the body, and increasing temperature of the room will have the same effect. It is a pronounced advantage of the construction that such temperature variations do not affect the thermo-electromotive force.

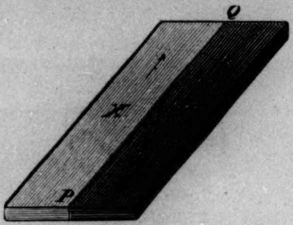

FIG. 2.

We may lay stress upon the fact that this result has been obtained (1) by giving to the body a large heat-capacity; (2) by keeping that of the strips and the surrounding air small; and (3) by securing an effective heat transport between the strips and the body. It is true that the means by which this transport is favoured (short strips, and small distance between the strips and the plate) are at the expense of the sensitivity. But we are convinced that quickness and stability are features so important that some sensitivity may be sacrificed. From the data which follow below it will be clear that the remaining sensitivity is still very high.

* This method is some years old; we gave a demonstration of these thermo-elements in 1913, during a Congress at Delft.

The new construction has the great advantage that it is not necessary to screen off the passive junctions; indeed the elements can be exposed to radiation along their full length. It is therefore possible to arrange the elements areally, and to construct surface-thermopiles.

Fig. 3 shows the grouping of 80 elements within a circle of 2 cm. diameter.

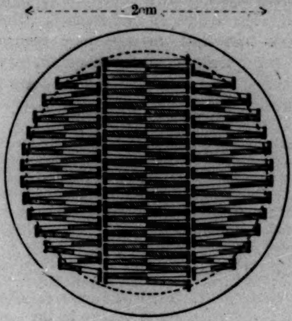

FIG. 3.

Such a thermopile has a resistance of about 50 ohms, and yields about 75 microvolts for the radiation of a candle at 1 metre, when closed by a plate of rocksalt.

The elements may also be arranged on a smaller surface; this increases the sensitivity per unit of area. Fig. 4 shows a type of construction, which in our investigations has largely fulfilled the requirements. This thermopile is closed by a window of fluorite having a circular opening of 6 mm., behind which 18 elements are arranged. Its resistance is about 25 ohms, and a candle at 1 metre generates about 13 microvolts. By providing the thermopile with a conical reflector the sensitivity may be largely increased.

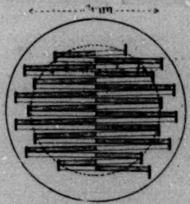

FIG. 4.

Both thermopiles are equally quick in response. If they are connected to a sufficiently rapid galvanometer (adjusted so as to have its indications dead beat), and radiation is periodically admitted and intercepted, the spot comes to a perfect standstill *within* 2 *seconds.*

It goes without saying that thermopiles built on this principle may be provided with a slit. In this case too the use of a fluorite window is recommended, in order to avoid air-currents.

A few examples of the working of this thermopile have been given in an earlier Paper, dealing with a registering microphotometer.[*] In this instrument the image of a strongly illuminated photographic plate is projected on to the slit of a small thermopile. The thermo-current is led to a rapid galvanometer, the deflections of which are photographically recorded. In that article some records of spectra were reproduced. From these the quickness and the accurate working of the thermopile will be evident.

DISCUSSION.

Mr. F. E. **Smith** commented on the strikingly dead-beat character of the indications given by both instruments. He suggested that the thermopile strips might be reduced to even a less thickness than ·005 mm. by solution in acid. The galvanometer seemed to have been designed to produce what, with a single fibre suspension, would have been an extremely sensitive instrument, but then the supporting wires were put in tension to reduce the period. Would it still be possible in these circumstances to obtain the required sensibility ?

Dr. R. T. **Beatty** recalled the time when it was necessary to work with a bismuth-iron couple of great heat-capacity and an insensitive Thomson galvanometer. Later came the Paschen astatic galvanometer ; but this instrument required such heavy protection from stray magnetic fields that few cared to use it. Dr. Moll's improvements should make possible an advance in measurements of the distribution of energy in the spectrum, for instance in the H and He spectra.

Mr. C. R. **Darling** suggested that the apparatus might usefully be applied to measure the distribution of energy in the spectrum of a glow lamp.

Mr. J. **Guild** inquired whether the present galvanometer differs materially from that exhibited by Dr. Moll to the Society three years ago, and what is the number of junctions per cm. in the linear form of the thermopile. If silicium bronze is similar in its properties to phosphor bronze the tension of the supporting wires of the galvanometer would have a serious effect on the sensibility of the instrument.

Mr. L. F. **Richardson** said that he thought Dr. Moll's thermopile admirable for indoor use. For work out of doors, where a rock salt cover could not be used and gusts of wind could not be entirely excluded, Mr. W. H. Dines had found it desirable to make the hot and cold junctions of as nearly the same thermal capacity as possible, so that air currents might affect both equally.

Mr. R. S. **Whipple**, in reply to the discussion, said that the difficulty of rolling the strips is immense, and Mr. Smith's suggestion as to solution in acid is well worth trying, though there might be difficulties due to unequal chemical action. He agreed that, although there is no instrument as sensitive as the Paschen galvanometer, there is none so troublesome to use.

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[*] Proc. Phys. Soc., Vol. 33, Part 4, p. 207 (1921).

DEMONSTRATION of (a) A New Balance for Compensating the Temperature Error of Watches and Chronometers, and (b) A Centre-seconds Marine Chronometer with Electric Contacts.

By PAUL DITISHEIM, *La Chaux de Fonds, Switzerland.*

(*a*) NEW BALANCE FOR COMPENSATING THE TEMPERATURE ERROR OF WATCHES AND CHRONOMETERS.

WHEN submitted to a rise in temperature, any uncompensated watch or chronometer fitted with the ordinary steel spring loses approximately 11 seconds per 1°C. in 24 hours. John Harrison first proposed a correcting curb method, so arranged that heat caused unequal expansion in two strips of brass and steel fastened together, the bending altering automatically the position of the pins and the effective length of the hair spring. At an early date, it was recognised that this correction should be effected by the balance-wheel itself, and not by the spring. In 1765 Pierre Le Roy accordingly applied the principle of the bimetallic rim, still used in chronometers and watches. This brass and steel compensating balance-whee was successfully improved by Arnold and Earnshaw. In this balance the action of increase of temperature on the elasticity of the hair spring is compensated by a corresponding reduction in the moment of inertia of the two circular bimetallic rims which curve in towards the centre of the balance-wheel.

In his Guthrie lecture* before the Physical Society of London, Dr. Guillaume presented a full account of the anomaly of the nickel-steels. By simply substituting a balance-wheel with nickel-steel and brass bimetallic rim, the Director of the International Bureau of Weights and Measures has solved the problem of the middle temperature error (Dent's error), the correction of which had been the subject of considerable endeavour and effort, especially among English watchmakers. The rates of high-class watches and chronometers were thereby greatly improved, the variation of rates now being only a quarter of the figures of 20 years ago. It should be noted that this Guillaume compensation balance can only be used with an ordinary steel hair spring.

The ordinary watch also benefits by use of the nickel-steel alloys. It can now be compensated solely through the action of a nickel-steel alloy substituted for the usual steel hair spring ; there is no bimetallic rim whatever—we come back to the plain, solid, uncut balance ring. By this means the temperature-coefficient is reduced to less than one second per day per 1°C. About 50 million watches have been manufactured with such auto-compensating hair springs since 1897, when Dr. Guillaume and the late Paul Perret made the first trial experiments.

The so-called *Invar* spring could not, however, be applied to the highest class of watches ; it had the disadvantage of causing a large middle temperature error, corresponding to a gain of 20 to 25 seconds per day, as compared with the rate at the extreme temperatures over an interval of 30°C. We should recall that an uncompensated watch would vary by five to six minutes per day in the same period.

A great advance, now to be recorded, consists in the use of hair springs made from Dr. Guillaume's new alloy *Elinvar.* The combination of this spring with the simple monometallic balance-wheel is now good enough to fulfil the requirements of higher-grade timekeepers.

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* Proc. Phys. Soc., Vol. 32 Pt. 5 (1920).

Another advantage of the elinvar hair spring is that, when made from a casting appropriate to a determinate linear expansion of the balance ring, very uniform rates over a wide range of temperature are obtained, without need of adjustment by highly skilled timing watchmakers.

The first temperature test made in England with an elinvar spring was carried out at the National Physical Laboratory, Teddington, from March 16 to April 29, 1920, with Watch No. 44811 Paul Ditisheim. Since that time other observatory and workshop tests have also proved the efficiency of the system.

It is no exaggeration to state that the new gift made to the horological industry will bring about a revolution in watch manufacturing methods. Dr. Guillaume, however, did not see how this combination could be adjusted, and feared it could not be used where the very highest accuracy was required. I have here specimens of balances which I have designed to overcome the difficulty. The principle consists in fitting on two opposite points of its rim two small bimetallic plates, which enable the timers to touch up and adjust in the usual manner.

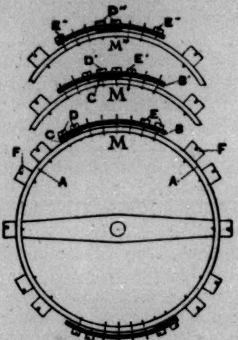

FIG. 1.—PAUL DITISHEIM COMPENSATING BALANCE, WITH MONOMETALLIC RING, FOR POCKET WATCHES.

A. Uncut monometallic ring. *B*. Compensating biblade. *C*. Removable heel holding the biblade. *D*. Fixing screws for the biblade. *E*. Compensating screws. *F*. Poising screws, without compensating influence—*M*, *M'*, *M"* varied arrangements of the screws and heel, showing the many facilities for adjusting the compensation through changing the position of the heel and the screws to *E*, *E'*, *E"*. (The aim has not been, as in Fig. 3, to reduce the barometric coefficient; the appearance is nearly the same as in the usual compensating balance.)

The tests I made of this combination have proved it to be very efficient, as was confirmed by the first-class pocket-chronometer trials to which it was submitted at Neuchâtel Observatory.

Some details of this new balance will now be given. The great practical interest of using the elinvar hair spring being precisely the suppression of the bimetallic balance, one may wonder wherein lies the advantage of the new combination, in which the bimetallic plate is reinstated. The difficulties normally encountered by timers in setting up a balance are to combine at all temperatures both its compensating action and its poise. The first depends on the combined actions of the two bimetallic rims; the second on the continuous regularity of their action. Furthermore, in calculating the end curves of the hair spring it is necessary to take into consideration the action of centrifugal force on the shapes of the blades, the effect of which in the association of a steel hair spring with an ordinary compensating balance may account for a difference of as much as 12 seconds in a day, as between large and small oscillations; this action is reduced to about a fifth by Guillaume's balance.

Now the coefficient of linear expansion of the metal of the balance fitted with our compensating bimetallic blades can always be chosen so that the uncompensated residue is reduced to a quantity not exceeding one-hundredth of that present in the compensating balance used with a steel hair spring; and the perturbing influences which the timer has to overcome will be reduced in a similar proportion. The poise at all temperatures is almost perfectly maintained by the monometallic rim of the balance, and in practice this result can easily be attained; the effect of centrifugal force on this part is almost negligible.

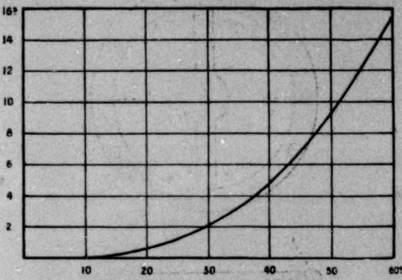

FIG. 2.—BALANCE WITH COMPENSATING AFFIX.

Abscissa: Angle subtended at centre of balance by biblades. Ordinate: Correcting effect in seconds per day.

With regard to the balance itself (Fig. 1), the first part manufactured is a plain, monometallic ring, to which small blades made of two metals of different expansibilities are adapted, the most expansible being inside or outside according to the necessity to correct fast or slow time when at the higher temperatures. In selecting these metals, the value of the quadratic coefficient of their equation of expansion can be utilised in order to neutralise any secondary error which might arise from the use of the elinvar alloy. Other means available for regulating the action of the biblades are their length and thickness.

In order to avoid any protuberance, the two blades can be inserted into grooves made in the rim of the balance. (Fig. 3.)

The screws needed to attain the equilibrium (poise) of the balance and to regulate its moment of inertia are level with the surface of the rim. Some of them, being screened, may be given a dissymmetric shape, permitting their rotation for the last touching up. This design has been adopted in order to reduce as much as possible air friction, which constitutes one of the main causes of variation in the timekeeping of watches now that the compensation has been so greatly improved, thanks to the Guillaume balance. In balances fitted with projecting screws the barometric coefficient is about 0·010 second per day for 1 mm. of mercury in the case of a chronometer of the " Marine " size, and about double this figure for one of the size of a large pocket watch. It can be gathered from observatory tests that timekeepers which have shown daily rates slightly inferior to one-tenth of a second would have lost their rank of classification had they been tested in periods of unstable atmospheric pressure. The experiments already made lead us to assign to the new balance a barometric coefficient about two-thirds of that of an ordinary compensating balance. This is a first attempt to reduce the barometric coefficient, the consequences of which will become still more important as new improvements are effected in chronometry.

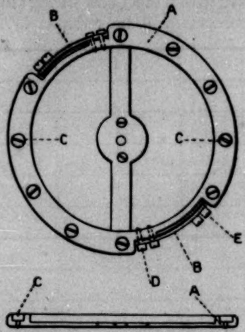

Fig. 3.—Paul Ditisheim Monometallic Balance, with Compensating Affix.

A. Monometallic independent uncut ring. *B*. Bimetallic compensating segment. *C*. Poising screws inserted in the level of the rings. *D*. Screws for fixing the biblade. *E*. Screws for adjusting the compensation.

The total absence of the centrifugal force effect reduces the timing to the pure Phillips curves, and thus facilitates the attainment of isochronism. The use of wireless signals for the daily checking of chronometers makes it no longer an absolute necessity to have regular timing over a long period, but, on the other hand, increases the importance of perfect timing during a diurnal period. In other words, the question of isochronism has become now the most important problem for the maker of chronometers, and it seems that the combination of the elinvar hair spring and balance with monometallic rim will equally constitute elements of progress in this direction.

Another advantage presented by the new form of compensation lies in the practically non-magnetic and rustless qualities of the elinvar alloys. The many facilities offered by the use of small detachable biblades, whose components can conveniently be changed or inverted, include the selection of a suitable non-magnetic metal or alloy for the larger uncut part of the plain balance-wheel above described.

(*b*) MARINE CHRONOMETER.

Two models of marine chronometer with large centre-seconds hand and electric contacts taken to a sounder were shown: (*a*) with lever escapement, five beats per second, only one of which is heard in the sounder; (*b*) with chronometer detent escapement. Once every five beats of the fast train lever escapement an electric contact is made, closing the circuit of a sounder by means of which a distinct tick is heard every second. At the sixtieth second one beat is omitted.

In an observatory the sounder can be fitted close to the transit instrument, the chronometer being at a point free from changes of temperature.

The mean variation of daily rate does not exceed 0·1 second.

XXVI. *A General Solution of the Problem of Finding the True Vertical for all Types of Marine and Aerial Craft.*

By J. G. Gray, *D.Sc., F.R.S.E., Cargill Professor of Physics, University of Glasgow.*

(Summary of a Lecture delivered June 8, 1923.)

In the first part of the lecture it was shown that the difficulties presented by this problem arise from the horizontal accelerations which result from the turning of vehicles. A gyroscopic pendulum to succeed must possess a real precessional period, or a virtual precessional period during turning motion of the vehicle on which it is measured, which is measured in hours.

Pioneer forms of Gray stabiliser were described. These were devised by the author and his brother for use in the Royal Naval Air Service. A system composed of a single gyroscope (mounted with its axis normally vertical) and an erector connected rigidly to one another, is pivoted to a gimbal frame by means of two cross pivots; and this frame is in turn attached by means of fore and aft pivots to uprights, or the equivalent, carried by the aeroplane. The system composed of the gyroscope and erector is mounted and balanced up so that the centre of gravity of the entire system coincides with the intersection of the pivot axes.

Various forms of erector made use of with these pioneer instruments were described, and their dynamical action discussed. One form consists of a circular track carried by the pivoted system, and so arranged that when the pivoted system is upright the track is horizontal. One, two, or more balls rotate on the track, each controlled by a pusher and a check carried by a member which rotates slowly (about 12 revolutions per minute) in the direction of spin of the gyroscope. When the system is upright the balls move round the track in contact with their pushers, and form a balanced system. When the system is inclined to the vertical the track is inclined to the horizontal, and each ball when ascending the slope of the track rests against its pusher, but after crossing the crest of the slope it is accelerated down the track and rests against its check. The motion of the balls relative to the pushers and checks results in the application to the pivoted system of an integral erecting couple.

It was shown how the contrivance could be arranged so that the stabiliser possessed the property that it was blind to the apparent vertical during turning motion of an aeroplane on which it was mounted, but conscious (so to speak) of the true vertical during the ordinary flight of the aeroplane.

These pioneer forms of the instrument were found to possess an accuracy, for bombing purposes, amounting to one-eighth or one-tenth of a degree, or about 20 feet on the ground from a height of 12,000 feet.

Previous work on the problem failed because the devices produced possessed the property that they left the true vertical quickly in the presence of the accelerations which accompany turning motions of the aeroplane, and returned to the true vertical only very slowly after the resumption of ordinary flight. The pioneer forms of Gray stabiliser (when properly used) moved only very slowly, if at all, towards the apparent vertical when the aeroplane was turning, and after the resumption of ordinary flight moved towards the true vertical (supposing an error to exist) relatively quickly.

Finally the latest forms of Gray stabiliser were described. These set themselves into the true vertical even when the vehicles on which they are mounted are turning, and this holds for all speeds of turning. Control is never lost. This result is obtained by constructing the apparatus so that a horizontal component of spin lies across the pivoted system, that is, parallel to the cross pivots. The pivoted system is mounted so as to be pendulous with respect to the pivots, and the direction of the horizontal spin, and its amount, is arranged so that when the vehicle turns there comes into existence a gyroscopic couple, applied about the fore and aft pivots, which is exactly equal and opposite to the so-called centrifugal couple applied to the pivoted system. Both couples are proportional to the angular speed at which the vehicle turns, and both change sign with that of the turning motion. Hence the compensation is correct for all speeds of turning.

This type of stabiliser when fitted with special erectors, a simple example of which was described, supplies a complete solution of the vertical problem. Assuming an error made (small) in adjusting the compensating component of spin the resting position of the device is one in which it is inclined to the true vertical at a very small angle. The devices solve the problem completely for the rapid and slow turning movements of aeroplanes, and for the turning motions of ships.

DISCUSSION.

Sir **James Henderson**, after complimenting Professor Gray on his lecture, referred to the great difficulties of the problem. These difficulties vary with the accuracy aimed at. If it be only required to maintain the vertical within one degree the problem is very easy, but if it be required to maintain it within one minute of arc the difficulties are very great, almost if not quite unsurmountable. The difficulties increase almost as the square of the accuracy aimed at.

Professor Gray had concentrated upon the problem of the bomb-dropping sight for aircraft, and had achieved considerable success during the war. The practical accuracy then aimed at in this connection was to maintain the vertical to about ¼ degree, and to this degree of accuracy Professor Gray had attained, as stated in his lecture.

The accuracy of any instrument of this type could be calculated accurately if the method of control were continuous. In Professor Gray's instrument the method of control involves a discontinuity which does not lend itself to easy computation. There are other methods of control, however, which are continuous and which enable accurate calculations to be made.

Professor Gray had said nothing in his lecture to give any idea of the amount of damping he employed. This determines the accuracy, because if the damping be very great the gyro follows the *virtual* vertical at corresponding rates and the deviation increases accordingly. On the other hand, if the damping be greatly diminished a considerable deviation arises from the rotation of the earth and a connection to the gyro-compass is required to compensate it. Thus when great accuracy is required the problem becomes excessively difficult.

Professor Gray's method of eliminating the effects of the centrifugal force during a turn was ingenious. If the speed adjustment be perfect, then any deviation which the pendulum has relatively to the vertical at the beginning of the turn turns in azimuth with the plane. The deviations due to any change of velocity either fore or aft or during a turn are, however, not so easily compensated.

M. **Paul Schilowsky** said that naval experts ask for a degree of accuracy which must be the despair of gyroscopic inventors, but accuracy to a degree or half a degree of angle is by no means unattainable. He had tried a different solution from that given by the lecturer, viz., that of suspending the gyroscopic pendulum in neutral equilibrium on knife edges passing through its centre of gravity. In time, however, precession takes place with this arrangement, and the pendulum errs progressively from the true vertical. What is required is that the pendular suspension should be neutral for curved motion but stable for rectilinear motion. Sir James Henderson has designed an instrument for indicating the curvature of the path of the aeroplane. A combination of the two devices should make it possible to meet the requirement mentioned. The lecturer's plan was open to the objection that very rapid changes of rotary speed in the gyroscope would be necessary to compensate for the rapidity with which an aeroplane changes its linear speed. His admiration for the lecturer's work was sincere, and he should expect great results if the latter would take up the problem of altering the suspension of the gyroscope according to the curvature of the path.

Mr. T. **Smith** said that all present would have followed the lecture with the greatest interest. It was not easy, however, for those less familiar than the lecturer with the practical handling of gyroscopic apparatus to appreciate all the points at issue in the absence of a formal mathematical treatment. Possibly mathematical difficulties would arise from the discontinuities which had been mentioned by Sir James Henderson, but the modern methods for dealing with discontinuous quantities developed in connection with the quantum theory might perhaps give a hint as to the line of attack to be adopted in the present case.

The **Lecturer**, in reply to the discussion, dwelt mainly on the point raised by M. Schilowsky, and claimed that his apparatus did become automatically "blind" during curved motion of the aeroplane in a manner satisfying M. Schilowsky's requirement. The effect of the earth's rotation on the gyroscope is very small provided the real precessional period is not great. The virtual precessional period, during turning motion, must of course be great.

The **President** congratulated the Lecturer on his solution of an important and difficult problem, and proposed a vote of thanks which was carried with acclamation.

XXVII. *The Fine Structure of Some Sodium Salts of the Fatty Acids in Soap Curds.*

By S. H. Piper, *D.S.O., B.Sc., F.Inst.P., and* E. N. Grindley, *B.Sc.*

Received June 8, 1923.

ABSTRACT.

X-ray photographs of certain sodium salts of the fatty acids (soap curds) show lines due to reflections from planes with very wide spacings of the order 40 A.U. These planar spacings increase uniformly with the number of CH_2 groups in the molecule, indicating an effective length of 1·25 A.U. for the CH_2 group. These and other lines can be accounted for by assuming that the curds are in the smectic state described by Friedel.

These experiments were originally undertaken to determine by X-ray methods whether the fibres observable in certain soap curds* were crystalline in nature. The observations are not yet complete, but some measurements already obtained appear to be of sufficient interest to merit publication.

Photographs were taken by the powder method, with a hot filament X-ray tube and camera modelled on those of Shearer. The wave-lengths used were those of copper K_α and K_β unfiltered. The watery nature of many of the specimens examined necessitated exposures of 2 to 4 hours with a tube current of 10 to 20 milli-amps. A thick layer of the substance has been used, usually about 2 mm., and in consequence the lines obtained are not usually sharp and fine, but measurements on the same plate of the best lines usually agree to about 1 per cent. or 2 per cent.

All the specimens examined show one very wide spacing of the order of 40 A.U. giving well defined lines. Up to five orders of these lines have been observed and measured. There are also two spacings of the order of 4 A.U. giving very black but diffuse lines. The wide spacing is in accord with the 43·5 A.U. observed by de Broglie† for some Oleates.

The principal salts and spacings observed are shown in Table I. The spacings are calculated from the best lines only.

Table I.

Salt.	No. of CH_2 groups.	Wide spacing.	Difference.	Close spacings.	
Na Laurate............	10	33·5 A.U.		4·22	4·88
			5·0 A.U.		
Na Myristate	12	38·5 A.U.		4·18	4·9
			5·0 A.U.		
Na Palmitate	14	43·5 A.U.		4·15	4·9

It will be observed that there is a regular increase in the wide spacing as the number of CH_2 groups in the molecule increases, and that the addition of two CH_2 groups to the molecule gives a constant difference of 5 A.U.

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* Darke, McBain and Salmon, p. 395, P.R.S. 98 (1921).

† C.R., No. 11, Vol. 176, p. 738, March (1923).

According to Langmuir[*] the length of the Palmitic Acid molecule is 24 A.U. This is of the order of one-half the wide spacing shown by the Sodium Palmitate curd, and suggests that the molecules are distributed in planes to which their axes are perpendicular, and that the molecules in successive planes face in opposite directions. In this case the reflections are probably from parallel planes containing Sodium atoms and spaced at distances equal to twice the length of the molecule. The common difference of 5 A.U. would then be equal to the effective length of four CH_2 groups.

The diffuse nature of the lines given by the closer spacings does not allow of an accuracy of measurement greater than about 4 per cent. Langmuir (*loc. cit.*) gives measurements of the effective cross-section of the heads of the molecules of two fatty acids in thin films as 21×10^{-16}cm^2 and 22×10^{-16}cm^2. This is assumed to be the area of cross-section of the —COOH group. Our end group only differs in nature from this by the substitution of sodium for hydrogen, and as one of the oxygen atoms is placed obliquely it is probable that —COOH and —COONa have approximately the same cross-section.

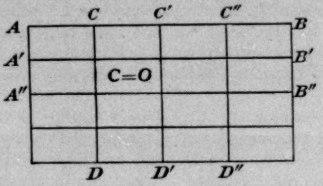
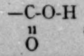


FIG. 1.

The distribution of the —COOH group is usually supposed to be  $\underset{\underset{O}{\|}}{-C\text{-}O\text{-}H}$ and it is quite conceivable that such a group would have an effective width in its plane greater than in the direction at right angles to it. The products of the close spacings given in Table I. is approximately 21×10^{-16}cm^2, suggesting that the lines from which these spacings are calculated are due to planes containing the lengths of the molecules and perpendicular to one another. The small rectangles are the effective cross-sections occupied by the COONa groups in the widely spaced planes. The planes referred to above are shown in section as *AB* for the 4·2 spacing and as *CD* for the 4·9 spacing. The molecules lie perpendicular to the plane of the diagram. This is perhaps made clearer by Fig. 1. This value of 21×10^{-16}cm^2 agrees with that given by N. K. Adams[†] for condensed films of fatty acids.

There are many other faint lines shown on the photographs which it has not yet been possible to analyse.

In addition to the three salts shown in the Table we have measured the lines given by sodium stearate both as a curd and in the dry condition. In this salt there are 16 CH_2 groups, and we should expect a wide spacing of about 48·5 A.U.

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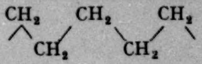
[*] J. Am. Chem. Soc., p. 1865, July-Dec. (1917).

[†] N. K. Adams, P.R.S., A., Vol. 101, p. 452 (1922).

Measurements of four plates give a mean value of 44 A.U., which is nearly that observed for the Palmitate. Langmuir (*loc. cit.*) gives 25, and Adams (*loc. cit.*) 26·2 A.U. as the length of the Stearic Acid molecule. Langmuir also gives 24 A.U. as the length of the Palmitic Acid molecule. There seems little doubt that the Stearic molecule is longer than the Palmitic, and we have not yet succeeded in accounting for our apparently anomalous values. We find that the values of the spacing as measured for the stearate show more variation among themselves than do the measurements on the strong lines of the other salts. There is also in the Stearate an indication of alteration in spacing as the amount of water in the substance changes. This substance is now being more thoroughly examined.

Assuming that these substances are in the smectic condition described by Friedel[*] (as is done by de Broglie[†] for the Oleates he measured), we can account for some of the properties of the smectic bodies. In many cases for bodies in the smectic state the molecules have a common direction and are distributed over parallel equidistant planes (planes of Grandjean) and have their lengths normal to these planes. The distance between the planes is assumed by Friedel to be equal to, or a multiple of, the thickness of the stratified layers of soap films measured by Wells[‡], that is, about 43 A.U. On our theory this minimum thickness is the length of two molecules perpendicular to the plane and so placed that the CH_3 groups are in contact inwards and the sodium atoms on the two free surfaces. The affinity of the $-CH_3$ groups for one another is greater than the affinity of the sodium atoms; the planes containing the sodium are therefore planes of slip. The structure attributed to the $-CH_2-$ chain by Langmuir (*loc. cit.*) is of the type



and if several of these are placed parallel the chains can lock and give a rigidity to the structure in the direction of the length. This suggests fluidity in the direction of the planes and rigidity at right angles, as mentioned by Friedel (*loc. cit.* p. 302).

That this is the true nature of the structure of the chain also receives support from the following considerations.

Assuming the angle between the lines joining the centres of successive Carbon atoms in the zig-zag chain is 109°28′ as in the diamond, and taking 1·54 A.U. as the size of the carbon atom, the distance between the centres of successive carbon atoms measured along the axis of the molecule will be 1·26 A.U. Four carbon atoms will account for a length of 5·04 A.U. agreeing with our measurement of 5·0 A.U. The hydrogen atoms are, of course, fixed to the sides of the chain. In addition to this, one plate taken with a rather high potential on the tube, shows clearly two absorption edges, which, if due to the silver in the film, correspond to spacings of 1·27 A.U. and 0·94 A.U. These correspond fairly well to the planes marked *AB* and *CD* in Fig. 2 below.

The diffuse nature of the reflections given by the 4·2 A.U. and 4·9 A.U. spacings can also be explained from considerations of the smectic state. The planes containing the lengths of the molecules are not crystalline planes, in fact the whole structure is looser than that of a crystal. One can therefore reasonably expect a much greater tolerance in the width of the spacing of these planes than in those of a true crystal.

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[*] G. Friedel, Annales de Physique, Nov. (1922).

[†] M. de Broglie, *loc. cit.*

[‡] Wells, Annales de Physique, Sept., p. 98 (1921).

Microscopic examination of the curds of sodium myristate shows the conical structure described by Friedel for the smectic body. The other substances have not yet been obtained in a state that permits of a satisfactory examination with the microscope.

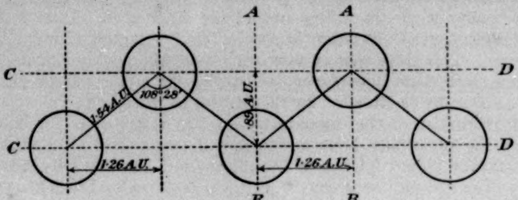

FIG. 2.

We are indebted to Professor Tyndall and Professor McBain for suggestions, and to the latter for the specimens. The experiments were carried out in the Physics Department of the University of Bristol with the aid of a grant from the Colston Research Society.

DEMONSTRATION on Intermittent Pressure with Boiling Water. By CHAS. R. DARLING, *F.Inst.P., F.I.C.*

IF a glass tube, open at both ends, and of about 5 mm. bore, be placed in a beaker of briskly boiling water so as to rest on the bottom of the beaker, steam bubbles will be observed to form at the point of contact, causing the water to rise to a definite height in the tube. The column of water thus raised sinks after a time, and then rises again, the rising and falling occurring at irregular intervals. If, however, the tube be narrowed to a bore of about 1 mm. near the top of the water, and widened out considerably just above the water surface, it will then be seen that the phenomenon becomes regular in action. After rising to the height of about 3 cm. in the widened part, the water discharges back again into the beaker, and after a short interval again rises and is discharged, the cycle being repeated indefinitely.

The explanation appears to be that the water is superheated at the points of contact of the tube and beaker, so that the steam produced can sustain a higher pressure of water. When the water reaches the widened part, however, it is cooled and increases in density until the extra steam pressure at the bottom of the tube is overcome, when it discharges completely. The capillary bore slows down the rate of flow in both directions, and so causes the movements to be steady. A separating funnel with open tap and short stem is well suited to the experiment.

It will be observed that the arrangement constitutes a simple heat engine, with source and sink, automatically passing through a regular cycle of operations, and forms a useful lecture demonstration of the conversion of heat into work.

DEMONSTRATION of A Novel Instrument for Recording Wireless Signals. By N. W. McLachlan, *D.Sc., M.I.E.E.*

The device consists essentially of a drum of Swedish iron with an annular recess in which are situated coils of fine wire, the ends of the coils being connected to corresponding slip rings. The periphery of the drum is faced with cast iron rings which are machined to run true to 0·0001 in. A small steel shoe rides on the rings, and side play is prevented by a brass guide-piece with a projection which fits into the annular recess. At each end of the guide-piece a hook is formed, and one of the hooks is connected by a light rod to a duralumin lever pivoted to turn in an horizontal plane. A silver syphon passes through the lever from an ink well overhead and rests lightly on a moving paper tape. To the other side of the lever is attached a strong spring whose tension can be varied, while the remaining hook is attached to a light spring which prevents the shoe from rocking on the drum.

The drum is revolved by a small electric motor, and when a current flows in one of the coils the shoe is attracted to the drum and a large pull is required to prevent relative motion of the two. This pull is used to actuate the syphon lever mechanism so that the transverse movement over the paper tape inscribes the incoming message as a series of rectangles, the tops of which represent the dots and dashes of the Morse code.

The tangential pull on the shoe is many times that calculated from the formula $\mu B^2 A/8\pi$, where μ=coefficient of friction, B=flux density at shoe contact, and A=area of contact. The ratio of the actual to the calculated pull depends on the flux density, and has a maximum value of about 80 for steel or cast iron.

Owing to this phenomenon, which gives amplification, and to the fact that there is no air-gap in the magnetic circuit through the shoe, the instrument is extremely sensitive, and will work at a speed of 150 words a minute with a current of 25 micro-amperes. It will work in a thermionic valve circuit, and for wireless it is fitted with relay contacts, so that incoming messages in Morse may be relayed to a printing or other machine direct.

THE SEVENTH GUTHRIE LECTURE,

Entitled "The Effect of Electric and Magnetic Fields on Spectral Lines."[*]

Delivered by NIELS BOHR, *University of Copenhagen,*

MARCH 24, 1922.

INTRODUCTION.

IN the characteristic effects on spectral lines observed when radiating substances are exposed to magnetic or electric fields we possess a valuable source of information in regard to the problem of atomic constitution. We have, indeed, in these effects a means of examining in detail the influence of controllable agencies upon intra-atomic processes. This fact has been generally realised by physicists ever since the fundamental discovery of Zeeman[†] 25 years ago of the characteristic effect of magnetic fields on spectral lines; and the problem was brought still more within the sphere of interest by the discovery of Stark,[‡] about 10 years ago, of the analogous effect of electric fields. Owing to the development of our ideas of electro-magnetic radiation in the interval, however, the way of approach along which explanations of these effects have been sought in connection with theories of atomic constitution has undergone fundamental change. It is the main object of this lecture to expose as clearly as possible the principal features of this development.

I. THE ZEEMAN AND STARK EFFECTS AND THE CLASSICAL THEORY OF RADIATION.

According to the classical theory of electrodynamics, the constitution of the radiation emitted by a system of electrified particles should depend directly on the motion of the particles. In fact, it follows from this theory that every constituent harmonic oscillation in the electric moment of the system must give rise to the emission of a train of waves of frequency coinciding with the frequency of oscillation, and of an intensity depending on the amplitude. Notwithstanding the difficulties of accounting in a simple way for the remarkable empirical laws which govern the frequencies of the spectra of the elements, the characteristic features of Zeeman's discovery obtained, as was shown by Lorentz, an immediate interpretation on this basis. Lorentz[§] assumed that each line in the spectrum of the undisturbed atom originates from the motion of an electrified particle performing a harmonic oscillation around a position of stable equilibrium within the atom, under the influence of an attraction directed towards this position and proportional to the first power of the displacement. The component of this displacement in a given direction in space may be expressed by the formula

$$\xi = C \cos 2\pi(\omega_0 t + \gamma) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where the frequency ω_0 is independent of the amplitude of oscillation C.

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[*] **In substance this report represents the contents of the Seventh Guthrie Lecture delivered before the Physical Society, March 24, 1922. Due to unavoidable circumstances the publication of this report has unfortunately been delayed until now. N.B. July, 1923.**

[†] **P. Zeeman, Phil. Mag., 5, Vol. 43, p. 226 (1897).** ***See*** **also Zeeman's collected Papers on magneto-optical phenomena, Leiden (1921).**

[‡] **J. Stark, Berliner Sitzungsber, Nov. (1913).** ***See*** **also Elektrische Spektral-analyse, Leipzig (1914).**

[§] ***See*** **H. A. Lorentz, The Theory of Electrons, Ch. 3, Leipzig (1909).**

Analysing the change in the motion of the particle caused by the presence of an external magnetic field, Lorentz showed that, if the problem is treated on the basis of the ordinary theory of electrodynamics, the original purely harmonic motion is changed in such a way that it can be considered as composed of three constituent harmonic components. One of these is a linear harmonic oscillation parallel to the magnetic field, with a frequency coinciding with that of the undisturbed motion. The two others are circular harmonic rotations in opposite directions in a plane perpendicular to the field, possessing frequencies given by

$$\omega = \omega_0 \pm \omega_H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where the double sign refers to the two opposite rotations. The expression for ω_H is

$$\omega_H = \frac{eH}{4\pi cm}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where H is the intensity of the magnetic field, e and m respectively the charge and mass of the oscillating particle, and c the velocity of light.

This result proved to be in most suggestive agreement with Zeeman's measurements. In fact, in a number of cases spectral lines were observed to be split up into three components, one of which was linearly polarised and appeared in the position of the original line, while the two others were placed symmetrically with regard to the original line and showed circular polarisation in opposite directions. Further, the ratio between charge and mass of the oscillating particles, calculated by means of the Lorentz expression from the displacements of these components, was found to be in close agreement with the value obtained from the deflection of cathode rays in electric and magnetic fields; and the sense of the polarisation observed for the outer components showed that the charge of these particles, like that of the cathode ray particles, possessed negative sign. This result was generally accepted as a most convincing proof of the electronic theory of matter, and it may certainly be said to have established beyond doubt the conclusion that the origin of these spectra is to be found in the motion of electrons within the atom.

For the general discussion of the Zeeman effect an electrodynamic theorem, first established by Larmor,* is of great importance. According to this theorem the motion of a system of electrons moving in a central field of force will, in the presence of a uniform magnetic field, undergo a change such that the motion of the system, to a first approximation, may be described as a possible motion of the electrons without field, on which is superposed a uniform rotation of the whole system around an axis parallel to the direction of the field, with frequency equal to ω_H, as given by formula (3).

This theorem will be seen to include Lorentz's results, since the effect of a superposed uniform rotation on a simple harmonic oscillation is to give rise to a motion of just the type described above. In fact, any elliptic harmonic oscillation may be resolved into a linear vibration in a given direction and an elliptic oscillation in a plane perpendicular to this direction; and the latter may again be considered as composed of two circular rotations with the same frequency, but opposite directions of revolution. Now a superposed rotation round a given axis will, of course, not influence a linear vibration in the direction of the axis ; it will, as regards a circular rotation in a perpendicular plane, simply augment or diminish the frequency by an amount equal to the frequency of the superposed rotation, according as the direction of this rotation is the same as or opposite to that of the original rotation.

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* J. Larmor, Aether and Matter, p. 341, Cambridge (1900).

However, although a number of spectral lines show Zeeman effects in conformity with the predictions of the theories of Lorentz and Larmor, the lines of many spectra show the so-called " anomalous " Zeeman effects. In such cases the lines are still resolved into components linearly polarised parallel to and components circularly polarised perpendicular to the field, which are arranged symmetrically with respect to the original line, and whose displacements are, at any rate for small intensities, proportional to the field. The number of components and the magnitude of their displacements may, however, differ considerably from what is the case in the " normal " effect. This variation in the character of the Zeeman effect has been found to be intimately connected with the structure of the spectra and the manner in which the lines of these spectra may be arranged in "series." Indeed, according to the rule announced by Preston,* Zeeman effects of the same type are shown not only by the lines of the same spectral series of one element, but also in respect of lines belonging to corresponding series in spectra of different elements exhibiting an analogous structure. Many efforts have been made to explain the appearance of the anomalous Zeeman effect on the basis of the classical theory of radiation. Amongst others may be mentioned the remarkable work of Voigt,† who succeeded in developing formal interpretations of a number of details of the observed phenomena. Nevertheless, great difficulties remained in the attempt to reconcile the anomalous effect with that theory; but it is hardly necessary to enter more closely into this problem here, inasmuch as new difficulties of a fundamental character arise when we try to explain the characteristic effect of electric fields on spectral lines on the basis of the classical theory, even in the case of spectra such as that of hydrogen, where the Zeeman effect is of the normal type.

As is well known, Stark discovered in 1913 that the lines of the hydrogen spectrum showed a resolution into a number of polarised components when radiating hydrogen atoms were exposed to a strong electric field. Regarded from the point of view of the classical theory of the origin of spectra, Stark's results were very surprising. Thus, if an electron performing oscillations round a position of stable equilibrium—as assumed in Lorentz's theory of the Zeeman effect—is exposed to the effect of a uniform electric field, the type of motion should not be changed at all, and the whole effect of the field should consist only in a displacement of the centre of the orbit by an amount proportional to the intensity of the field. Any possible effect on the spectral lines must therefore, on the classical theory, be due to deviations from a central attraction proportional to the displacement exhibited by the forces keeping the electron in its position in the atom. An effect of this kind—which had been considered by Voigt‡ several years before Stark's discovery—should clearly be proportional to the square or higher powers of the intensity of the external fields. In contrast to this expectation an essential feature in Stark's results was that the effect of the electric field on the hydrogen lines was, to a close approximation, directly

* Th. Preston, Nature, 59, p. 224 (1899).

† C . W. Voigt, Magneto-und Elektrooptik, Leipzig (1908).

‡ W. Voigt, Ann. d. Phys., 43, p. 410 (1991).

proportional to the field. Viewed as a whole the Stark effect, indeed, constitutes a most complex phenomenon which not only differs greatly for different spectra and different series of lines, but which even changes essentially from line to line within one and the same spectral series.

As is well known, the difficulties here alluded to are only a few examples of the breakdown of the ideas of the classical electrodynamics when applied to atomic phenomena. Moreover, as a result of the fundamental researches of Rutherford on the phenomena of radioactivity, we may consider it as proved that the atom consists of a positively charged central nucleus surrounded by a distribution of electrons. On the theory of classical electrodynamics it is clear that such a picture of the atom does not allow of static configurations of stable equilibrium, and that we are forced to assume that the electrons within the atom rotate with high velocities. This gives rise to new difficulties, however, since on the classical theory such motions should be accompanied by a continuous emission of electromagnetic radiation, which would go on until so much energy was radiated that the electrons would fall into the nucleus. It is unnecessary to argue at length the impossibility of explaining the stability of atoms and the emission of spectra consisting of sharp lines on this atomic model, if we confine our considerations to the classical ideas of electrodynamics.

II. QUANTUM-THEORY OF SPECTRA.

A short time before Stark's discovery I ventured to propose a theory of spectra which constitutes a definite break with classical electrodynamics.* This theory is based on considerations of atomic stability and emission of radiation arising from conceptions of the so-called quantum-theory, which was originated by Planck's famous theory of temperature-radiation put forward about 20 years ago, and to the development of which Einstein made fundamental contributions in its earlier stages. The application of the quantum theory to atomic problems rests upon the two following postulates :—

1. Among the conceivably possible states of motion in an atomic system there exist a number of so-called *stationary states* which, in spite of the fact that the motion of the particles in these states obeys the laws of classical mechanics to a considerable extent, possess a peculiar, mechanically inexplicable stability, of such sort that every permanent change in the motion of the system must consist in a complete transition from one stationary state to another.

2. While, in contradistinction to the classical electromagnetic theory, no radiation takes place from the atom in the stationary states themselves, a transition between two stationary states can be accompanied by the emission of electromagnetic radiation, which will have the same properties as that which would be sent out according to the classical theory from an electrified particle executing an harmonic vibration with constant frequency. This frequency ν has, however, no simple relation to the motion of the particles of the atom, but is given by the relation

$$h\nu = E' - E'' \quad \ldots \ldots \ldots \ldots \quad (4)$$

where h is Planck's constant, and E' and E'' are the values of the energy of the atom in the two stationary states which form the initial and final states of the radiation process. Conversely, irradiation of the atom with electromagnetic waves of this frequency can lead to an absorption process, whereby the atom is transferred back from the latter stationary state to the former.

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* N. Bohr, Phil. Mag., 26, pp. 1, 476, 857 (1913).

I shall not enter here on the philosophical problem of the possibility of reaching a satisfactory description of nature by application of such formal postulates, but shall endeavour to show that they allow us to construct a theory which gives a simple and consistent interpretation of spectroscopic phenomena, for the explanation of which the classical ideas of electrodynamics have not proved themselves directly suited.

As the first application we shall consider the so-called "principle of combination of spectral lines," which was brought to light by the researches of Balmer,[*] Rydberg,[†] and Ritz[‡] on series spectra, and which in recent years has proved to be of general validity in regard to spectra of very different types. According to this principle the frequency of each of the lines of a spectrum may be represented by the formula

$$\nu=T_2-T_1, \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (5)$$

where T_1 and T_2 are two among a manifold of so-called spectral terms. This law, which holds with an accuracy unrivalled in physics, has hitherto resisted any interpretation on the classical ideas, at any rate in a form suitable as a foundation for a detailed discussion of spectroscopic evidence. On our postulates, on the other hand, it is seen that the combination principle can be directly interpreted by identifying the spectral terms with the numerical values of the energy in the possible stationary states, divided by Planck's constant, and by supposing that each spectral line originates from a transition between two of these states.

At first sight this interpretation of the combination principle might be considered as being of a very formal character, since it is not only admittedly at variance with the ideas of classical electrodynamics, but also involves a radical departure from the conceptions on which physical phenomena have hitherto been based. This appears particularly in the assumption that the constitution of the radiation emitted during a process at the outset of which an atom finds itself in a certain stationary state, depends not only on this state but also on that state of the atom which appears as the result of the process. In fact, spectral lines which appear as combinations of various spectral terms with one and the same term are attributed to various possible processes of transition from a certain state of the atom to other stationary states. In the present state of the theory the mode of occurrence of these transitions is considered to be a question of probability, in the sense that an atom in a given stationary state is assumed to possess a certain probability of passing spontaneously in unit time to any of the other stationary states under consideration. This view, which exhibits a marked analogy to the theory of radioactive disintegration, conforms with the assumptions used by Einstein[§] in his suggestive deduction of the law of temperature-radiation on the basis of the fundamental postulates stated above.

Notwithstanding the fundamental nature of the departure from the classical electrodynamics which is involved in the quantum theory of spectra, we shall see that it seems possible in a certain sense to regard this theory as a natural generalisation of our ordinary ideas of radiation. Thus it is possible to correlate each of the various processes of transition giving rise to the emission of harmonic trains of waves with one of the various harmonic oscillations which occur in the electric moment of the atom, in such a way that the probability of the occurrence of a transition of a given type is to be ascribed to the presence of a corresponding harmonic oscillation in this moment. This feature of the quantum-theory of spectra, which is termed the "correspondence principle," plays an essential part in the interpretation of the spectral evidence. It may especially be emphasised that it has been possible, by means of this principle, to remove the mystery which has hitherto hung over the application of the combination principle owing to the apparent capriciousness which attends the appearance of the predicted spectral lines.

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[*] Balmer, Ann. d. Physik, 25, p. 80 (1885).

[†] J. R. Rydberg, Handl. Akad., Stockholm, 23 (1890).

[‡] W. Ritz, Physik. Zeit., 9, p. 521 (1908).

[§] A. Einstein, Phys. Zeit., 18, p. 121 (1917).

We shall also see how the correspondence principle has been of great use in developing explanations of the effects of magnetic and electric fields on spectral lines. Before entering into the detailed discussion of these problems, it is necessary to consider briefly the application of the postulates to the simple case of the interpretation of the hydrogen spectrum—which was the starting point of the theory—as well as the main features of the subsequent development of the general principles of the quantum-theory.

III. THEORY OF THE HYDROGEN SPECTRUM.

As is well known, the frequencies of the lines of the hydrogen spectrum may be represented to a high degree of approximation by the simple formula

$$\nu=K\left(\frac{1}{(n'')^2}-\frac{1}{(n')^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For $n''=1$ and $n'=2, 3, 4$ we get a series of lines in the extreme ultra-violet, first observed a few years ago by Lyman[*]; for $n''=2$ and $n'=3, 4, 5$ . . . the formula represents the Balmer series in the visible region; $n''=3$ and $n'=4, 5, 6$ . . . . gives the infra-red series, of which the first two members were observed by Paschen[†] several years ago; finally, $n''=4$ and $n'=5, 6$ . . . . correspond with a series quite recently observed in the far infra-red by Brackett.[‡]

Comparing (6) with formulæ (4) and (5), we may conclude, according to the general considerations of the former section, that the hydrogen spectrum is emitted by an atom which possesses a sequence of stationary states, such that the numerical value of the energy in the nth state is given by $\frac{Kh}{n^2}$. Now, according to Rutherford's theory, the hydrogen atom consists of one electron rotating round a positive nucleus of unit charge. Apart from the small effect due to the change of mass of the electron with velocity, the motion of the atom will be a very simple one, the particles describing simple periodic, elliptic orbits with the centre of gravity at the common focus. For such a motion the frequency of revolution of the particles and the dimensions of the orbits will, according to the well-known Keplerian laws, be related to the values of the total energy of the system by the simple formulæ

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[*] T. Lyman, Phys. Rev., 3, p. 504 (1914).

[†] F. Paschen, Ann. d. Phys., 27, p. 565 (1908).

[‡] F. Brackett, Nature, 109, p. 209 (1922).

$$\omega=\sqrt{\frac{2W^3}{\pi^2e^4m}}\,,\quad 2a=\frac{e^2}{W},\quad\ldots\ldots\ldots\ldots \tag{7}$$

where ω is the frequency of revolution and $2a$ the major axis of the orbit of the electron, while W is the work necessary to remove the particles to an infinite distance from each other. As before, e and m denote the charge and mass of the electron, while for brevity we have considered the mass of the nucleus as infinitely large. Putting now for the stationary states

$$W=\frac{Kh}{n^2},\quad\ldots\ldots\ldots\ldots \tag{8}$$

we obtain for the frequency of revolution and the major axis in these states

$$\omega=\frac{1}{n^3}\sqrt{\frac{2Kh^3}{\pi^2e^4m}}\,,\quad 2a=n^2\frac{e^2}{Kh}.\quad\ldots\ldots\ldots\ldots \tag{9}$$

These formulæ give a picture of the formation of the atom by means of a step-wise process in which an electron is bound with emission of radiation in a sequence of stationary orbits of rapidly increasing frequencies and decreasing dimensions, until a state is reached in which the energy of the atom is a minimum and the process of binding is brought to a final stop; this state corresponds to $n=1$ in formula (9). Introducing the empirical values for K as well as e, m and h in (9) we get values for the frequency of revolution and the major axis of the orbit in this "normal" state of the atom, which are of the same order as the values for atomic frequencies and dimensions derived from consideration of the optical and mechanical properties of gases.

On our fundamental postulates there is no question, however, of a closer direct comparison between such formulæ as (9) and formulæ derived on the classical theory of electrodynamics. In particular, we can have no direct comparison between frequencies in the stationary states and frequencies of the spectral lines, as we have assumed that each of these lines corresponds to the radiation emitted during a transition between two states, for which the frequencies of revolution may in general have quite different values. An opportunity for tracing a connection between the spectrum and the motion is offered, however, by the circumstance that the ratio between the frequencies of revolution in two successive stationary states approaches unity when the values of n gradually increase. From (6) we obtain (to a first approximation) for the frequency of the radiation emitted by a transition between two successive states corresponding to large values of n

$$\nu\doteq\frac{2K}{n^3}.$$

Comparing with (9), we find that this expression will coincide asymptotically with the frequency of revolution in the two states if

$$K=\frac{2\pi^2e^4m}{h^3}.\quad\ldots\ldots\ldots\ldots \tag{10}$$

As I have shown in the Paper referred to, this condition is actually fulfilled within the limits of experimental error, if we introduce the empirical values for K as wel' as h, e and m.*

The connection thus established between the hydrogen spectrum and the quantities describing the model of the hydrogen atom is evidently as close as could be expected, considering the fundamental departure from classical electrodynamics involved in our interpretation of the spectrum. This divergence becomes clear, as soon as we seek an explanation of the effects of magnetic and electric fields on the hydrogen lines; here we meet a problem quite different from that with which we are presented when regarding the matter from the point of view of the classical theory of radiation. A detailed explanation of the Stark effect could not be attained by means of such simple considerations as sufficed for the interpretation of the main features of the hydrogen spectrum; it was found possible nevertheless not only to explain the direct proportionality of the displacement of the components to the intensity of the electric force, but also to account for the absolute magnitude of the effect and the characteristic way in which it varies from line to line in the spectrum.† A closer explanation of the details of the Stark and Zeeman effects, however, demanded the development of methods for the fixation of the stationary states of the atom under the influence of external fields, as well as the formulation of further rules governing the transitions between stationary states and the polarisation of the ensuing radiation. It is of interest to point out that in the case of the Zeeman effect, for the main features of which the classical theory had offered so simple an interpretation, doubt prevailed for a time as to whether this effect could be accounted for at all on the basis of the postulates in the form stated above. In fact it is easily seen that the frequencies of the components into which the lines are split up in the field cannot be represented by a complete combination-diagram of spectral terms. As we shall presently see, a detailed theory of the Stark effect as well as of the Zeeman effect for the hydrogen lines has been established in course of the development of the quantum-theory in recent years.

IV. STATE RELATIONS FOR PERIODIC AND MULTIPLE-PERIODIC MOTIONS.

The formal basis of the application of the quantum-theory to atomic problems consists of a number of formulæ which, together with the formula (4), which has often been termed the "frequency relation," allow us to select the stationary states from among the mechanically possible motions of the particles of the atom. These latter formulæ, the so-called "state-relations," may be considered as rational generalisations of the assumption originally used by Planck regarding the possible values of the energy of a system consisting of a particle executing simple harmonic oscillations. Although it is naturally an essential object of the theory to determine the energy of the stationary states, the energy function itself is not, for more complicated systems, well suited to a general formulation of state-relations. A suitable basis for this formulation is found, however, in the so-called integral of action, which plays such an important part in analytical dynamics.

Let us first consider the simple case when the motion of the particles of the atom is simply periodic, independently of the initial conditions. In this case the displacement ξ of each particle in a given direction may be expressed as a function of the time, in the well-known way, as the superposition of a number of harmonic oscillations

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* *Cf.* also R. A. Millikan, Phil. Mag., 34, p. 1 (1917).

† N. Bohr, Phil. Mag., 27, p. 506 (1914); 30, p. 394 (1915). *Cf.* also E. Warburg, Verh. Deut. Phys. Ges., 15, p. 1259 (1913).

$$\xi = \Sigma C_\tau \cos 2\pi(\tau\omega t + \gamma_\tau) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where ω is the frequency of the periodic motion, and the summation is over all positive integral values of τ. A similar expression holds also, of course, for the component in a given direction of the electric moment of the atom, whose variation with time determines on the classical theory the constitution of the emitted radiation. For such simply periodic systems the stationary states are fixed by a single condition which may be written

$$I = nh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where h is Planck's constant, and n a positive integer, the so-called *quantum-number*. The quantity I is defined by

$$I = \int_0 A\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where the integral is the so-called integral of action taken over a complete period of the motion. If the motion is assumed to be governed by the laws of Newtonian mechanics, the integrand A is equal to twice the kinetic energy of the moving particles ($A = \Sigma mv^2$); while, if the modifications demanded by the theory of relativity are taken into account, A is given by the expression

$$A = \Sigma mv^2 \left(1 - \frac{v^2}{C^2}\right)^{-\frac{1}{2}}$$

For the sake of the later discussion it may be noted that this definition of I is identical with the condition

$$I\omega = \bar{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where $\bar{A}$ denotes the mean value of the function A during the motion.

While the relation between I and the total energy ε for different systems may take very different forms, this relation will always obey the simple differential equation

$$\delta E = \omega \delta I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where δE and δI denote the difference in E and I for two mechanically possible motions of the system which differ but little from each other.

From (4) and (15) it is seen at once that, in the case of a simple harmonic with oscillator constant frequency ω_0 the motion of which is represented by (1), the equation (12) is equivalent to Planck's well-known relation

$$E = nh\omega_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

We can also easily show that in the case of the hydrogen atom (12) is equivalent to the formula (8), if in this formula the value of K given by (10) is introduced. In fact, noticing that $W = \infty$ for $I = 0$, we get from (7) and (15) by simple integration

$$W = \frac{2\pi^2 e^4 m}{I^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The general relations (12) and (15) allow us, further, to trace on a broader basis the formal asymptotic connection between the motion of an atomic system and the spectrum deduced on the quantum-theory, established in the former section in the special case of the hydrogen atom.

Let us consider a transition between two states for which the values of n in formula (12) are equal to n' and n'' respectively. From the frequency relation (4) we get by means of (15)

$$\nu=\frac{1}{h}(E'-E'')=\frac{1}{h}\int_{n=n''}^{n=n'}\omega\delta I \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

If now the numbers n' and n'' are large compared with their difference and consequently the motions in the two states differ comparatively little from each other in frequency and dimensions, we may in (18) consider ω as approximately constant and thus obtain, using (12) also, the asymptotical relation

$$\nu\sim(n'-n'')\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

In the limit, for large quantum numbers, the frequency of the radiation emitted during a transition will consequently coincide asymptotically with the frequency of one of the harmonic constituents of the radiation which, on the classical theory, would be emitted from the atom owing to variation of the electric moment with time; namely, with the frequency of the trains of waves which would result from the harmonic component in the expression (11) for which $\tau=n'-n''$. Now of course there is no question of a gradual approximation of the quantum-theory in the limit of large quantum-numbers to the classical ideas of the origin of radiation. Indeed, just as in the case where these numbers are not large compared with their difference, we assume in this limit that the various harmonic components of the radiation, which would be emitted simultaneously on the classical theory, will originate in quite different processes of transition between different pairs of stationary states. It is exactly this circumstance, however, which leads us to consider the coincidence of frequencies traced in the limit as evidence of a general law which underlies the occurrence of transitions between stationary states.

This law, which has been called the "correspondence principle," states that the occurrence of each transition between two stationary states accompanied by emission of radiation is correlated to one of the constituent harmonic oscillations into which the electric moment of the atom considered as a function of the time can be resolved, to the extent that the occurrence of the transition is conditioned by the presence of the "corresponding" harmonic oscillation. This correlation demands that the probability of the occurrence of a transition shall depend on the amplitude of the corresponding harmonic oscillation of the atom, in such a way that in the limit when the quantum-number is large, the intensity of the emitted radiation in unit time in the mean shall be the same as that which would follow from the classical laws of electrodynamics. A similar connection with the classical theory will be exhibited by the polarisation of the emitted radiation. If, for instance, the corresponding harmonic oscillation in all states of the atom is a linear vibration or a circular rotation, the radiation will have the same constitution as that which on the classical theory would be emitted by an electron executing a harmonic motion of that type.

In the case above considered of a periodic system, the correspondence principle states that the appearance of a transition between two stationary states, in which the quantum-number changes from n' to n'', is conditioned by the presence in the electric moment of the atom of an $(n'-n'')$th harmonic. This result allows us to throw light on a marked difference between the rules governing the occurrence of transitions between stationary states in the case of a Planck oscillator, on the one hand, and of the hydrogen atom on the other. In Planck's theory of temperature-radiation it is an essential assumption that, as in the classical theory, the frequency of the radiation emitted or absorbed by an oscillator shall be always equal to the characteristic frequency of oscillation. In terms of our theory of spectra this means, as seen from the comparison of (4) and (16), that the oscillator can, in a single step, only pass between two stationary states for which the quantum-number n differs by one unit. On the other hand the interpretation of the hydrogen spectrum necessarily requires that transitions between any pairs of the stationary states of the hydrogen atom shall be possible. On the correspondence principle this remarkable difference is directly accounted for by the fact that the Keplerian elliptic motion of the electron in the hydrogen atom, in contrast to the purely harmonic motion of the Planck oscillator, possesses a complete sequence of upper harmonics.

Owing to the recent development of the quantum-theory it has been possible to establish state-relations not only for simple-periodic systems, but also for systems in which the motion is of the so-called multiple-periodic type. For such systems the displacement of each particle, as well as the variation of the electric moment, may still be represented as a superposition of harmonic oscillations. In contrast to a simple-periodic system the frequencies of these oscillations are not multiples of a single fundamental frequency, but are for a multiple-periodic system of a "degree of periodicity" equal to s, built up as linear expressions of s independent fundamental frequencies in the way shown by the following formula—

$$\xi=\Sigma C_{\tau_1,\ldots\tau_s}\cos 2\pi\left[(\tau_1\omega_1+\ldots+\tau_s\omega_s)t+\gamma_{\tau_1,\ldots\tau_s}\right]\quad .\quad .\quad (20)$$

where ω_1 to ω_s are the fundamental frequencies, and the summation is to be extended over all negative and positive values of the integers τ_1 to τ_s.

In such a case the stationary states will be fixed by the s state relations:

$$I_1=n_1h,\quad\ldots\quad I_s=n_sh,\quad .\ .\ .\ .\ .\ .\ .\ .\ .\ .\quad (21)$$

where n_1 to n_s are positive integers. The I's are a number of quantities expressing certain properties of the motion, which by analogy with the definition of the quantity I for a periodic system are related to the energy and fundamental frequencies of the motions through the differential equation

$$\delta E=\omega_1\delta I_1+\ldots+\omega_s\delta I_s,\quad .\ .\ .\ .\ .\ .\ .\ .\quad (22)$$

expressing the energy difference for two mechanically possible motions of the system which differ very little from each other. These relations fix the quantities $I_1\ldots I_s$ save for an arbitrary constant in each, which, however, is fixed by the condition

$$I_1\omega_1+\ldots+I_s\omega_s=\overline{A},\quad .\ .\ .\ .\ .\ .\ .\ .\ .\ ;\quad (23)$$

where $\overline{A}$ as in formula (14) is the mean value of the function A which appears in the integral of action.

From formula (22) we find that, for the radiation emitted during the transition of the system between two states for which the quantum-numbers in the relations (21) are given by $n'_1 \ldots n'_s$ and $n''_1 \ldots n''_s$ respectively, the frequency, in the limit where these numbers are large compared with their differences, and where the motions in the two stationary states differ comparatively little from each other, will be given by the asymptotical relation

$$\nu \doteq (n'_1 - n''_1)\omega_1 + \ldots + (n'_s - n''_s)\omega_s. \quad . \quad . \quad . \quad . \qquad (24)$$

According to the correspondence principle we shall consequently assume that a transition between two stationary states of a multiple-periodic system will be dependent on the presence in the expression for the electric moment of the system of a constituent harmonic oscillation for which in (20) we have

$$\tau_1 = n'_1 - n''_1, \ \ldots \ \tau_s = n'_s - n''_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

The establishment of the state-relations for periodic and multiple-periodic systems depends upon the work of a great number of physicists, including Planck himself. It may be of interest to note that a general condition equivalent to (12) was used for the first time by Debye,[*] and that conditions of a similar type to those in (21) were proposed simultaneously by Wilson[†] and Sommerfeld.[‡]

Among the contributions to the later development of the theory we may mention the work of Ehrenfest[§] on the adiabatically invariant character of the state-relations. He considers the action on the motion in the stationary states which results from a slow and uniform transformation of the field of force in which the particles of the system are moving, and points out that if the stationary states are fixed by conditions of the type (21) and (22), the effect of this transformation can be described by means of the ordinary laws of mechanics. This so-called " adiabatic principle " constitutes a natural generalisation of the application of mechanics to the description of the motion in the stationary states themselves, which is obviously not at variance with the non-mechanical character of the stability of these states. These problems are discussed in detail in a treatise by the lecturer published a few years ago, in which the correspondence principle was also developed.[||]

In the application of the theory of multiple-periodic systems to spectral problems the first essential progress was made by Sommerfeld in his interpretation of the fine-structure of the hydrogen lines as revealed when these lines are observed by instruments of high dispersive power, and which is due to the fact that the motion of the electron in the hydrogen atom is no longer strictly periodic when the change of mass of the electron with the velocity is taken into consideration. This work was closely followed by the interpretation of the details of the Stark effect of the hydrogen lines carried out simultaneously by Epstein[¶] and Schwarzschild,[**] and

[*] P. Debye, Wolfskehl Vortrag Göttingen (1913).

[†] W. Wilson, Phil. Mag., 29, 795 (1915); 31, p. 156 (1916).

[‡] A. Sommerfeld, Sitz. der Münchener Akod., p. 425 and 459 (1915).

[§] P. Ehrenfest, Proc. Acad. Amsterdam, 16, 591 (1914); Phil. Mag., 33, 500 (1914); *cf.* also J. M. Burgers, Phil. Mag., 33, 514 (1917).

[||] N. Bohr, On the Quantum-Theory of Line Spectra. D. Kgl. Danske Videnskabernes Selskabs Skrifter, 8, iv., 1 (1918) (quoted hereafter as Q.L.S.). For a brief survey of the present state of the theory compare also N. Bohr, Ann. d. Phys., 71, p. 277 (1923). An English translation of this Paper will appear shortly in Proc. Camb. Phil. Soc.

[¶] P. Epstein, Phys. Zs., 17, p. 148 (1916); Ann. d. Phys., 50, p. 489 (1916).

[**] K. Schwarzschild, Berliner Sitzungsber, April (1916).

by the work of Sommerfeld* and Debye† on the interpretation of the Zeeman effect for the hydrogen lines. The theories of these effects were completed by the application of the correspondence principle, which allows us to account in detail for the limited number of components observed, as well as for their polarisation and intensities.

The method of representation of the state-relations used by these authors was based on the procedure called "separation of variables" in the integral of action. Quite apart from the more limited applicability of this procedure, the method of representing the state-relations followed here, in which the properties of periodicity of the motion are brought into the foreground, gives us in many cases a more direct insight into the physical meaning of the theoretical considerations. In the succeeding discussions of the application of the general theory we shall therefore not follow the historical order of development, but shall treat the problems in the manner which seems best suited to illustrate the main features of the theory.

V. THE EFFECT OF MAGNETIC AND ELECTRIC FIELDS ON THE HYDROGEN LINES.

On the basis of the general considerations in the preceding section we shall now consider in detail the effect of a magnetic and of an electric field on the spectral lines of hydrogen. For this purpose we shall for simplicity neglect the fine-structure of these lines; this is legitimate since the influence on the motion of the electron of variation of mass is very small compared with the effect of external magnetic and electric forces of the intensities used in experiments on the Zeeman and Stark effects. This is clearly shown by the fact that the distance between the fine-structure components of the undisturbed hydrogen lines is much smaller than the displacement of the components into which the lines are resolved in these experiments.

As in Section III., we shall therefore assume the orbit of the electron in the undisturbed atom to be a simple-periodic Keplerian ellipse, for which the frequency of revolution and the major axis are related to the energy as given by (7). Introducing the quantity I defined by (13) we get from (17)

$$E=-W=-\frac{2\pi^2e^4m}{I^2},\quad \omega=\frac{4\pi^2e^4m}{I^3},\quad 2a=\frac{I^2}{2\pi^2e^2m}\quad \ldots\ldots \quad (26)$$

For the stationary states we obtain, therefore, introducing $I=nh$, according to the state-relation (12),

$$E_n=-\frac{1}{n^2}\,\frac{2\pi^2e^4m}{h^2},\quad \omega_n=\frac{1}{n^3}\,\frac{4\pi^2e^4m}{h^3},\quad 2a_n=n^2\,.\,\frac{h^2}{2\pi^2e^2m}\quad \ldots \quad (27)$$

which are, of course, equivalent to the formulæ (8) and (9) when the value for K is as given by (10). Finally, by the relation (4) we obtain for the frequency of the radiation emitted during a transition between two states for which n is equal to n' and n'' respectively,

$$\nu=\frac{2\pi^2e^4m}{h^3}\left(\frac{1}{(n'')^2}-\frac{1}{(n')^2}\right)\,.\quad \ldots\ldots\ldots \quad (28)$$

Effect of a Magnetic Field.

In considering the effect of a magnetic field we shall in the first place assume, according to Larmor's theorem, that the motion of the electron in the hydrogen atom in the presence of the field differs from a possible motion of the atom without field, in having superposed a uniform rotation around an axis through the nucleus and parallel to the field, with a frequency ω_H given by formula (3). As a consequence of this the displacement of the electron in a given direction is no more represented by an expression of the type (11) holding for a purely periodic orbit, but its motion will contain harmonic components of three different types. One type of component will consist of linear vibrations parallel to the field with frequencies $\tau\omega$, where ω is the frequency of revolution in the periodic orbit which the electron describes in a system of reference which partakes of the superposed rotation impressed by the field. The two other types of harmonic components will be circular rotations in a plane perpendicular to the field with frequencies $\tau\omega+\omega_H$ and $\tau\omega-\omega_H$ respectively; the sense of rotation of the former being the same as, and that of the latter opposite to, the sense of the superposed rotation. Denoting the components of electric moment in directions parallel and perpendicular to the field by ξ and η respectively, we have

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* A. Sommerfeld, Phys. Zs., 17, p. 491 (1916).

† P. Debye, Phys. Zs., 17, p. 507 (1916).

$$\xi=\Sigma C_\tau \cos 2\pi(\tau\omega t+\gamma_\tau)$$

and

$$\eta=\Sigma C_{\tau,\pm 1} \cos 2\pi((\tau\omega\pm\omega_H)t+\gamma_{\tau,\pm 1}) \quad . \; . \; . \; . \; . \; . \; . \quad (29)$$

The motion of the atom in the field is thus a typical double-periodic motion with the fundamental frequencies ω and ω_H. According to the considerations in the former section the stationary states will therefore be subject to two conditions, which may be written

$$I=nh,\; I_H=n_H h. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

Here I is equal to the quantity defined by (13) when applied to the periodic motion of the atom relative to a system of reference partaking of the superposed rotation, while I_H is equal to 2π times the component M of the angular momentum of the electron around the axis of this rotation. In fact, the change in kinetic energy of the electron due to the superposed rotation is easily seen to be equal, as a first approximation, to $2\pi\omega_H M$. Since the magnetic field does not effect the potential energy of the atom, the energy difference between two neighbouring motions will therefore be expressed by the relation

$$\delta\varepsilon=\omega\delta I+2\pi\omega_H\delta M=\omega\delta I+\omega_H\delta I_H \quad . \; . \; . \; . \; . \; . \; . \; . \quad (31)$$

which corresponds to the condition (15). At the same time the condition

$$\omega I+\omega_H I_H=\overline{A}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (32)$$

which corresponds to (14), is seen to be fulfilled by any motion of the atom in the field. For the total energy of the atom as a function of I and I_H we get from (3) and (26)

$$E=-\frac{2\pi^2e^4m}{I^2}+\frac{eH}{4\pi mc}I_H \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (33)$$

which, on inserting (30) gives for the energy in the stationary states

$$E=-\frac{2\pi^2e^4m}{h^2}\frac{1}{n^2}+\frac{heH}{4\pi mc}n_H \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (34)$$

It is of interest to point out that the dynamical property of the stationary states expressed by the first of the conditions (30) might have been obtained by a direct application of Ehrenfest's adiabatic principle. In fact, as shown by Langevin* in his work on atomic magnetism, a slow and uniform establishment of an external magnetic field will, on account of the induced electric forces, affect the motion of a system of electrons revolving in a central field in such a way that the motion at any moment will differ from the original motion only by a superposed rotation possessing the Larmor frequency. On the other hand, it lies in the nature of the problem that the appearance of the second of the quantum conditions (30) can in no way be accounted for by considerations depending on ordinary mechanical and electrodynamical ideas. Indeed the appearance of this condition may be considered as a consequence of the fact that the presence of the external field impresses a new fundamental frequency on the motion of the atom, and thereby calls into play the unknown quantum mechanism governing the stability of the stationary states; the effect being that the energy differences between the various possible states corresponding to the same stationary state of the undisturbed atom will exhibit a relation to the new frequency of the same kind as that between energy and frequency in the stationary states of simple-periodic systems.† In the special case under consideration the additional periodic motion impressed on the atom by the field is of a simple-harmonic character; and it is of interest to note that the second term on the right side of equation (34), which is equal to $n_H \omega_H h$, is quite analogous to Planck's original formula (16) for the possible value of the energy of a harmonic oscillator, with the sole difference that, in accordance with the nature of the problem, n_H can take negative as well as positive values.

Since the maximum value M_0 which the angular momentum P of the electron round the nucleus can take is obviously equal to $I/2\pi$, we see that the second of the conditions (30) is equivalent to the condition

$$M = \frac{n_H}{n} M_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

from which it follows at once that the numerical value of n_H can never be larger than n. We must assume that n_H can take any of the values $\pm 1, \pm 2, \ldots \pm n$, while a consideration, which would take us too long to develop here, leads us to the conclusion that no stationary state corresponds to the value $n_H = 0$. For the frequency of radiation emitted by a transition from a state for which $n = n'$ and $n_H = n'_H$ to one for which $n = n''$ and $n_H = n''_H$, we find by (4)

$$\nu = \frac{2\pi e^4 m}{h^2}\left(\frac{1}{(n'')^2} - \frac{1}{(n')^2}\right) + \frac{eH}{4\pi mc}(n'_H - n''_H) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

According to the correspondence principle the possibility of such a transition is dependent on the presence in the electric moment of the atom of a harmonic component of frequency $(n' - n'')\omega + (n'_H - n''_H)\omega_H$. Recalling the above analysis of the motion of the electron represented by (29), we see in the first place that, as in the case of the undisturbed atom, there exist possibilities of transitions in which n changes by any number of units. Such transitions, however, will no longer give rise to the usual hydrogen lines, the frequencies of which are given by (28). Instead, as seen from (36), we obtain for each of these lines a number of components corresponding to the possible simultaneous changes of the quantum-number n_H. These components are of three types. In the first type, which is dependent on the linear harmonic oscillation parallel to the field, n_H remains unchanged, and the components take the same positions in the spectrum as the original lines. On the correspondence principle the radiation corresponding to these components will have the same constitution as the radiation emitted according to classical electrodynamics by an electron performing linear oscillations parallel to the field. In the two other types, which are dependent on the circular harmonic rotations perpendicular to the field, n_H decreases or increases by one unit respectively, and we get for each hydrogen line two components, which are placed symmetrically with respect to the original position of the line, and which, if observed in a direction parallel to the field, will show circular polarisation in opposite directions.

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* P. Langevin, Annales de Physique et de Chimie, 5, p. 70 (1905).

† *Cf.* Q. L. S., p. 11.

It will be seen that this interpretation of the Zeeman effect for the hydrogen lines exhibits a formal analogy with the original theory of Lorentz, discussed in Section I.; which is really remarkable, when we think of the great divergence between the ideas of classical dynamics and the postulates of the quantum-theory. In the matter, however, of the relative intensities of the components of the Zeeman effect the fundamental departure of the quantum-theory from classical electrodynamics comes to light in an interesting way. According to the classical theory the intensities of these components are determined by the condition that the total radiation of each triplet of components shall not exhibit any sensible resultant polarisation, since the orientation of the atom in the field is not subject to any limitations. On the quantum-theory, on the other hand, where the existence of such limitations is an absolutely essential feature, we should be prepared to find a resultant polarisation of the total light of each triplet, even in weak magnetic fields. Such a polarisation has actually been recorded by various investigators of the Zeeman effect, and it is especially interesting to note that Traubenberg* has, in recent experiments on the hydrogen spectrum emitted by positive rays in a magnetic field, succeeded in observing a resultant polarisation of the kind discussed.

Effect of an Electric Field.

In dealing with the influence of a uniform electric field on the hydrogen spectrum, our first problem will be to examine the effect of the field on the motion of the atom. As in the case of a magnetic field, we meet here with the question of small perturbations in a periodic orbit. In the former case Larmor's theorem allowed us at once to perceive the character of the perturbations; the problem is, however, more complicated in the case of an electric field, which not only produces alterations in the orientation of the orbit in space, but also a continuous change in the shape of the orbit. Nevertheless the problem admits of a simple solution by making use of a general theorem in the theory of perturbations.

Consider a system in which every motion is periodic, and let us imagine that the system is subject to a small external field of force. In this case the motion may be described as a periodic motion, which at any moment differs from a possible motion of the undisturbed system by small quantities proportional to the intensity of the external forces, and which at the same time undergoes slow alterations as regards shape and position of orbit at a rate which is also proportional to these forces.

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* R. v. Traubenberg, Naturwissenschaften, 10, p. 791 (1922).

A study of these alterations of motion over long time intervals, which in celestial mechanics are known as "secular perturbations," allows a direct insight into the effect of the external field on the periodic properties of the motion. A fundamental law governing the course of the secular perturbation produced by a fixed field of force is now available in the general theorem referred to above, which states, that, neglecting small quantities proportional to the square of the perturbing forces, the mean value of the potential energy of the system relative to the external field, taken over an approximate period of the motion, will remain unaltered through time intervals long enough for these forces to produce a finite change in the shape and position of the orbit. If we further imagine the external field to be slowly established at a uniform rate, this mean value will, with the same approximation, represent the difference between the total energy of the perturbed system and the original value of the energy of the system before the establishment of the field.*

Reverting to the case of a hydrogen atom perturbed by a uniform electric field, we find by a simple calculation that the mean value of the potential energy of the atom relative to the field is the same as if the electron were placed on the major axis of the orbit at a point dividing the distance between the nucleus and the other focus in the ratio 3 : 1. This point may be called the "electrical centre" of the orbit, and it is an immediate consequence of the general theorem stated above that during the secular perturbation this centre will to a first approximation move in a plane perpendicular to the direction of the external electric force. Now a closer examination of the secular displacement of the electrical centre of the orbit in its plane, which is easily carried out by the simple consideration of the secular changes of angular momentum of the electron round the nucleus due to the external force, shows that the electrical centre performs a purely harmonic, in general elliptical, oscillation, symmetrically placed with regard to an axis through the nucleus parallel to the external force. Moreover the frequency of this oscillation is independent of the shape and orientation of the electron orbit and dependent only on the quantity I, defined for a periodic orbit by (13), which, neglecting small quantities proportional to the external force, will, of course, remain unaltered during the perturbations. Denoting the frequency by ω_F, we have

$$\omega_F = \frac{3I}{8\pi^2 em} F, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

where F is the intensity of the electric field.†

Before proceeding to the fixation of the stationary states, we will examine what bearing these results have on the resolution of the motion of the electron into its harmonic components. For this purpose consider the motion relative to a frame of reference which performs a uniform rotation around the axis of the system in the same sense as the rotation of the electrical centre and with a frequency equal to σ. Owing to the harmonic character of the oscillation of the electrical centre this can obviously be described as motion in a periodic orbit, the shape and position of which will vary with frequency equal to $2\omega_F$. Such a motion will be double-periodic with frequencies ω_1 and ω_2, where ω_2 may be taken equal to $2\omega_F$, while ω_1 is equal to the mean frequency of revolution of the electron in its approximately periodic orbit, calculated from perihelion to perihelion ; which in the system of reference used is obviously equal to the mean frequency of revolution of the electron round the axis. The motion may therefore be considered as a superposition of elliptical harmonic vibrations of frequencies $\tau_1\omega_1+\tau_2\omega_2$, where τ_1 and τ_2 are integers. Returning now to a fixed frame of reference, the motion may be resolved into a sequence of linear harmonic vibrations parallel to the axis with frequencies $\tau_1\omega_1+\tau_2\omega_2$, and two sequences of circular harmonic rotation around this axis with frequencies $\tau_1\omega_1+\tau_2\omega_2\pm\omega_F$. Let us now introduce as fundamental frequencies for the perturbed motion the quantities ω and ω_F, where ω is the mean frequency of revolution of the electron round the axis, and therefore equal to either $\omega_1+\omega_F$ or $\omega_1-\omega_F$, according to whether the electrical centre rotates in the same direction round the axis as the electron itself or in the opposite direction. Denoting the displacement of the electron parallel to the axis by ξ, we obtain consequently

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* Q.L.S., p. 46.

† Q.L.S., p. 73.

$$\xi=\Sigma C_{\tau,}\tau_F, \cos 2\pi[t(\tau\omega+\tau_F\omega_F)+C_{\tau,}\tau_F], \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where $\tau+\tau_F$, being equal to $2(\tau_1+\tau_2)$ or to $2\tau_2$, is always an *even* integer. For a displacement η perpendicular to the axis we find similarly :

$$\eta=\Sigma D_{\tau\omega,\,\tau F}\cos 2\pi[t(\tau\,\omega+\tau_F\omega_F)+d\tau,\,\tau_F] \quad . \quad . \quad . \quad . \quad . \quad (39)$$

where $\tau+\tau_F$, being equal to $2(\tau_1+\tau_2)\pm1$ or $2\tau_2\pm1$, is always an *odd* integer.

The stationary states of this double-periodic motion will be fixed by two quantum conditions, which can be written in the form

$$I=nh, \quad I_F=n_Fh, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where the quantities I and I_F are related to the total energy of the system and to the action function through the equations

$$\delta E=\omega\delta I+\omega_F\delta I_F \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

and

$$\omega I+\omega_F I_F=\bar{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

which correspond to the conditions (22) and (23). Consider for the moment the especially simple case where the electron moves in a circular orbit in a plane perpendicular to the axis. For such orbits the dependence of E and A on ω, so far as quantities proportional to the external force are concerned, is obviously the same as that holding for a simple Keplerian motion. It follows therefore from (41) and (42) that I_F vanishes in this case, while I coincides with the quantity defined by (13) for a simple periodic orbit. Since ω_F to a first approximation depends only on I, we deduce from this result the following general expression for the energy of the perturbed atom :

$$E=E_0(I)+\omega_F I_F, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

where $E_0(I)$ represents the energy of a simple Keplerian orbit expressed as a function of I. From (26) and (37) we find, therefore.

$$E=-\frac{2\pi^2e^4m}{I^2}+\frac{3II_F}{8\pi^2em}F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The kinematical significance of the second of the quantum conditions (40) follows simply from the general theorem of the adiabatic invariance of the quantum conditions for multiple-periodic systems. In fact, from a consideration of a slow establishment of the external field, it follows according to the general theorem of p. 291 that the change of the energy of the atom due to the field is equal to ζeF, where ζ is equal to the distance of the electrical centre from a plane through the nucleus perpendicular to the axis. The maximum value ζ_0, in the limiting case in which the orbit degenerates into a straight line parallel to the direction of the field, is equal to $3a/2$, where $2a$ is the major axis of the Keplerian orbit, the dependence of which on I is expressed by the last of the equations (26). Comparing with the second term in (44), we get therefore the simple relation

$$\zeta=\zeta_0 \cdot \frac{I_F}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

This relation imposes an obvious limit to the values which n_F can take in the stationary states corresponding to a given value of n. We conclude that n_F can take any of the values 0, ± 1, ± 2, . . . $\pm(n-1)$, while the limiting values $\pm n$ have to be excluded on account of the singularity of the corresponding motion.

In connection with the state-relations (40), it may still be of interest to point out that just as in the case when a magnetic field is applied, the second quantum condition ensures a relation between the additional fundamental frequency ω_F, impressed on the atom by the field, and the possible values of the energy of the atom relative to the field, which is completely analogous to formula (16) for the possible energy values of a simple Planck oscillator. This remark illustrates the physical side of the problem of the influence of the field on the stationary states of the atom. It need hardly be emphasised that neither in considering the effect of the magnetic field nor that of the electric field does it suffice to base the treatment on the application of the adiabatic principle to the problem of the slow establishment of the field; this is seen directly from the fact of the entire freedom of orientation in space of the atom in the absence of the field.

Proceeding now to the consideration of the effect of the electric field on the hydrogen lines, we obtain from (40) and (44) for the energy in the stationary states of the atom

$$E=-\frac{2\pi^2e^4m}{h^2}\frac{1}{n^2}+\frac{3h^2F}{8\pi^2em}n\,n_F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

By means of the general frequency-relation this gives, for the radiation emitted by a transition from a state for which $n=n'$ and $n_F=n'_F$ to a state for which $n=n''$ and $n_F=n''_F$,

$$\nu=\frac{2\pi^2e^4m}{h^3}\left(\frac{1}{(n'')^2}-\frac{1}{(n')^2}\right)+\frac{3hF}{8\pi^2em}(n'n'_F-n''n''_F). \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

According to the correspondence principle, the occurrence of such a transition is conditioned by the presence in the electric moment of the atom of a harmonic component of frequency $(n'-n'')\omega+(n'_F-n''_F)\omega_F$. Comparing this with our analysis of the motion in the field, we are consequently led to infer that each of the spectral components into which the hydrogen lines are split up will show a characteristic polarisation, such that all components for which $(n'-n'')+(n'_F-n''_F)$ is an even integer will show linear polarisation parallel to the field, while components for which this expression is an odd integer will exhibit a characteristic polarisation in a direction perpendicular to the field. These results are fully confirmed by Stark's experiments; not only can the positions of the observed components for each hydrogen line be accounted for by formula (47) within the limits of experimental error, but also the polarisation of the components is found to conform to the above rules.[*] Moreover, by theoretical estimation, based on a calculation of the amplitudes of the corresponding harmonic oscillations of the probabilities of transitions giving rise to the various spectral components, it has even been possible to account in detail for the laws of distribution of intensities of the different components, which show great differences from line to line. The latter problem has been treated by Kramers in a Paper which contains a thorough discussion of the problem of intensities of spectral lines in relation to the correspondence principle.[†]

In view of these results, we may say that the Stark effect for the hydrogen lines, when properly interpreted, reveals every detail of the action of the electric field on the motion of the hydrogen atom. In contrast to the Zeeman effect, however, the image of the motion of the electron which we observe in the spectrum is so distorted that it would hardly have been possible to recognise it on the basis of our ordinary ideas of the origin of electromagnetic radiation. At the same time the fundamental departure of the quantum-theory from classical electrodynamics comes to light in a most striking way in a feature of the effect recorded by Stark.[‡] While under usual conditions the components of each hydrogen line exhibit complete symmetry with respect to the position of the original line, a remarkable asymmetry is observed when the spectrum is excited under such conditions that the atom in the main receives impacts from electrons moving in the same or in the opposite direction to that of the electric force. In fact, in the latter condition the components on the long-wave side of the original line are much more intense, or less intense, respectively, than the components on the short-wave side. On the quantum-theory this observation receives immediate interpretation if we assume that under these conditions the probability of the plane in which the electric centre moves being displaced from the nucleus in the same direction as the motion of the impacting electrons, is markedly larger than the probability of this displacement being in the reverse direction. The effect under consideration has often been considered as affording a serious objection to the quantum-theory explanation of the Stark effect.[§] We see, however, that, on the contrary, it must be regarded as a most direct evidence of the complete independence of the processes which give rise to the appearance of the various spectral components; and this, according to our postulates, is just an essential feature of the quantum-theory of spectra.[||]

Effect of Simultaneous Electric and Magnetic Fields.

The considerations applied above allow of direct application to more complicated problems. One such problem is presented when we investigate the simultaneous effect of uniform electric and magnetic fields on the hydrogen lines.

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[*] Q.L.S., p. 77. *Cf.* also Zs. für Phys., 2, p. 446 (1920).

[†] H. A. Kramers, Intensities of Spectral Lines, D. Kgl. Danske Vidensk. Selsk. Skrifter, 8, 3, 287 (1919).

[‡] J. Stark, Ann. d. Physik, 56, 569 (1918).

[§] *See* J. Stark, Jahrbuch d. Ra. u. El., 17, p. 161 (1921).

[||] *Cf.* N. Bohr, Phil. Mag., 30, p. 402, (1915); *see* also A. Rubinowiz, Zs. f. Phys., 5, p. 331 (1921).

In the case when the two fields are parallel the perturbations of the original periodic motion will be a simple superposition of the perturbations considered above in the case of separate fields, and the stationary states will obviously be fixed by three conditions :

$$I=nh,\quad I_H=n_Hh,\quad I_F=n_Fh\quad .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ (48)$$

where the second condition, which determines the angular momentum of the electron round the axis of the system, as well as the third condition, which determines the position of the plane perpendicular to this axis in which the electrical centre of the orbit moves, are in every respect completely analogous to the additional quantum conditions in the formulæ (30) and (40) respectively. The energy of the atom in the stationary states will consequently be given by

$$E=-\frac{2\pi^2e^4m}{h^2}\frac{1}{n^2}+\frac{ehH}{4\pi mc}n_H+\frac{3h^2F}{8\pi^2em}n\,n_F\quad .\ .\ .\ .\ .\ .\ .\ .\ (49)$$

Moreover, it follows directly from the correspondence principle that the effect of the fields will consist in the resolution of every hydrogen line : partly into one set of components polarized parallel to the fields, and placed at the positions of the parallel components of the Stark effect which would appear in the absence of the magnetic field ; partly into two sets of components exhibiting circular polarization in opposite directions and placed symmetrically with respect to the perpendicular components of the Stark effect, in the same way as the circularly polarized components of the usual Zeeman effect are placed with respect to the original line.[*] This consequence of the theory seems adequately supported by the experiments.[†]

In many experiments on the Zeeman effect we are concerned with the disturbing effect of small electric fields which possess a component perpendicular to the direction of the magnetic force. This effect may be discussed by considering the resultant motion as a small perturbation of the motion holding in the presence of the magnetic field alone, and the problem may be treated by a method closely analogous to that applied above to the perturbations of a periodic motion. In the present case it may be shown that the electric forces will, in a first approximation, not give rise to the appearance of new fundamental frequencies in the secular changes of the periodic orbit; nor—so far as quantities proportional to the intensity of the electric field are concerned—will this field have any effect on the energy in the stationary states of the atom. The presence of this field, nevertheless, will give rise to the appearance of new harmonic oscillations with amplitudes proportional to the intensity of the field, and with frequencies equal to the sum or difference of two frequencies appearing in the atom when the magnetic field alone is present. On the correspondence principle this will, in addition to probabilities of transitions responsible for the components of the usual Zeeman effect, give rise to small probabilities of the occurrence of new types of transitions. Besides irregularities in the polarisation of the usual components, the electric field may therefore be expected to cause the appearance of new weak components, at distances from the original lines twice that of the outer components in the normal effect. Such effects have actually been observed.*

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[*] Q.I.S., p. 92.

[†] Garbasso, Phys. Zs., 15, p. 729 (1914).

Before concluding our consideration of the effect of external fields on the hydrogen spectrum, it may be of interest to characterise, in a few words, the difference between the treatment given here and the method applied in their original investigations on the Stark and Zeeman effects for the hydrogen lines by the authors mentioned in Section IV. These methods were based mainly on the so-called procedure of separation of variables, which in every case inevitably leads to a number of quantum conditions equal to the number of degrees of freedom of the system. As we have seen, however, a treatment of the effect of magnetic and electric fields on the hydrogen lines, adapted to illustrate the physical side of the problem, can be given by using a smaller number of quantum conditions, equal to the degree of periodicity of the motion. The principal objection to the use of a higher number of quantum conditions is that by their use the inherent stability of the spectral phenomena under consideration does not come to light. In fact, these conditions imply the formal fixation of certain properties of the motion of the atom which, in contrast to those fixed by our treatment, are unstable in the presence of external forces which are yet so small that they cannot essentially influence the spectrum. Apart from this, the method of separation of variables has a more limited applicability. This is seen, for instance, on taking into consideration the influence of variability of mass of the electron, which for simplicity has been neglected in the foregoing analysis. The problem of the fine structure of the hydrogen lines can be treated by separation of variables, and so also can the effect of magnetic fields on this fine structure, as was shown by Sommerfeld.† But this is no longer true when we approach the problem of the effect of electric fields on the fine-structure. In this case the motion is of such a complex nature that no set of generalised co-ordinates can be found which allows of a separation of variables. On the other hand, as shown by Kramers,‡ the problem can be successfully treated by considering the motion as a perturbed periodic motion, and examining the periodic properties of the secular perturbations. In this way it is possible to follow theoretically the details in the transformation of the fine-structure of the hydrogen lines which accompanies a gradual increase in the electric field from very small intensities to intensities of the order of magnitude usually applied in experiments on the Stark effect, in which the effect of variability of mass of the electron plays only a very small part. Experiments allowing of a test of these theoretical predictions would be of great interest.

VI. THE EFFECT OF EXTERNAL FIELDS ON THE SPECTRA EMITTED BY ATOMS CONTAINING SEVERAL ELECTRONS.

Notwithstanding their greater complexity the so-called series spectra of many elements show a marked analogy with the hydrogen spectrum. On the quantum-theory of spectra this is accounted for by the assumption that in the stationary states concerned in the emission of these spectra one of the electrons in the atom moves, at any rate during the greater part of its path, at a distance from the nucleus large compared with the distances of the other electrons. According to this view the spectral lines are emitted by transition processes in which the motion of this electron alone undergoes essential changes, while the orbits of the other electrons coincide quite closely with their orbits in the normal state of the atom. Just as the hydrogen spectrum may be regarded as evidencing a process by which the neutral atom is formed by the binding of the electron by the nucleus, so a series spectrum of this type for another element may be considered as evidencing the last stage of a process in which the atom is formed by the successive capture and binding of electrons in the field of the nucleus. Time will not allow me to enter here into the details of the results it has been possible to attain regarding the general features of atomic constitution by an elaboration of this point of view.* I shall confine myself to showing how these ideas on the origin of series spectra allow us to account for certain main features of the structure of these spectra as well as of the effect of electric and magnetic fields on their lines.

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* Q.L.S., p. 98.

† A. Sommerfeld, Phys. Zeitschr., 17, p. 497 (1916).

‡ H. A. Kramers, Zeitschr. f. Phys., 3, p. 199 (1920).

In so far as it remains at distances from the nucleus large compared with the dimensions of the orbits of the inner electrons the force exerted by the rest of the atom on the outer electron will coincide very nearly with the force due to a nucleus of unit charge. In the case in which the outer electron remains always outside the region in which the inner electrons move, its motion may therefore be considered as a Keplerian motion undergoing slow secular perturbations, and of much the same type as the motion in the hydrogen atom under the influence of an external field. In the case when the outer electron penetrates at intervals during its revolution into the inner region, its motion will be composed of a sequence of outer loops, each coinciding closely with part of a Keplerian ellipse and connected with the next by an inner orbital loop in which the motion may differ essentially from a Keplerian motion.

The inherent stability of atomic structure—brought to light so strikingly by experiments on the impact of atoms and free electrons—suggests in the first place that this penetration into the inner region will involve no interchange of energy between the outer electron and the rest of the atom, in the sense that for one and the same electron orbit the various outer loops will coincide closely with parts of ellipses corresponding to the same value of the energy of the hydrogen atom. Moreover, the general central symmetry of the electronic arrangement in the nuclear atom suggests that consecutive loops will as a first approximation have the same shape and be spaced at equal angles to each other in the orbital plane. This means that the motion may be considered as a plane periodic motion on which is superposed a uniform rotation in its plane, a description which may be assumed to hold in a first approximation for the outer orbit, whether it penetrates into the inner region or not. On this motion may be superposed again a slow precession of the orbital plane around the invariant axis of angular momentum of the atom.

The resolution of a motion of this type into its harmonic components may be simply effected in the following way. In a system of reference partaking of the rotation of the orbit in its plane and the precession of the orbital plane, the motion of the electron will be composed of a sequence of elliptic harmonic oscillations of frequencies $\tau\omega$, where τ is an integer and ω the frequency of revolution. It is seen that each of these oscillations, as a consequence of the uniform rotation of the orbit in its plane, will be resolved into two circular harmonic rotations in opposite directions, with frequencies $\tau\omega \pm \omega_{\mathrm{R}}$, where ω_{R} is the frequency of the orbital rotation. As in the consideration of the Zeeman effect, each of these will, on account of the precession of the orbital plane, be resolved into a linear harmonic vibration parallel to the fixed axis with unaltered frequency, and two circular harmonic rotations in opposite directions with frequencies increased or diminished by the frequency of precession. Denoting this frequency by ω_{P} we find consequently for the displacement of the outer electron in directions parallel and perpendicular to the invariant axis respectively,

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* *Cf.* The Theory of Spectra and Atomic Constitution, Camb. Univ. Press (1922), containing three essays of which the first two deal in a general way with the problems considered in the former sections of this lecture, while the third gives a detailed discussion of the theory of the constitution of atoms. *See also* my Paper Linienspectren und Atombau (Ann. d. Physik, 71, p. 229, 1923) which contains a more detailed account of the interpretation of spectra with complete reference to the literature.

$$\xi = \Sigma C_{\tau,\pm 1} \cos 2\pi[(\tau\omega \pm \omega_{\mathrm{R}})t + \gamma_{\tau,\pm 1}], \quad \eta = \Sigma D_{\tau,\pm 1,\pm 1} \cos 2\pi \\ [(\tau\omega \pm \omega_{\mathrm{R}} \pm \omega_{\mathrm{P}})t + \delta_{\tau,\pm 1,\pm 1}]. \quad . \quad . \quad . \quad . \quad (50)$$

By analogy with the general theory of the state-relations for periodic and multiple-periodic systems we shall assume that the motion of the outer electron in the stationary states is determined by a set of conditions which may be written

$$I = nh, \; I_{\mathrm{R}} = n_{\mathrm{R}}h, \; I_{\mathrm{P}} = n_{\mathrm{P}}h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

where the quantities I, I_{R} and I_{P} are correlated with the fundamental frequencies in the triple-periodic motion of the outer electron by the relation

$$\delta E = \omega\delta I + \omega_{\mathrm{R}}\delta I_{\mathrm{R}} + \omega_{\mathrm{P}}\delta I_{\mathrm{P}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

which refers to two states of the atom for which the orbits of the inner electrons retain their shape and relative configuration while the shape of the orbit of the outer electron as well as its orientation relative to the inner orbits differ slightly.

Since the perturbations of the outer orbit from a Keplerian ellipse occur almost entirely in the region close to perihelion, where the electron remains only a small fraction of the period required to traverse an orbital loop, this period will to a high degree of approximation be equal to that required for the description of the Keplerian ellipse of which the outer loop forms a part. With this approximation we have therefore

$$\delta E = \omega\delta I_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

where I_0 is the quantity defined by (13) as applied to this ellipse. Comparing (52) with (53) we may therefore write

$$I_0 = I + \Phi(I_{\mathrm{R}}, I_{\mathrm{P}}), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

where Φ is a function of I_{R} and I_{P} subject to the relation

$$\omega\delta\Phi = \omega_{\mathrm{R}}\delta I_{\mathrm{R}} + \omega_{\mathrm{P}}\delta I_{\mathrm{P}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

from which it follows that the ratios $\omega_{\mathrm{R}}/\omega$ and $\omega_{\mathrm{P}}/\omega$, with the approximation under consideration, are independent of I. For the dependence of the work W required to remove the outer electron from the atom we obtain now in accordance with (26)

$$W=\frac{2\pi^2e^4m}{I_0}=\frac{2\pi^2e^4m}{(I+\Phi)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

Writing for simplicity $E=-W$ we get with reference to (51) for the energy in the stationary states

$$E=-\frac{2\pi^2e^4m}{h^2}\cdot\frac{1}{[n+\alpha(n_{\mathrm{R}}, n_{\mathrm{P}})]^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

where α stands for Φ/h. This formula accounts at once for the series structure of the spectra under consideration. In fact, the empirical expressions for the spectral terms within each series have in a first approximation just the same form as (57), if α is taken as a constant characteristic of each series, and n is given a sequence of consecutive integral values. In other words, each series of spectral terms may be correlated to the stationary states corresponding to a sequence of integral values of the "principal" quantum number n and constant values of the "subordinate" quantum integers n_{R} and n_{P}.

So far the considerations have been independent of our special assumptions as regards the deviations of the orbit of the outer electron from a Keplerian orbit and of the dynamical significance of the symbols I_{R} and I_{P}. Applying a relation analogous to (32) it is simply shown, however, that with the assumptions used as to the character of the perturbations, $2\pi I_{\mathrm{R}}$ will be the angular momentum of the outer electron around the nucleus, while $2\pi I_{\mathrm{P}}$ will be the resultant angular momentum of the whole atom around the invariant axis. On the basis of this result a detailed classification of the manifold of terms has been obtained correlating each series of "terms" in extended sense with a given value of n_{R}, while the complex structure of these terms (doublets, triplets, &c.) is accounted for by correlating each term "component" to a given value of n_{P}. This classification, which is due principally to Sommerfeld, has received most convincing support from the application of the correspondence principle. According to this principle the appearance of a transition from a stationary state characterised by n', n'_{R}, n'_{P} to another state characterised by n'', n''_{R}, n''_{P}, is conditioned by the presence in the motion of a constituent harmonic vibration of frequency $(n'-n'')\omega+(n'_{\mathrm{R}}-n''_{\mathrm{R}})\omega_{\mathrm{R}}+(n'_{\mathrm{P}}-n''_{\mathrm{P}})\omega_{\mathrm{P}}$. In view of the analysis of the motion expressed by (50) we conclude therefore that while for a given transition n can change by an arbitrary number of units, n_{R} can only increase or decrease by a single unit, and n_{P} remains unchanged or changes by one unit. The classification of the empirical manifold of spectral terms has indeed been effected in such a way that these theoretical rules of combination are obeyed.

Proceeding now to the effect of electric and magnetic fields on series spectra, we find that the application of the same principles which have guided us in examining these effects in the case of the hydrogen spectrum leads us to a number of theoretical predictions which have been found to be fulfilled to a large extent.

In the case of the electric field we meet at once with a typical difference from the conditions encountered in hydrogen. Owing to the periodic character of the electronic orbit in hydrogen, the external field produces a finite change in the shape and position of the orbit, due to an accumulation of the effects of the secular perturbations. In the motion involved in the series spectra of other elements, on the other hand, we have in the undisturbed atom to do with an electronic orbit which is continuously undergoing regular changes as regards its position in space, of a type that limits a cumulative effect of the perturbations to time intervals of the same order of magnitude as the periods characteristic of these changes of position. As long as these periods are short compared with the period of the changes which the same field would produce in a purely Keplerian orbit of the same dimensions, the character of the motion will undergo only small periodic changes, and notably there will appear no secular perturbations chacterised by a new frequency proportional to the first power of the external forces. In the case of the series spectra of elements other than hydrogen there will, therefore, be no question of a splitting up of the spectral lines into components with a displacement proportional to the field, at any rate so far as the spectral terms involved differ from the hydrogen terms with the same principal quantum number by an amount which is large compared with the effect of the same field on the hydrogen terms. In this case any resolution or displacement of the spectral lines will be proportional to the second power of the electric forces, and the effect will be the smaller, the more the spectral terms involved deviate from the hydrogen terms, which deviations, according to (55) and (56), indeed give a measure of the frequencies of the changes of position of the orbit in space.

These theoretical expectations are completely confirmed by the experiments of Stark and other investigators, which have shown that an effect of the electric field on the lines, of the same order of magnitude as exists in hydrogen, occurs only for lines for which at least one of the two spectral terms involved coincides closely with the hydrogen term of the same quantum number; while for lines where both terms deviate considerably from hydrogen terms, the effect is very small, if measureable at all.

The problem of the influence of electric fields on the spectral lines can be followed in great detail, both as regards the theoretical predictions as well as their confirmation by experiment. It will, however, carry us too far here to enter more closely into these questions. I shall, however, mention one very important feature brought out by Stark's experiments, namely, the production of new combination lines under the influence of the field. This phenomenon receives an immediate explanation on the theory. In fact, although, as mentioned, the electric field does not change the type of the motion of the electron in a first approximation, there will nevertheless, on account of the perturbations, appear new constituent harmonic oscillations in the motion, with amplitudes proportional to the electric forces, and with frequencies equal to the sums or differences of frequencies belonging to harmonic constituents present in the undisturbed motion. Owing to these new oscillations, which are analogous to the "combination tones" well known in acoustics, the atom will acquire, apart from the transitions giving rise to the usual spectral lines in the presence of the field, possibilities of new transitions giving rise to new spectral lines, the frequencies of which will be equal to the sum or difference of the frequencies of lines which appeared in the undisturbed spectrum.* As far as experimental evidence is available, these expectations are fulfilled both as regards the position of the new lines, and as regards their intensities estimated on the correspondence principle. The observation of such "true" combination lines has been generally considered among the strongest supports of the combination principle, although at the same time the apparent capriciousness of their appearance threw a veil of mystery over the application of this principle. To-day it is seen, however, that the quantum-theory has not only afforded a formal interpretation of the combination principle, but that it has also contributed materially to the clearing up of the mystery surrounding its applications.

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* *Cf.* Q.L.S., pp. 36 and 108.

Considering next the effect of a uniform magnetic field we find that the application of the laws of electrodynamics, together with the correspondence principle, leads to very simple deductions. In fact, quite independently of the character of the motion of the electrons in the absence of the field, we should expect from Larmor's theorem that the effect of the field would consist simply in the superposition of a uniform rotation of the whole atom around an axis parallel to the field. Just as in the case of hydrogen, the superposed rotation will give rise to the appearance of a new quantum condition, to the effect that only those orientations of the atom relative to the field are possible in which the component of the total angular momentum of the atom parallel to the field is equal to an integral multiple of $h/2\pi$. Moreover, on the correspondence principle, the effect of the superposed rotation on each of the harmonic constituents in the motion of the atom, in the absence of the field, would involve the resolution of each line into a normal Lorentz triplet.

These theoretical expectations are, however, as already mentioned at the beginning of this lecture, only partly fulfilled. While all spectra consisting of single lines, indeed, show the normal effect, the series spectra of more complicated types exhibit, as is well known, the so-called anomalous Zeeman effect. On the correspondence principle this may be considered as proving that, in contrast to the laws of classical electrodynamics, the magnetic field will, for spectra of such types, not only affect the motion of the atom as a whole, but will also directly influence the interplay between the various electrons in the atom. This is especially clearly shown by the way in which the anomalous Zeeman effects, as first observed by Paschen and Back, are gradually transformed by increasing field intensity; as well as by the appearance observed by the same authors of new components in the complex structure of the series lines in the presence of the magnetic field.* The latter phenomenon may be considered as the complete analogue of the appearance of new series lines in the presence of external electric fields. At the same time these effects show clearly that the magnetic field does not directly influence those properties of the motion which are fixed by the principal quantum number n as well as by the subordinate quantum-number n_R. This is also satisfactory, since not only the approximately Keplerian character of the orbital loops but also the rotation of these loops in their plane, depends only on the simple assumption that the effects on the motion of the outer electron exerted by the rest of the atom conform approximately to that of a central field of force. On the other hand the properties of the motion fixed by the quantum number n_P involve directly the dynamic character of the configuration of inner electron orbits, and may be considered as representing primarily the finer interplay of the outer electron and the atomic residue. In view of the above considerations the anomalous Zeeman effect suggests that these features of the interplay cannot even in a first approximation be described by the laws of classical electrodynamics. Indeed, only on this view does the breaking down of Larmor's theorem in these cases seem intelligible; and it is therefore most satisfactory that the other features in regard to the complex structure of series spectra also point definitely to this conclusion.*

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* F. Paschen und E. Back, Ann. d. Phys., 39, p. 897 (1912), and Physica, 1, p. 261 (1921).

A most suggestive clue to the examination of this problem may be considered as afforded by Preston's rule, which was referred to at the beginning of this lecture, as well as by the rules first established by Runge concerning the simple numerical relations between the displacements of the components of the anomalous effects and those of the normal Zeeman effect. In this respect a step of great significance has recently been made by Landé, who has succeeded from the empirical rules in deriving general laws governing the way in which a given spectral term, under the influence of a magnetic field, is split up into a number of term-components, as well as the manner in which these components combine with each other to give rise to the observed resolution of the spectral lines.† It is to be hoped that these beautiful results will help to clear up the still unsolved secrets of electron interplay in the atom. Ingenious and suggestive attempts have already been made in this direction, but a satisfactory solution is hardly yet in sight. As indicated above, such a solution will presumably demand a still greater departure from the classical conceptions, even though it may be expected to conform with the general ideas regarding the stability of atoms and the radiation emitted by them, the illustration of which forms the principal purpose of this lecture.

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* *Cf.* N. Bohr, Ann. d. Phys., 71, p. 277 (1923).

† A. Landé, Zeitschr. f. Phys., 5, p. 231 (1921), and 15, p. 189 (1923).

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